# PROPERTIES OF LIGHT Hg, Pb AND Po ISOTOPES

I. Muntian and Z. Patyk

A. Soltan Institute for Nuclear Studies
Hoża 69, 00-681 Warsaw, Poland

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*Dedicated to Adam Sobiczewski in honour of his 70th birthday*

Quality of mass description for three different theoretical mass models is studied. Masses and deformations for Po, Pb and Hg isotopes are compared with experimental data. Gap in the proton single particle energy spectrum is discussed.

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## 1. Introduction

The knowledge of magic numbers is important in any finite Fermi system, especially for superheavy nuclei *e.g.* [1-3]. The heaviest known yet nucleus with two magic numbers, $^{208}$Pb, has 82 protons and 126 neutrons. Nuclei around the nucleus $^{208}$Pb are intensively studied experimentally [4,5] and theoretically [6]. Recently, masses and equilibrium shapes [7] for Pb isotopes have been determined at GSI-Darmstadt [4] and at ISOLDE/CERN [5]. The shapes of the Pb isotopes can be understood as a balance of two tendencies: the gap in the proton single particle energy spectrum forces protons to have a spherical shape, while neutrons with neutron number between 82 and 126 prefer deformed shape. From experimental data we know that light Pb nuclei are spherical [7]. Also according to macroscopic–microscopic calculations they are spherical, and have approximately 4 MeV energy gap in the proton single particle spectrum. Analysis of Pb isotopes can help us to understand the properties of superheavy elements.

## 2. Description of the calculations

In the present paper (PP) mass of nucleus is described by the macroscopic–microscopic model [2]. The deformation-dependent macroscopic part is treated within the Yukawa plus-exponential approach. The microscopic part

is the Strutinski shell correction with the Woods–Saxon single particle potential. The pairing part is calculated in the framework of the BCS theory. The ground states for even–even nuclei are found in 4-dimensional deformation space ($\beta_\lambda$), $\lambda = 2, 4, 6, 8$. Five deformation-independent parameters in the macroscopic part of the mass formula have been fitted to 273 measured masses of even–even nuclei in a range of atomic numbers from $Z = 50$ up to $Z = 92$. As a result, experimental masses are described with accuracy (rms) of 0.7 MeV. Masses of nuclei measured at GSI-Darmstadt or ISOLDE/CERN are not included in the fit procedure.

## 3. Results and discussion

In the present paper, we compare masses and equilibrium deformation parameters obtained within three approaches: the Extended Thomas–Fermi model with Strutinski Integral [8] (ETFSI), the Hartree–Fock method (HF-BCS) [9] and the macroscopic–microscopic model (PP). Mass models with a set of parameters fitted for large area of measured masses are chosen for comparison. In all these models pairing has been treated in the BCS approach.

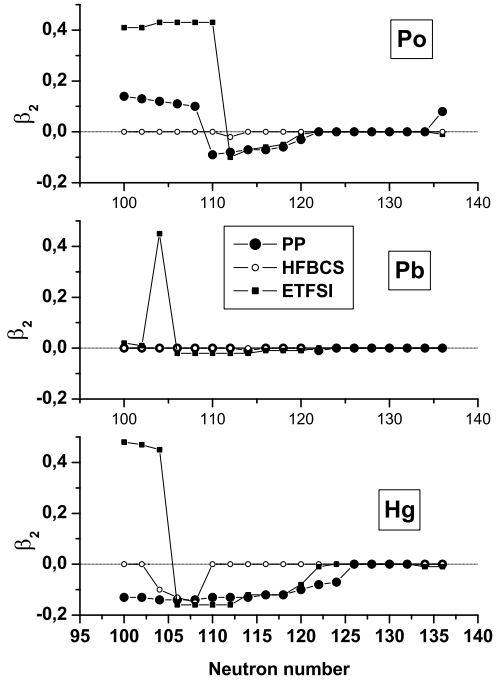


Fig. 1. Equilibrium deformation parameter $\beta_2$ for macroscopic–microscopic (full circle), ETFSI (full square) and HFBCS (open circle) models plotted as a function of neutron number. The comparison is done for Hg, Pb and Po isotopes.

In Fig. 1 equilibrium deformation parameter $\beta_2$ for Hg, Pb and Po isotopes is presented as a function of neutron number. Full circles, full squares and open circles denote macroscopic–microscopic (PP), ETFSI and HFBCS calculations, respectively. The ground state shapes are predicted spherical in the Hartree–Fock approach for three mentioned isotope chains. The exceptions are three light Hg isotopes calculated to be oblate, but with the corresponding deformation energies less then 0.2 MeV.

For the ETFSI model and the macro–micro method (PP), one can see drastic shape changes for light Po and Hg isotopes (predicted to be oblate or prolate) compared with spherical shapes of neighboring Pb isotopes. The deformation energy reaches 2 MeV for light Hg isotopes in both models, as seen in Fig. 2. The ground states of the nucleus $^{186}$Pb and heavier even–even Pb isotopes are spherical as recently has been shown experimentally [7].

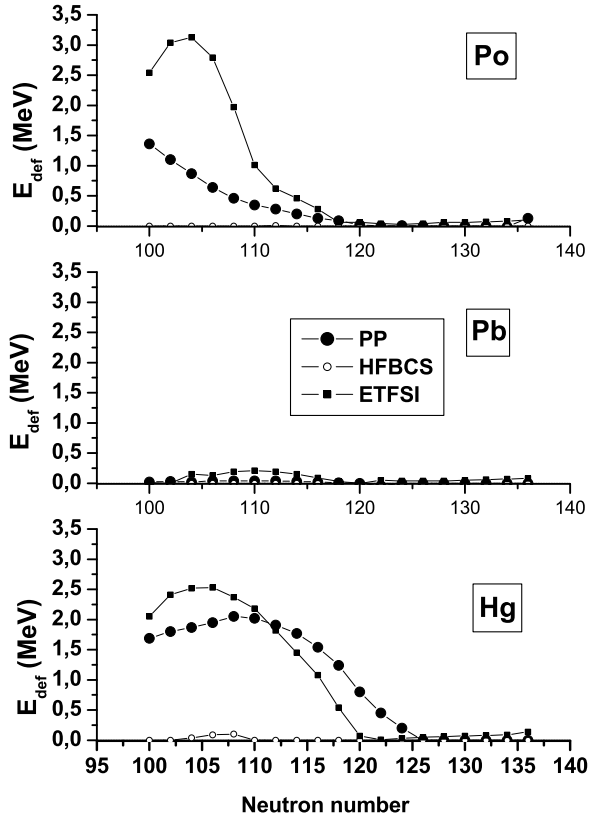


Fig. 2. Deformation energy (in MeV) for macroscopic–microscopic (full circle), ETFSI (full square) and HFBCS (open circle) models plotted as a function of neutron number. Hg, Pb and Po isotopes are studied.

In Fig. 3, the ratio of two experimental excited states $E_4$ and $E_2$ is plotted as a function of neutron number. The ratio equal to one describes perfect harmonic oscillator spectrum, while for the ratio approaching 3.3 (perfect rotor) the spectrum has rotational character and the nucleus is deformed. For light Hg isotopes we observe that experimental ratio approaches 3 and we conclude that the light Hg isotopes are deformed, what supports the results of ETFSI and PP calculations.

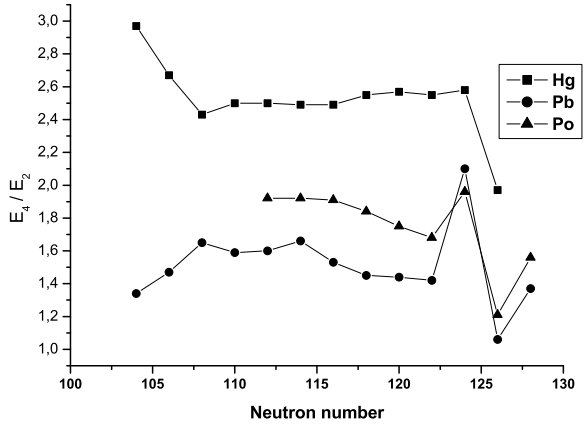


Fig. 3. The experimental ratio [10] $E_4$ to $E_2$ for Hg (square), Pb (circle) and Po (triangles) isotopes illustrated as a function of neutron number.

In Fig. 4, the difference between calculated and measured masses is presented for three models. The largest deviations (absolute value 2.6 MeV) are obtained for light Pb isotopes in HFBCS approach. For Hg isotopes, absolute values of mass differences are less then 1 MeV. For Po isotopes, only the macroscopic–microscopic model describes masses with discrepancy smaller then 1 MeV.

Having three masses, see the Eq. (1), we compute $G_p$, which is interpreted as a gap in the proton single particle spectrum.

$$2G_p = (M(Z-2,N) + M(Z+2,N) - 2M(Z,N))c^2 \,. \qquad (1)$$

Similar equation can be constructed for the neutron gap Fig. 5 presents the quantity $2G_p$, extracted from experimental and theoretical masses for Pb isotopes. Additionally, the gap around the Fermi level in the Woods–Saxon spectrum is illustrated. Substantial weakening of $G_p$ is observed for the experimental data, what could suggest that light Pb isotopes have small gap around the Fermi level and they are not magic nuclei. However, experimental data [7] for light Pb isotopes supports the opposite idea: light Pb isotopes have strong enough proton gap to keep neutrons within spherical shape.

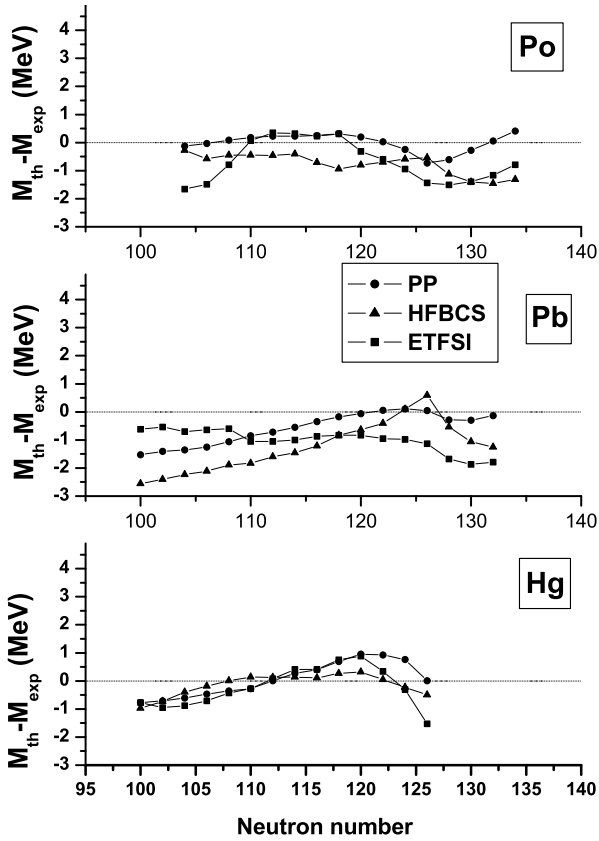


Fig. 4. Deviation of theoretical nuclear mass (in MeV) for macroscopic–microscopic (circle), ETFSI (square) and HFBCS (triangle) models plotted as a function of neutron number. The experimental mass has been chosen as a reference. Isotopes of Hg, Pb and Po are presented.

Also the calculated Woods–Saxon proton gap shows that the gap is strong for light Pb isotopes. If three nuclei connected by the formula (1) have similar structure at the ground state, the value $G_p$ can be interpreted as the gap around the Fermi level in the single particle spectrum. Light Pb and Hg isotones have different shapes at the ground state, so the latter condition is not fulfilled. We cannot interpret $G_p$ for light Pb isotopes as the gap in the single particle spectrum. Moreover, in Fig. 5 we see that the calculated gap in the proton Woods–Saxon spectrum (WS) increases as the neutron number decreases.

When we use the same Woods–Saxon spectrum to calculate masses in the macroscopic–microscopic approach, the gap extracted from formula (1) decreases for light Pb isotopes (full circles).

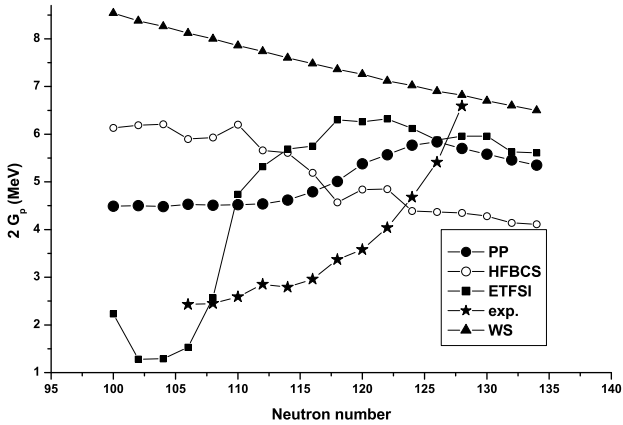


Fig. 5. The double proton gap (in MeV) calculated according to Eq. (1) in the macroscopic–microscopic (full circles), ETFSI (full squares) and HFBCS (open circles) approaches plotted as a function of neutron number. The experimental proton gap is indicated by stars. Also the gap in the proton single particle spectrum of the Woods–Saxon (WS) potential around the Fermi level (triangles) is presented.

Finally we conclude, that light Pb isotopes have spherical ground state shapes and they are still magic nuclei. The quantity $G_p$ defined by the Eq. (1) cannot be interpreted as a gap in the proton single particle spectrum. The latter is very important for predictions of magic numbers in self-consistent studies of superheavy nuclei using $G_p$ as a magicity indicator (*cf. e.g.* [12]).

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