

PROPERTIES OF HEAVIEST NUCLEI*

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Masses and α -decay energies are calculated for very heavy nuclei with proton number $Z = 102$ – 109 . A macroscopic–microscopic approach is used. Much attention is also given to experimental values of these two quantities, deduced from energies of α particles observed in α -decay chains and from masses of finite nuclei in these chains. A comparison between calculated and experimental values is discussed. Other properties of the nuclei, such as deformations, shell-correction energies and deformation energies, are also studied.

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1. Introduction

We concentrate in this paper on two important properties of heaviest nuclei: masses M and α -decay energies Q_α . Main attention is given to calculation of these quantities. Their experimental values, however, are also discussed. We also perform some discussion of the comparison between calculated and experimental results. This especially concerns Q_α , as for masses, such a discussion has been performed already earlier [1,2].

2. Theoretical model used

The ground-state mass of a nucleus is calculated within a macroscopic–microscopic approach. The Yukawa-plus-exponential model is used for the macroscopic part of mass and the Strutinski shell correction, based on the Woods-Saxon single-particle potential, is taken for its microscopic part. A large, 7-dimensional deformation space $\{\beta_\lambda\}$, $\lambda=2,3,\dots,8$, is used to obtain the equilibrium deformation of a nucleus. More details of the model and the calculations are given in Refs. [1,3].

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3. Results

Table I gives masses (more precisely, mass excesses) and Q_α for nuclei with proton number $Z = 102$ – 109 . For heavier nuclei, with $Z = 110$ – 120 , masses and Q_α have been presented earlier [1]. For each nucleus, its equilibrium deformation parameters β_λ^0 , $\lambda=2,4,6,8$, deformation energy E_{def} and shell correction energy E_{sh} are also given in Table I. Only β_λ^0 with even multipolarity are presented, as those with odd λ , $\lambda=3,5,7$, are found to be zero, for nuclei considered in the table. The deformation energy E_{def} is defined as: $E_{\text{def}} = E(0) - E(\beta_\lambda^0)$, *i.e.* as a difference between the energy of a nucleus at its spherical and equilibrium shapes. Thus, it is the gain in its energy due to its deformation. The shell correction energy E_{sh} is defined as: $E_{\text{sh}} = E(\beta_\lambda^0) - E_{\text{macr}}(0)$. Thus, it is the gain in energy of a nucleus due to its shell structure and is a global characteristics of a nucleus.

One can see in Table I that all nuclei given in it are deformed, with $E_{\text{def}} > 2.1$ MeV.

TABLE I

Deformation parameters β_λ^0 , deformation energies E_{def} , shell correction energies E_{sh} , masses M and α -decay energies Q_α , calculated for nuclei with $Z = 102$ – 109 .

N	A	β_2	β_4	β_6	β_8	E_{def}	E_{sh}	M	Q_α
—	—	—	—	—	—	MeV	MeV	MeV	MeV
Z=102									
146	248	0.246	0.046	-0.042	-0.012	12.0	-3.4	81.07	9.27
147	249	0.247	0.044	-0.045	-0.011	12.9	-3.1	81.91	9.12
148	250	0.248	0.035	-0.047	-0.005	12.5	-4.0	81.75	9.00
149	251	0.249	0.029	-0.050	-0.002	13.2	-3.7	82.85	8.83
150	252	0.251	0.024	-0.051	0.003	12.7	-4.6	82.95	8.53
151	253	0.252	0.019	-0.054	0.005	13.4	-4.5	84.19	8.19
152	254	0.254	0.014	-0.057	0.007	12.7	-5.2	84.70	8.06
153	255	0.254	0.009	-0.059	0.012	12.8	-4.6	86.70	8.32
154	256	0.253	0.004	-0.056	0.014	11.8	-5.0	87.76	8.36
155	257	0.253	-0.003	-0.056	0.019	11.8	-4.3	90.07	8.19
156	258	0.251	-0.008	-0.053	0.020	10.7	-4.8	91.37	7.99
157	259	0.252	-0.016	-0.055	0.026	10.7	-4.2	93.86	7.71
158	260	0.247	-0.021	-0.048	0.024	9.6	-4.7	95.34	7.45
159	261	0.243	-0.029	-0.044	0.026	9.6	-4.1	97.98	7.10
160	262	0.237	-0.034	-0.035	0.021	8.7	-4.7	99.64	6.86
161	263	0.235	-0.042	-0.032	0.024	8.8	-4.4	102.35	6.45
162	264	0.228	-0.048	-0.024	0.020	7.9	-5.0	104.19	6.25

Table I cont.

N	A	β_2	β_4	β_6	β_8	E_{def}	E_{sh}	M	Q_α
—	—	—	—	—	—	MeV	MeV	MeV	MeV
Z=103									
148	251	0.248	0.026	-0.046	-0.004	12.8	-3.2	87.96	9.51
149	252	0.248	0.024	-0.049	-0.002	13.6	-3.0	88.82	9.32
150	253	0.250	0.016	-0.050	0.003	13.1	-3.9	88.65	9.03
151	254	0.248	0.013	-0.052	0.006	13.9	-3.8	89.67	8.71
152	255	0.253	0.008	-0.056	0.008	13.2	-4.6	89.92	8.57
153	256	0.253	0.003	-0.057	0.013	13.4	-4.0	91.69	8.85
154	257	0.252	-0.002	-0.055	0.015	12.3	-4.4	92.53	8.92
155	258	0.253	-0.008	-0.056	0.020	12.4	-3.8	94.59	8.71
156	259	0.252	-0.013	-0.054	0.022	11.3	-4.3	95.67	8.47
157	260	0.253	-0.020	-0.055	0.028	11.3	-3.7	97.94	8.16
158	261	0.249	-0.024	-0.050	0.026	10.3	-4.2	99.22	7.88
159	262	0.254	-0.033	-0.045	0.031	10.2	-3.6	101.73	7.61
160	263	0.239	-0.037	-0.037	0.023	9.3	-4.2	103.12	7.26
161	264	0.236	-0.045	-0.033	0.025	9.5	-3.9	105.63	6.84
162	265	0.230	-0.050	-0.025	0.021	8.5	-4.5	107.29	6.62
Z=104									
148	252	0.245	0.020	-0.042	-0.002	12.0	-3.6	93.34	9.85
149	253	0.247	0.016	-0.045	0.000	12.8	-3.4	93.98	9.64
150	254	0.248	0.009	-0.047	0.004	12.4	-4.4	93.55	9.37
151	255	0.249	0.008	-0.050	0.006	13.2	-4.4	94.32	9.04
152	256	0.251	0.002	-0.052	0.009	12.6	-5.2	94.30	8.93
153	257	0.252	-0.004	-0.054	0.014	12.8	-4.7	95.82	9.21
154	258	0.250	-0.009	-0.051	0.016	11.8	-5.2	96.41	9.29
155	259	0.251	-0.015	-0.053	0.021	11.9	-4.6	98.21	9.08
156	260	0.249	-0.021	-0.050	0.023	10.9	-5.2	99.02	8.84
157	261	0.251	-0.026	-0.052	0.029	11.0	-4.6	101.02	8.53
158	262	0.247	-0.032	-0.046	0.027	10.0	-5.2	102.04	8.24
159	263	0.245	-0.038	-0.044	0.029	10.1	-4.8	104.18	7.90
160	264	0.239	-0.042	-0.035	0.025	9.1	-5.4	105.41	7.64
161	265	0.237	-0.049	-0.033	0.026	9.3	-5.1	107.67	7.27
162	266	0.231	-0.054	-0.025	0.023	8.4	-5.8	109.11	7.05
Z=105									
150	255	0.247	0.000	-0.046	0.007	12.8	-3.8	100.00	9.62
151	256	0.248	-0.001	-0.048	0.008	13.6	-3.8	100.55	9.31
152	257	0.249	-0.006	-0.051	0.010	13.0	-4.7	100.29	9.22
153	258	0.250	-0.011	-0.052	0.015	13.3	-4.2	101.58	9.48
154	259	0.249	-0.016	-0.050	0.017	12.4	-4.7	101.91	9.57

Table I cont.

N	A	β_2	β_4	β_6	β_8	E_{def}	E_{sh}	M	Q_α
—	—	—	—	—	—	MeV	MeV	MeV	MeV
155	260	0.250	-0.021	-0.051	0.021	12.5	-4.2	103.48	9.36
156	261	0.249	-0.026	-0.049	0.024	11.5	-4.8	104.06	9.11
157	262	0.250	-0.031	-0.050	0.029	11.5	-4.3	105.84	8.83
158	263	0.247	-0.036	-0.046	0.028	10.6	-4.9	106.63	8.53
159	264	0.245	-0.041	-0.046	0.030	10.7	-4.5	108.56	8.20
160	265	0.241	-0.045	-0.037	0.026	9.8	-5.1	109.61	7.97
161	266	0.239	-0.051	-0.034	0.027	10.0	-4.8	111.68	7.52
162	267	0.234	-0.056	-0.026	0.025	9.0	-5.4	112.95	7.41
163	268	0.226	-0.061	-0.018	0.020	8.7	-4.5	115.85	7.80
164	269	0.223	-0.060	-0.017	0.018	7.2	-4.6	117.89	8.17
Z=106									
152	258	0.248	-0.011	-0.050	0.013	12.3	-5.3	105.58	9.61
153	259	0.250	-0.016	-0.051	0.018	12.5	-4.8	106.63	9.89
154	260	0.248	-0.022	-0.049	0.021	11.7	-5.4	106.68	9.95
155	261	0.249	-0.027	-0.050	0.025	11.9	-5.0	107.99	9.74
156	262	0.249	-0.032	-0.048	0.027	11.0	-5.6	108.33	9.49
157	263	0.250	-0.037	-0.050	0.032	11.0	-5.1	109.85	9.21
158	264	0.246	-0.042	-0.044	0.030	10.2	-5.8	110.39	8.94
159	265	0.247	-0.047	-0.045	0.034	10.3	-5.4	112.08	8.63
160	266	0.240	-0.051	-0.036	0.029	9.4	-6.1	112.89	8.42
161	267	0.238	-0.057	-0.033	0.030	9.6	-5.8	114.70	8.09
162	268	0.233	-0.061	-0.025	0.026	8.7	-6.5	115.73	7.89
163	269	0.227	-0.065	-0.018	0.022	8.4	-5.7	118.42	8.32
164	270	0.224	-0.064	-0.016	0.020	6.9	-5.7	120.27	8.74
165	271	0.218	-0.067	-0.011	0.018	6.3	-4.8	123.32	8.71
166	272	0.213	-0.067	-0.007	0.015	5.2	-4.9	125.29	8.50
Z=107									
154	261	0.248	-0.030	-0.045	0.019	12.0	-5.0	113.03	10.31
155	262	0.247	-0.034	-0.046	0.025	12.2	-4.5	114.10	10.09
156	263	0.246	-0.040	-0.044	0.027	11.4	-5.2	114.18	9.84
157	264	0.248	-0.044	-0.046	0.032	11.6	-4.8	115.49	9.59
158	265	0.246	-0.048	-0.043	0.032	10.7	-5.5	115.75	9.27
159	266	0.245	-0.052	-0.041	0.035	10.9	-5.2	117.21	8.95
160	267	0.240	-0.056	-0.034	0.031	10.0	-5.9	117.81	8.75
161	268	0.239	-0.061	-0.032	0.032	10.3	-5.7	119.39	8.40
162	269	0.235	-0.065	-0.026	0.029	9.4	-6.4	120.22	8.19
163	270	0.229	-0.068	-0.020	0.025	9.1	-5.5	122.73	8.63
164	271	0.228	-0.067	-0.018	0.023	7.5	-5.5	124.45	9.07
165	272	0.224	-0.068	-0.015	0.021	6.9	-4.5	127.36	9.08
166	273	0.220	-0.069	-0.012	0.019	5.8	-4.6	129.21	8.89

Table I cont.

N	A	β_2	β_4	β_6	β_8	E_{def}	E_{sh}	M	Q_α
—	—	—	—	—	—	MeV	MeV	MeV	MeV
167	274	0.214	-0.073	-0.006	0.016	5.1	-3.5	132.38	8.83
168	275	0.209	-0.070	-0.005	0.014	4.2	-3.8	134.36	8.64
Z=108									
154	262	0.244	-0.036	-0.039	0.020	10.9	-5.3	119.01	11.00
155	263	0.245	-0.041	-0.040	0.025	11.1	-4.9	119.86	10.81
156	264	0.242	-0.046	-0.037	0.026	10.4	-5.6	119.69	10.59
157	265	0.245	-0.049	-0.039	0.030	10.5	-5.2	120.78	10.36
158	266	0.242	-0.054	-0.035	0.030	9.7	-6.0	120.80	10.04
159	267	0.240	-0.059	-0.034	0.032	9.9	-5.7	122.02	9.75
160	268	0.237	-0.062	-0.028	0.029	9.1	-6.5	122.31	9.49
161	269	0.237	-0.067	-0.026	0.031	9.5	-6.4	123.64	9.14
162	270	0.233	-0.071	-0.021	0.029	8.7	-7.1	124.18	8.87
163	271	0.228	-0.074	-0.015	0.025	8.4	-6.3	126.42	9.29
164	272	0.225	-0.074	-0.013	0.023	6.8	-6.4	127.95	9.80
165	273	0.221	-0.076	-0.009	0.020	6.3	-5.4	130.62	9.78
166	274	0.217	-0.077	-0.005	0.018	5.2	-5.5	132.25	9.55
167	275	0.211	-0.080	0.001	0.014	4.6	-4.5	135.15	9.41
168	276	0.205	-0.079	0.003	0.012	3.8	-4.8	136.91	9.19
169	277	0.189	-0.075	0.007	0.008	3.2	-3.9	139.99	9.03
170	278	0.180	-0.073	0.009	0.007	2.6	-4.3	141.82	8.77
Z=109									
156	265	0.238	-0.049	-0.030	0.022	9.8	-4.4	127.19	11.74
157	266	0.237	-0.054	-0.029	0.024	10.0	-4.0	128.06	11.54
158	267	0.236	-0.059	-0.027	0.025	9.4	-4.9	127.82	11.21
159	268	0.236	-0.063	-0.025	0.027	9.6	-4.7	128.79	10.88
160	269	0.233	-0.067	-0.021	0.026	8.9	-5.6	128.80	10.62
161	270	0.233	-0.071	-0.020	0.029	9.2	-5.4	129.91	10.27
162	271	0.229	-0.075	-0.015	0.026	8.6	-6.3	130.15	9.91
163	272	0.225	-0.077	-0.010	0.022	8.4	-5.6	132.07	10.25
164	273	0.222	-0.079	-0.007	0.020	6.9	-5.7	133.38	10.73
165	274	0.218	-0.083	-0.001	0.018	6.5	-4.8	135.78	10.63
166	275	0.215	-0.082	0.000	0.016	5.4	-5.0	137.21	10.34
167	276	0.211	-0.088	0.007	0.012	4.9	-4.0	139.87	10.09
168	277	0.207	-0.085	0.008	0.012	4.0	-4.3	141.47	9.84
169	278	0.197	-0.084	0.011	0.009	3.5	-3.3	144.36	9.55
170	279	0.189	-0.082	0.011	0.007	2.8	-3.7	146.06	9.28
171	280	0.142	-0.051	0.003	0.005	2.6	-3.0	148.90	8.88
172	281	0.132	-0.051	0.005	0.004	2.1	-3.7	150.53	8.49

4. Experimental masses and Q_α of heaviest nuclei

Table II gives α -decay energies Q_α and masses M deduced from kinetic energies of α particles of 4 decay chains observed and studied at GSI-Darmstadt, with initial nuclei: $^{269}\text{110}$, $^{271}\text{110}$, $^{272}\text{111}$ and $^{277}\text{112}$ [4–9] (for decay of ^{269}Hs and ^{270}Hs , see also Ref. [10]). Two of the chains ($^{271}\text{110}$ and $^{277}\text{112}$) are linked to nuclei (^{255}No and ^{257}No , respectively) with well established masses [11]. The other two ($^{269}\text{110}$ and $^{272}\text{111}$) end at nuclides (^{249}Fm and ^{252}Md , respectively), masses of which have not yet been measured and may be estimated only from systematic trends [11], what obviously introduces some uncertainty.

TABLE II

Experimental Q_α and masses (both in MeV) for nuclei of the four considered α -decay chains. The notation ‘[11]s’ is explained in the text.

	^{249}Fm	^{253}No	^{257}Rf	^{261}Sg	^{265}Hs	$^{269}\text{110}$
Q_α		8.42 [8]	8.84 [4]	9.71 [4]	10.73 [4]	11.28 [4]
M	73.61 [11]s	84.45	95.71	107.84	120.99	134.69
		^{255}No	^{259}Rf	^{263}Sg	^{267}Hs	$^{271}\text{110}$
Q_α			9.03 [7]	9.39 [7]	10.03 [7]	10.91 [7]
M		86.84 [11]	98.29	110.10	122.55	135.88
	^{252}Md	^{256}Lr	^{260}Db	^{264}Bh	^{268}Mt	$^{272}\text{111}$
Q_α		8.60 [5, 9]	9.29 [5, 9]	9.77 [5]	10.45 [9]	11.19 [9]
M	80.70 [11]s	91.72	103.43	115.62	128.49	142.10
	^{257}No	^{261}Rf	^{265}Sg	^{269}Hs	$^{273}\text{110}$	$^{277}\text{112}$
Q_α	8.47 [6]	8.65 [6]	8.82 [10]	9.34 [6, 9]	11.37 [9]	11.62 [6]
M	90.22 [11]	101.29	112.53	124.29	138.08	152.12

Another uncertainty in masses of nuclides of Table II is introduced by the fact that they are of the odd- A or odd-odd types. In such nuclei, in distinction to even–even ones, the main α -decay branch is usually not the ground-state to ground-state transition, because of various hindrances due to differences in the structure of the ground state in parent and daughter nuclei. A rather large density of low excited states in these nuclei is favorable for such situation (*cf. e.g.* Ref. [12]).

An evidence that the α -transitions are not only the ground-state to ground-state transitions, is the observation of α -particles, emitted from the

same nucleus, with energies differing by more than the accuracy of the measurement of it. Usually, spectrum of α -particles is divided into a number of groups, with energies differing rather little (less than the accuracy of the measurement) within one group, and differing more between different groups. Each group is connected with definite lifetime of emitter of the particles.

We expect that the transition is usually from the ground state of a parent nucleus to the ground or excited state of its daughter. Transition from excited (isomeric) state of a parent nucleus is rather seldom. Due to this, to be as close as possible to the ground-state-ground-state transition, we take (average) energy of the group with the highest α energy. The resulting Q_α are given in Table II. At each value, there is specified the reference from which the α energy is taken (notation ' [11]s' in the table stresses that mass, taken from Ref. [11] for a given nucleus, comes from estimation based on systematic trends and not from measurement; this also concerns masses denoted by AW97 in Fig. 1). With respect to Ref. [7], where similar analysis has been performed, we have at our disposal later data.

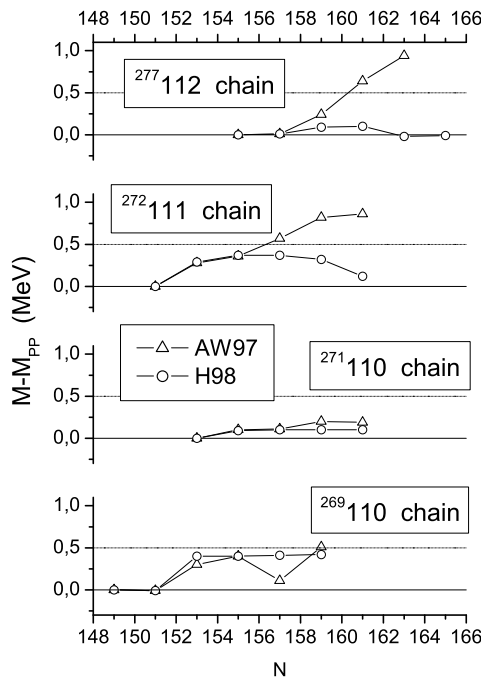


Fig. 1. Differences between masses obtained in Refs. [7, 11] and in this paper.

With a given Q_α and mass of a daughter nucleus, mass of the parent nucleus has been directly obtained.

As we do not know, what exactly is the transition which we took for the calculation of Q_α , it is difficult to estimate the accuracy of masses M obtained by the described procedure. To get some idea of it, we calculated differences between masses obtained in this paper (Table II) and those obtained in the analysis of Ref. [7] and from extrapolation of systematics of well established masses [11]. The differences are shown in Fig. 1. M_{PP} are the masses taken from Table II, H98 are the masses of Ref. [7] and AW97 denote the extrapolated masses of Ref. [11]. One can see that the differences are quite large, *e.g.* up to about 0.5 MeV for nuclei in the $^{269}_{110}$ chain.

Table III gives Q_α deduced from the data of Refs. [13–17], assuming that the observed decay corresponds to the ground-state – ground-state transitions. Here, masses of the considered nuclei cannot be obtained, as the observed chains are not linked to nuclei with measured masses. The chains end by fission.

TABLE III

Experimental Q_α (in MeV) for nuclei which are not linked to nuclides with known masses.

	$^{266}_{108}$		$^{270}_{110}$
Q_α	10.45 [13]		11.15 [13]
	$^{287}_{114}$		
Q_α	10.44 [14]		
	$^{281}_{110}$	$^{285}_{112}$	$^{289}_{114}$
Q_α	8.96 [15]	8.79 [15]	9.85 [15]
	$^{284}_{112}$	$^{288}_{114}$	$^{292}_{116}$
Q_α	9.27 [16, 17]	9.97 [16, 17]	10.71 [17]

Finally, let us look how well are the experimental values given in Tables II and III reproduced by theory. This is illustrated in Fig. 2 for heaviest nuclei with $Z = 110$ –116. Four theoretical approaches are considered: semi-empirical (SE) [18], our macroscopic–microscopic approach specially adapted to heavy nuclei (HN) [3], Thomas–Fermi (TF) [19] and Hartree–Fock (HF) [20] approaches. One can see that the discrepancy between theoretical and experimental values is quite large, up to more than 1 MeV, and is similar in its value to the discrepancy between various theoretical approaches, themselves.

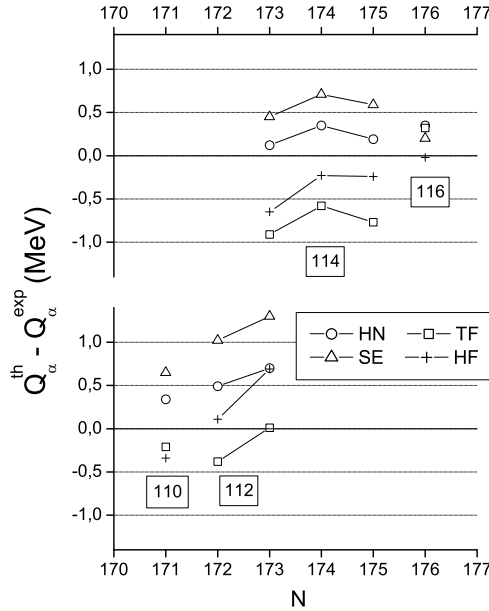


Fig. 2. Differences between calculated and experimental values of Q_α .

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