NUCLEAR MATTER APPROACH TO THE INTERACTION POTENTIAL BETWEEN HEAVY IONS*

J. Dąbrowski

Theoretical Division, A. Sołtan Institute for Nuclear Studies
Hoża 69, 00-681 Warsaw, Poland

H.S. Köhler

University of Arizona, Tucson, AZ 85721, USA

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Dedicated to Adam Sobiczewski in honour of his 70th birthday

A simple theory of the interaction potential between heavy ions $\mathcal{V}$, based on the local density approach and the frozen density model, is presented for nuclei with neutron excess. The energy density needed for calculating $\mathcal{V}$ is expressed in a simple way through the known properties of nuclear matter.

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1. Introduction

The theoretical description of the synthesis of the heaviest elements in heavy ion collisions requires the knowledge of the interaction potential $\mathcal{V}$ between the two colliding ions. The calculation of $\mathcal{V}$, based on a simple nuclear matter (NM) approach was presented in Refs. [1–4]. We call the approach simple, because it allows to determine $\mathcal{V}$ directly from the known properties of NM. In a simplified form, the approach was applied a long time ago by Brueckner *at al.* [5] (see also [6]).

The simple NM approach was restricted in Refs. [1–4] to heavy ions with equal numbers of neutrons and protons, and could not be applied to really heavy ions with an appreciable neutron excess. In the present paper we extend our simple NM approach to $\mathcal{V}$ to the scattering of ions with neutron excess, *i.e.*, to the case relevant in the synthesis of the heaviest elements.

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The paper is organized as follows. In Section 2, the interaction potential $\mathcal{V}$ is defined as the energy difference between overlapping and separated nuclei. In Section 3 the local density approximation and the frozen density model are presented and the density and momentum distributions of neutrons and protons in the system of two colliding nuclei are described. Final comments, especially concerning results obtained previously for nuclei with equal number of neutrons and protons, are presented in Section 4. Properties of the energy and the single particle (s.p.) energies in NM with neutron excess, in particular their density dependence, are described in the Appendix.

2. The definition of $\mathcal{V}$

We consider nuclei 1(target) and 2 (projectile) (with masses M_1, M_2, and with the reduced mass $\mu = M_1M_2/(M_1 + M_2)$, moving with relative momentum $\boldsymbol{K}_{\rm REL}$ (in units of $\hbar$). We denote by $\boldsymbol{R}$ the relative position vector between the centers of mass of 1 and 2 (directed from 1 to 2). The definition of $\mathcal{V}$ as the difference between the energies of the overlapping and spatially separated nuclei 1 and 2 does not depend on the reference frame. As the most convenient one, we chose the rest frame of 1, which we call the "laboratory" (lab) frame (at $R = \infty$, it coincides with the laboratory (LAB) frame of the two nuclei). In this frame

$$\mathcal{V}(E,R) = \mathcal{E}_{\rm lab}(K_{\rm REL},R) - \hbar^2K_2^2/2M_2 - \mathcal{E}_{\rm in}(1) - \mathcal{E}_{\rm in}(2)\ , \qquad (1)$$

where $\mathcal{E}_{\rm in}(i)$ is the intrinsic nuclear energy of the isolated nucleus i, $\mathcal{E}_{\rm lab}$ is the nuclear energy of the total system in the lab frame, and

$$\boldsymbol{K}_2 = (M_2/\mu)\boldsymbol{K}_{\rm REL} \qquad (2)$$

is the projectile momentum in the lab frame.

The conservation of the total energy implies that the instantaneous relative momentum $\boldsymbol{K}_{\rm REL} = \boldsymbol{K}_{\rm REL}(R)$ is changing with R:

$$\hbar^2K_{\rm REL}(R)^2/2\mu + \mathcal{V}(E,R) + \mathcal{V}_{\rm C}(R) = \hbar^2K_{\rm REL}(\infty)^2/2\mu = E\ , \qquad (3)$$

where $\mathcal{V}_{\rm C}(R)$ is the Coulomb potential between nuclei 1 and 2, and E is the CMS kinetic energy .

3. The NM approach to $\mathcal{V}$

We apply the local density approximation, and write $\mathcal{E}_{\rm lab}$ in the form:

$$\mathcal{E}_{\rm lab}(K_{\rm REL},R) = \int d\boldsymbol{r}\, H_{\rm lab}(K_{\rm REL},R;\boldsymbol{r}), \qquad (4)$$

where $H_{\rm lab}$ is the energy density (in the lab frame) at $\boldsymbol{r}$. For a given distance R between the two nuclei, the system is approximated locally (at each point $\boldsymbol{r}$) by a piece of NM with the neutron and proton densities ρ_n and ρ_p, and with the corresponding momentum distributions $n_n(\boldsymbol{k}'_n)$ and $n_p(\boldsymbol{k}'_p)$. Obviously, we have

$$\rho_n = [2/(2\pi)^3] \int d\boldsymbol{k}'_n n_n(\boldsymbol{k}'_n), \qquad \rho_p = [2/(2\pi)^3] \int d\boldsymbol{k}'_p n_p(\boldsymbol{k}'_p)\,. \tag{5}$$

(The assumed spin saturation leads to the factors 2. Neutron and proton momenta in the lab frame are denoted by $\boldsymbol{k}'_n$ and $\boldsymbol{k}'_p$.)

Instead of ρ_n and ρ_p, as an alternative description of the the local NM we also shall use the total density $\rho = \rho_n + \rho_p$ and the neutron excess parameter $\alpha = (\rho_n - \rho_p)/\rho$.

For ρ_y and $n_y(\boldsymbol{k}'_y)$, we apply the frozen density model (the sudden approximation) in which all degrees of freedom are frozen, except for R. Here and further on, we use the notation $y = n, p$. The neutron (proton) density of the combined system at $\boldsymbol{r}$ is equal to the sum of the original neutron (proton) densities of nuclei 1 and 2:

$$\rho_y(\boldsymbol{r}) = \rho_{1y}(r) + \rho_{2y}(|\boldsymbol{r} - \boldsymbol{R}|)\,. \tag{6}$$

(The origin of the position vector $\boldsymbol{r}$ is the same as that of $\boldsymbol{R}$, *i.e.*, the center of 1. We assume that both nuclei are spherically symmetric.)

The motion of each of the colliding nuclei resembles that of a rigid body: the instantaneous velocity of each point of the nucleus 1 (2) is the same. Thus in the lab frame the velocity of each point of 1 vanishes, and the velocity of each point of 2 is $\hbar\boldsymbol{K}_2/M_2 = \hbar\boldsymbol{K}_{\rm REL}/\mu$, and the average momentum $\boldsymbol{K}_r$ of neutrons and protons in nucleus 2 is

$$\boldsymbol{K}_r = (m/\mu)\boldsymbol{K}_{\rm REL}\,, \tag{7}$$

where m is the nucleon mass. Consequently in the lab frame (see Fig 1), the local momentum distributions at $\boldsymbol{r}$ of neutrons and protons in nucleus 1 are the respective Fermi spheres (surfaces $F_{1n} = F_{10n}$ and $F_{1p} = F_{10p}$) centered in O_1, with the local Fermi momenta

$$k_{F10y} = k_{F10y}(r) = [3\pi^2\rho_{1y}(r)]^{1/3}\,, \tag{8}$$

and those of neutrons and protons in 2 are the respective Fermi spheres (surfaces $F_{2n} = F_{20n}$ and $F_{2p} = F_{20p}$) centered in O_2 (with $\overrightarrow{O_1O_2} = \boldsymbol{K}_r$), with the local Fermi momenta

$$k_{F20y} = k_{F20y}(\boldsymbol{r}) = [3\pi^2\rho_{2y}(|\boldsymbol{r} - \boldsymbol{R}|)]^{1/3}\,. \tag{9}$$

For the combined system of nuclei 1 and 2, we obtain the local neutron and proton momentum distributions $n_y(\boldsymbol{k}'_y)$ consisting of two Fermi spheres F_{1y} and F_{2y}: $n_y(\boldsymbol{k}'_y) = 1$ for $\boldsymbol{k}'_y$ within $F_y = F_{1y} + F_{2y}$ and $n_y(\boldsymbol{k}'_y) = 0$ otherwise.

As long as $K_r > k_{F10y} + k_{F20y}$ (Fig. 1(a)), our definition of $n_y(\boldsymbol{k}'_y)$ presents no problems. If however $K_r < k_{F10y} + k_{F20y}$ (Fig. 1(b)), the two Fermi spheres F_{10y} and F_{20y} overlap, and we face the problem of the double occupancy in the overlap region. We resolve this problem by increasing $k_{F10y} \to k_{F1y}$ and $k_{F20y} \to k_{F2y}$, and obtain our final momentum distributions $n_y(\boldsymbol{k}'_y)$ with the Fermi surfaces $F_y = F_{1y} + F_{2y}$, with a single occupancy inside F_y. This reshuffling of neutrons and protons from the original distri-

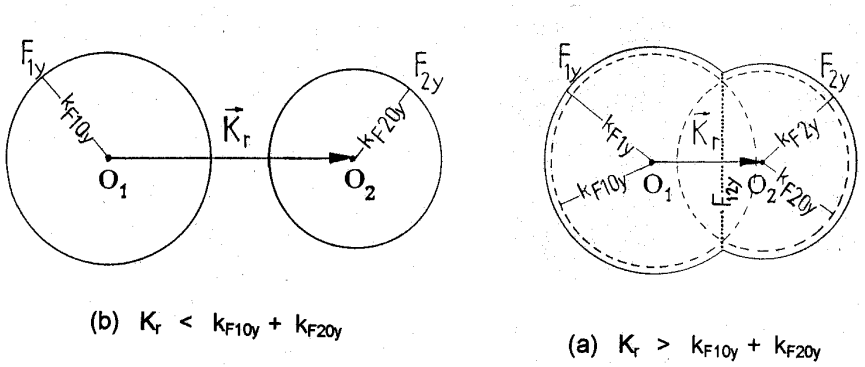

Fig. 1. The local momentum distribution of neutrons or protons ($y = n$ or p) in two colliding nuclei.

butions $\tilde{n}_y$ with the Fermi surfaces $F_{10y} + F_{20y}$ (with the double occupancy in the overlap region) to our final distributions n_y should leave the neutron and proton densities unchanged:

$$\rho_y = \rho_{1y} + \rho_{2y} = [2/(2\pi)^3]V_{F_y} = [2/(2\pi)^3](V_{F_{1y}} + V_{F_{2y}})\,, \qquad (10)$$

where V_{F_y} is the volume within F_y and $V_{F_{1y}}$ ($V_{F_{2y}}$) is the volume within F_y to the left (right) of the plane F_{12y}. We have ($i = 1, 2$):

$$\begin{aligned} V_{F_y} &= \pi\{\tfrac{2}{3}(k^3_{F1y} + k^3_{F2y}) + \tfrac{1}{2}K_r(k^2_{F1y} + k^2_{F2y}) + \tfrac{1}{4}K_r^3(x_y^2 - \tfrac{1}{3})\}\,, \\ V_{F_{iy}} &= \pi\{\tfrac{2}{3}k^3_{Fiy} + \tfrac{1}{2}K_r(1 + \eta_i x_y)k^2_{Fiy} - \tfrac{1}{3}[\tfrac{1}{2}K_r(1 + \eta_i x_y)]^3\}\,, \end{aligned} \qquad (11)$$

where $\eta_1 = 1$, $\eta_2 = -1$, and $x_y = (k^2_{F1y} - k^2_{F2y})/K_r^2$.

To determine k_{F1y} and k_{F2y}, we need one condition more for each value of y (n and p) in addition to Eq. (10)[1]. Let us denote by $\boldsymbol{k}_{Gy}$ the average

[1] Here, we apply the prescription of Ref. [7] (see also [4]).

neutron ($y = n$) or proton ($y = p$) average momentum in the local NM at $\boldsymbol{r}$ in the lab frame:

$$\boldsymbol{k}_{Gy} = \frac{\int d\boldsymbol{k}'_y n_y(\boldsymbol{k}'_y)\boldsymbol{k}'_y}{\int d\boldsymbol{k}'_y n_y(\boldsymbol{k}'_y)} = \begin{cases} \boldsymbol{K}_r \rho_{2y}/\rho_y & \text{for } K_r > k_{F10y} + k_{F20y}\,, \\ \boldsymbol{K}_r V_{F_2 y}/V_{F_y} & \text{otherwise}\,. \end{cases} \tag{12}$$

Before reshuffling neutrons and protons, in the original distributions $\tilde{n}_y$, we have $\boldsymbol{k}_{Gy} = \boldsymbol{K}_r \rho_{2y}/\rho_y$ also for $K_r < k_{F10y} + k_{F20y}$. Let us insist that the reshuffling does not change the average neutron and proton momenta $\boldsymbol{k}_{Gy}$:

$$\frac{V_{F_{2y}}}{V_{F_y}} = \frac{\rho_{2y}}{\rho_y}\,. \tag{13}$$

Eqs. (13) and (10) are equivalent to the following system of two equations for both neutrons and protons:

$$V_{F_{1y}} = \frac{4\pi}{3} k_{F10y}^3\,, \qquad V_{F_{2y}} = \frac{4\pi}{3} k_{F20y}^3\,, \tag{14}$$

from which both (k_{F1n}, k_{F2n}) and (k_{F1p}, k_{F2p}) may be determined (numerically)[2].

To simplify the presentation, we go over to the rest frame of NM, in which the total momentum vanishes. The nucleon momenta in this frame are denoted by $\boldsymbol{k}_y$. This frame depends on $\boldsymbol{r}$ (is local). The energy density in this frame is denoted by H, and — in agreement with the König theorem of the classical mechanics — we have (see Ref. [4]):

$$H_{\text{lab}} = H + \frac{\hbar^2}{2m} \frac{K_r^2 \rho_2^2}{\rho}\,, \tag{15}$$

where we use the notation $\rho_i = \rho_{\text{in}} + \rho_{\text{ip}}$.

Let us consider normal NM (*i.e.*, NM in its ground state) with the same neutron density ρ_n and proton density ρ_p as the local neutron and proton densities of our system or equivalently with the same total density ρ and neutron excess α as the local total density and neutron excess of our system. In this normal NM, the neutron and proton momentum distributions are $n_{0y}(k_y) = \theta(k_{Fy} - k_y)$, where $k_{Fy} = (3\pi^2\rho_y)^{1/3}$, and the energy density is

$$H_0^{\text{NM}} = \frac{E_{\text{NM}}}{A}\rho = f(\rho, \alpha)\rho\,, \tag{16}$$

where $f(\rho, \alpha) = E_{\text{NM}}/A$ is the energy per nucleon in our normal NM. The NM by which our system is locally approximated, differs from the normal

[2] As it turns out, one always finds $|k_{F1y} - k_{F2y}| < K_r$.

NM by the momentum distributions n_y (the two sphere distributions in Fig. 1). We denote its energy density by $H^{\rm NM}$, and use the approximate relation

$$H^{\rm NM} \cong H_0^{\rm NM} + \frac{2}{(2\pi)^3} \sum_{y=n,p} \int d\boldsymbol{k}_y [n_y(\boldsymbol{k}_y) - n_{0y}(k_y)] e_{0y}(\rho, \alpha; k_y)\,, \qquad (17)$$

where e_{0y} is the s.p. energy of the y-nucleon in normal NM. Eq. (17) represents the change in the energy density caused by the redistribution of nucleons in the momentum space with unchanged s.p. energies. If we expressed the energy density through an effective two-body interaction, then we would obtain expression (17) by neglecting the change in the effective interaction induced by the change in the neutron and proton momentum distributions.

If we assume for e_{0y} the effective mass approximation [see Appendix, Eq. (A.1)], then Eqs. (16,17) lead to the following result for $H^{\rm NM}$:

$$H^{\rm NM} = f(\rho, \alpha)\rho + \tfrac{1}{\nu}(\tau_n + \tau_p - \tau_{0n} - \tau_{0p})\,, \qquad (18)$$

where τ_y and τ_{0y} are the kinetic energy densities (in the rest frame of NM) in our system and in normal NM:

$$\tau_y = [2/(2\pi)^3] \int d\boldsymbol{k}_y n_y(\boldsymbol{k}_y) \varepsilon(k_y)\,, \qquad \tau_{0y} = \tfrac{3}{5}\varepsilon(k_{Fy})\rho_y\,. \qquad (19)$$

Let us notice that with the help of Eqs. (A.8), (A.10) we may write expression (18) in the alternative form:

$$H_{\rm NM} = f_0(\rho)\rho + \tfrac{1}{\nu}(\tau_n + \tau_p - \tau_0) + \tfrac{1}{2}\alpha^2 \tfrac{1}{4} V_1 \rho\,, \qquad (20)$$

where $\tau_0 = \frac{3}{5}\varepsilon(k_F)\rho$.

To take into account density gradient corrections, we follow Brueckner *et al.* [8] and add to the energy density of the local NM the gradient correction H_∇:

$$H = H^{\rm NM} + H_\nabla\,, \qquad (21)$$

where

$$H_\nabla = H_\nabla(\rho) = \eta_{\rm W} \frac{(\nabla\rho)^2}{\rho} + \eta_V (\nabla\rho)^2, \qquad (22)$$

where $\eta_{\rm W} = \hbar^2/72m$. The first term in (22) is the Weizsäcker correction to the kinetic energy density, and the second one is the gradient correction to the potential energy density, in which η_V is treated as a phenomenological parameter.

In calculating the intrinsic energies $\mathcal{E}_{\rm in}(i)$ we apply the expression:

$$\mathcal{E}_{\rm in}(i) = \int d\boldsymbol{r}\{f(\rho_i,\alpha_i)\rho_i + H_\nabla(\rho_i)\}\,, \tag{23}$$

where $\alpha_i = (\rho_{\rm in} - \rho_{\rm ip})/\rho_i$.

Following the procedure explained in Ref. [4] we insert our results presented in Eqs. (23,21,18,15,4) into definition (1) of $\mathcal{V}$, and write our final expression for the interaction potential $\mathcal{V}$ in the form:

$$\mathcal{V}(E,R) = \int d\boldsymbol{r} v(K_r,R;\boldsymbol{r}), \tag{24}$$

$$\begin{aligned} v(K_r,R;\boldsymbol{r}) &= f(\rho,\alpha)\rho + \tfrac{1}{\nu}(\tau_n+\tau_p-\tau_{0n}-\tau_{0p}) + H_\nabla(\rho) \\ &\quad - \sum_{i=1,2}[f(\rho_i,\alpha_i)\rho_i + H_\nabla(\rho_i)] - \frac{\hbar^2K_r^2}{2m}\frac{\rho_1\rho_2}{\rho}\,, \end{aligned} \tag{25}$$

where $\rho_1 = \rho_1(r)$, $\rho_2 = \rho_2(|\boldsymbol{r}-\boldsymbol{R}|)$, and $\rho = \rho_1 + \rho_2$.

Let us notice that $K_r = K_r(R) = (m/\mu)K_{\rm REL}(R)$, and this R dependence of K_r and $K_{\rm REL}$ is determined by relation (3) wich contains $\mathcal{V}(E,R)$. One may solve this problem by applying the iteration procedure described in Refs. [2,3].

4. Final comments

In the limiting case of $\alpha_i = 0$, the reliability of our approach was discussed in Refs. [1–3]. In this case we could compare our results with those obtained by the Faessler group (see [7,9] and references quoted there). In this group, the "exact" energy density $H^{\rm NM}$ was calculated within the Brueckner theory. Our results for $\mathcal{V}$ — based on $H_0^{\rm NM}$ fitted to known properties of NM, and on relation (17) — turned out to be in a satisfying agreement with the most extensive calculations of the Faessler group. It should be pointed out, that these extensive calculations would become extremely complicated in the case of heavy ions with neutron excess (even normal NM with neutron excess presents serious computational problems in *ab initio* calculations — see *e.g.* [10]). On the other hand, our approach to heavy ions with neutron excess remains simple.

Recently, the static nucleus–nucleus potential has been calculated by Skalski [11] who applied the Hartree–Fock method. In particular, he considers the case of ^{40}Ca-^{40}Ca potential, which we also considered in Refs. [3,4]. Our result obtained for the total potential $\mathcal{V}_{\rm tot} = \mathcal{V} + \mathcal{V}_{\rm C}$, calculated with $K_r \equiv K_r(\infty) = 0$, is shown in Fig. 2. In calculating $\mathcal{V}$, we used for ^{40}Ca standard Woods–Saxon density, $\eta_V = 22$ MeV fm^5 was fitted to binding

energies of ^{40}Ca and ^{16}O, and $\nu_0 = .83$ was taken from [12]. The charge distributions in the two ^{40}Ca nuclei were approximated by equivalent uniform distributions, and $\mathcal{V}_{\rm C}(R)$ was calculated as the Coulomb interaction between these two uniform charge distributions. Our potential $\mathcal{V}_{\rm tot}$ turns out to be similar to the potential calculated in [11]. Especially the maximum of $\mathcal{V}_{\rm tot}$ equal to 54.5 MeV at $R = 9.4$ fm is close to the fusion barrier of 53 MeV obtained in [11] (and is consistent with the threshold barrier of 50.2±0.2 MeV estimated in [13]). This agreement between the essentially adiabatic approach of Ref. [11] and our approach involving the sudden approximation takes place at distances R at which the tails of the two densities overlap. At smaller distances our frozen density exceeds the adiabatic density and our potential is bigger (more repulsive) than the potential determined in [11].

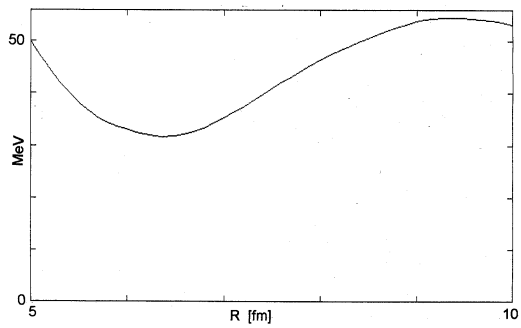

Fig. 2. The total interaction potential $\mathcal{V}(R) + \mathcal{V}_{\rm C}(R)$ for ^{40}Ca–^{40}Ca at $K_r = 0$.

The application of the scheme described in this paper to a number of heavy ion partners will be presented in the future. This application requires the knowledge of both proton and neutron distributions in the colliding partners, of which the neutron distribution is empirically less known and its detailed shape has to be discussed.

Appendix A

The energy and the s.p. potential in normal NM

For the s.p. energies e_{0y}, we assume the following effective mass approximation:

$$\begin{aligned} e_{0n}(\rho,\alpha;k_n) &= \frac{\varepsilon(k_n)}{\nu(\rho)} + C(\rho,\alpha)\,, \\ e_{0p}(\rho,\alpha;k_p) &= \frac{\varepsilon(k_p)}{\nu(\rho)} + C(\rho,-\alpha)\,, \end{aligned} \tag{A.1}$$

where $\varepsilon(k_y) = \hbar^2 k_y^2/2m$, and $\nu = m^*/m$ is the ratio of the effective to the real nucleon mass. The dependence on $\pm\alpha$ of C follows from the charge symmetry of nuclear forces. For the dependence of ν on ρ, we use the relation (ρ_0 is the equilibrium density of NM):

$$\frac{1/\nu(\rho) - 1}{1/\nu_0 - 1} = \frac{\rho}{\rho_0}\,, \tag{A.2}$$

where $\nu_0 = \nu(\rho_0)$. This relation follows from the assumption that $\partial V_{0y}/\partial k_y$ is proportional to ρ, where V_{0y} is the s.p. potential:

$$V_{0y}(\rho, \alpha; k_y) = e_{0y}(\rho, \alpha; k_y) - \varepsilon(k_y)\,. \tag{A.3}$$

We assume that our s.p. energies e_{0y} lead to the correct energy per nucleon $E_{\rm NM}/A$ in normal NM:

$$\frac{E_{\rm NM}}{A} = f(\rho, \alpha) = \tfrac{1}{2} \sum_{y=n,p} \int_0^{k_{Fy}} dk_y k_y^2 [\varepsilon(k_y) + \tfrac{1}{2} V_{0y}(\rho, \alpha; k_y)] / \tfrac{1}{3} k_F^3\,, \tag{A.4}$$

where $k_F = (3\pi^2\rho/2)^{1/3}$, $k_{Fn} = (3\pi^2\rho_n)^{1/3} = k_F(1+\alpha)^{1/3}$, and $k_{Fp} = (3\pi^2\rho_p)^{1/3} = k_F(1-\alpha)^{1/3}$.

Let us now introduce the linear approximation in α of V_{0y}:

$$\begin{aligned} V_{0n}(\rho, \alpha; k_n) &= V_0(\rho; k_n) + \tfrac{1}{4}\alpha V_1(\rho)\,, \\ V_{0p}(\rho, \alpha; k_p) &= V_0(\rho; k_p) - \tfrac{1}{4}\alpha V_1(\rho)\,, \end{aligned} \tag{A.5}$$

where

$$V_0(\rho; k_y) = \left(\frac{1}{\nu(\rho)} - 1\right) \varepsilon(k_y) + C(\rho, 0), \tag{A.6}$$

and the Lane potential V_1 is:

$$V_1(\rho) = 4 \frac{\partial C(\rho, \alpha)}{\partial \alpha}\bigg|_{\alpha=0}\,. \tag{A.7}$$

For the energy per nucleon in normal NM, $f = E_{\rm NM}/A$, we use on the left hand side of Eq. (A.4) the quadratic[3] approximation in α:

$$f(\rho, \alpha) = f_0(\rho) + \tfrac{1}{2}\alpha^2 f_1(\rho)\,. \tag{A.8}$$

On the right hand side of Eq. (A.4), we insert expressions (A.5) for V_{0y} and expand this side in powers of α up to quadratic terms (terms linear

[3] Terms linear in α vanish for charge symmetric nuclear forces.

in α vanish). By comparing the corresponding terms on both sides of the resulting equation, we get:

$$\begin{aligned} f_0(\rho) &= \tfrac{1}{2}[\tfrac{3}{5}(1+1/\nu)\varepsilon(k_F) + C(\rho,0)]\,, \qquad \text{(A.9)} \\ f_1(\rho) &= \tfrac{2}{3}\varepsilon(k_F) + \tfrac{1}{3}[k\partial V_0(\rho,k)/\partial k]_{k=k_F} + \tfrac{1}{4}V_1(\rho) \\ &= \tfrac{2}{3}\varepsilon(k_F)/\nu(\rho) + \tfrac{1}{4}V_1(\rho)\,. \qquad \text{(A.10)} \end{aligned}$$

For f_0, we assume the form:

$$f_0(\rho) = \frac{3}{5}\varepsilon(k_F) + \sum_{j=3}^{5} a_j (k_F/k_{F0})^j\,, \qquad \text{(A.11)}$$

where k_{F0} is the Fermi momentum at equilibrium density ρ_0, and the coefficients a_j are determined by k_{F0}, by the volume energy of NM, $\varepsilon_{\rm vol} = f_0(\rho_0)$, and by the compressibility $K_{\rm c} = k_{F0}^2 (d^2 f_0/dk_F^2)_{k_{F0}}$:

$$\begin{aligned} a_3 &= 10\varepsilon_{\rm vol} - \tfrac{9}{5}\varepsilon(k_{F0}) + \tfrac{1}{2}K_{\rm c}\,, \\ a_4 &= \tfrac{9}{5}\varepsilon(k_{F0}) - 15\varepsilon_{\rm vol} - K_{\rm c}\,, \\ a_5 &= \tfrac{1}{2}K_{\rm c} - \tfrac{3}{5}\varepsilon(k_{E0}) + 6\varepsilon_{\rm vol}\,. \qquad \text{(A.12)} \end{aligned}$$

With the assumed form of $f_0(\rho)$ one could use Eq. (A.9) to determine $C(\rho,0)$, although in our present calculation of $\mathcal{V}$ the value of $C(\rho,0)$ is not required. On the other hand, we shall use Eq. (A.10) to determine the function $f_1(\rho)$. Empirically we only know the symmetry energy $\varepsilon_{sym} = f_1(\rho_0)$. To fix the dependence of f_1 on ρ we use Eq. (A.10) in which we assume that $V_1(\rho)$ is proportional to ρ:

$$V_1(\rho) = \frac{\rho}{\rho_0} V_1(\rho_0)\,, \qquad \text{(A.13)}$$

where $V_1(\rho_0)$ is determined by:

$$\varepsilon_{\rm sym} = \frac{2}{3}\frac{\varepsilon(k_{F0})}{\nu_0} + \frac{1}{4}V_1(\rho_0)\,. \qquad \text{(A.14)}$$

(For $k_{F0} = 1.35$ fm^{-1}, $\nu_0 = 0.7$, and $\varepsilon_{sym} = 60$ MeV, we get $V_1(\rho_0) = 96$ MeV, in good agreement with existing estimates of the Lane potential.) Thus our final expression for $f_1(\rho)$ is:

$$f_1(\rho) = \frac{2}{3}\varepsilon(k_F)\left[1 + \frac{(1/\nu_0 - 1)\rho}{\rho_0}\right] + \frac{1}{4}V_1(\rho_0)\frac{\rho}{\rho_0}\,. \qquad \text{(A.15)}$$

One consequence of the density dependence of f_1 is that the equilibrium density is shifted by the neutron excess (towards lower densities). It is easy to estimate this shift (with an α^2 accuracy):

$$\Delta k_F/k_{F0} = -\frac{1}{2}\frac{[k_F df_1/dk_F]_{k_{F0}}}{K_c}\alpha^2 \,. \tag{A.16}$$

For $K_c = 235$ MeV (and with the remaining NM parameters quoted above) we get $\Delta k_F/k_{F0} = -0.375\alpha^2$. This agrees with other estimates [10, 14], which indicates that our expression (A.15) for $f_1(\rho)$ appears to be a reasonable approximation.

REFERENCES

[1] J. Dąbrowski, H.S. Köhler, *Nucl. Phys.* **A489**, 303 (1988).

[2] J. Dąbrowski, H.S. Köhler, *Nucl. Phys.* **A499**, 413 (1989).

[3] J. Dąbrowski *Acta Phys. Pol. B* **20**, 61 (1989).

[4] J. Dąbrowski *Acta Phys. Pol. B* **21**, 223 (1990); *Phys. Lett.* **B240**, 33 (1990).

[5] K.A. Brueckner, J.R. Buchler, M.M. Kelly, *Phys. Rev.* **173**, 944 (1968).

[6] F. Beck, K.-H. Mueller, H.S. Köhler, *Phys. Rev. Lett.* **40**, 837 (1978).

[7] M. Trefz, A. Faessler, W.W. Dickhoff, *Nucl. Phys.* **A443**, 499 (1985).

[8] K.A. Brueckner, J.R. Buchler, S. Jorna, R.J. Lombard, *Phys. Rev.* **171**, 1188 (1968).

[9] N. Ohtsuka, A. Faessler, *Z. Phys.* **A239**, 89 (1988).

[10] K.A. Brueckner, S.A. Coon, J. Dąbrowski, *Phys. Rev.* **168**, 1184 (1968).

[11] J. Skalski, *Acta Phys. Pol. B*, **34**, 1977 (2003).

[12] C.H. Johnson, D.J. Horen, C. Mahaux, *Phys. Rev.* **C36**, 2252 (1987).

[13] K. Siwek-Wilczyńska, J. Wilczyński, *Phys. Rev.* **C64**, 024611 (2001).

[14] M. Dworzecka, *Acta Phys. Pol.* **29**, 783 (1966).