MEAN FIELD PLUS PARTICLE-VIBRATION COUPLING IN STABLE AND EXOTIC NUCLEI*

G. Colò, P.F. Bortignon, D. Sarchi, and E. Vigezzi

Dipartimento di Fisica, Università degli Studi, and INFN,
Via Celoria 16, 20133 Milano, Italy

(Received November 18, 2002)

Recent mean field calculations with effective interactions are discussed, and the relevance of corrections on top of these calculations, associated with the particle-vibration coupling, is emphasized.

PACS numbers: 21.10.Pc, 21.60.Jz, 24.30.Cz

1. Introduction

The study of "exotic" nuclear systems, which is made possible by the new radioactive beam facilities, has different motivations and aims. Leaving aside the important aspect of providing clues to other branches of science, from the nuclear structure point of view one hopes on one hand, to elucidate the peculiar features of the weakly bound nuclei and on the other, to improve our capability to design nuclear models with a better predictive power (eventually, to solve the longstanding basic puzzles of nuclear physics).

Along this line, there is nowadays a renewed interest in the properties of the nuclear average potential. If it is generally accepted that the mean field approximation is a good starting point to describe the atomic nuclei, many basic questions still wait for an answer, including the properties of the average potential far from the stability line.

The mean field approach can be formulated, *e.g.*, within the density functional framework. The total energy $E = \langle \Phi | H_{\rm eff} | \Phi \rangle$ can be written as a functional of the nucleon density, $E[\varrho]$, by assuming a ground state made up with an anti-symmetrized product of single-particle (or quasi-particle) wave functions, and by using an effective nucleon-nucleon interaction [1]. Once this is done, the ground state can be obtained by finding the absolute minimum of E, the vibrational states can be described as small oscillations

* Presented at the XXXVII Zakopane School of Physics "Trends in Nuclear Physics", Zakopane, Poland, September 3–10, 2002.

around this minimum, and many other nuclear phenomena can find a model description (shape coexistence, large fluctuations induced by nuclear reactions and so on). There are many ways to build the energy functional E. We will not focus in the present contribution on the relativistic functionals. In the non-relativistic framework, there are mainly two classes of effective interactions which are widely used, the Skyrme [2] and Gogny [3] forces.

Even if the basic theory of the mean field calculations is known since many decades, a number of technical and numerical difficulties associated with their implementation are still to be solved. Recently, there has been a renewed interest in this topic.

In particular, we will review in the present contribution the work done along this line by the nuclear theory group of the University of Milano, mentioning the results obtained by other groups as well. One of our main activities has been to study how the mean field results are changed, when additional correlations are included. In particular, much attention has been devoted to the effect — on the single-particle levels or on the giant resonances — of the coupling with the surface fluctuations. Although this activity started many years ago [4], a number of problems related to this topic have not yet been explored and will constitute a part of this contribution.

2. Mean field theory and beyond

2.1. Mean field calculations using Skyrme and Gogny interactions

The Skyrme forces have been extensively used in Hartree–Fock (HF) calculations of the ground state of magic nuclei. The oscillations around the ground state are described by the well-known Random Phase Approximation (RPA), which is the small amplitude limit of the time-dependent extension of HF. One of the obvious limitations of the HF–RPA model is the neglect of pairing correlations. The study of the mean field evolution far from stability obliges us to calculate the open-shell isotopes. Recently, there have been a number of attempts to extend the HF calculations to HF–Bardeen–Cooper–Schrieffer (HF–BCS) or HF–Bogoliubov (HFB) — and correspondingly, the RPA to Quasi-particle RPA (QRPA) [5–10]. We have recently built codes to solve first the HF–BCS equations in coordinate space and then the QRPA equations in the configuration space, by setting the system in a box [11]. The use of the BCS approximation and of the discretized continuum should not be a severe drawback if we do not study extremely weakly bound systems [12]. On the other hand, we have included consistently in our calculations the full (proton–neutron) two quasi-particle interaction.

In most of the extensions of Skyrme HF–RPA, in the pairing channel a zero-range density-dependent force is introduced [13],

$$V = V_0 \left[1 - \left(\frac{\varrho(\vec{r})}{\varrho_0} \right)^{\gamma} \right] \delta(\vec{r} - \vec{r}\,') \,, \tag{1}$$

whose parameter may be fixed with different criteria. In our case, we have kept for simplicity ϱ_0=0.16 fm^{-3} and γ=1 and we have adjusted V_0 to reproduce the empirical ground state pairing gap along an isotope chain. The divergence associated with the zero-range is neither a practical problem [14] nor a serious matter of principle [15]. Some preliminary results of our calculations will be reported in Sec. 3.3.

On the other hand, it is true that the Gogny force has the advantage that it allows treating the particle–hole and particle–particle channel on the same footing without introducing *ad hoc* parameters. Many nuclear properties (ground state energies and radii, deformations *etc.*) have been investigated using the Gogny interaction. On the other hand, until so far we have been lacking a consistent QRPA code using the Gogny force and this has prevented the study of the effect of the Gogny pairing on the low-lying and giant resonance states. Therefore, we have built such a code and performed a detailed investigation of quadrupole and dipole states in the O, Ni and Sn isotopes. We shall briefly mention some of the results, referring to [16] for a complete report of these calculations.

2.2. Particle-vibration coupling

The importance of the coupling between the (mean field) single-particle degrees of freedom and the vibrational states, has been recognized since the early days of nuclear structure physics. Especially the Copenhagen school has emphasized this aspect and constructed a nuclear field theory (NFT), in which the interplay of the single-particle and collective degrees of freedom like vibrations or rotations is the central concept [17]. It must be stressed that other theories are built on similar principles, and we mention in particular the large amount of work done by the Russian school [18].

These theories usually start from a phenomenological mean field (typically from a Woods–Saxon potential). On the other hand, as discussed above, the calculations using effective interactions have recently improved and have reached a high level of reliability. They have the advantage that they can be correlated to basic nuclear matter properties (like the effective mass or the nuclear incompressibility) and, at least qualitatively, they retain a relationship with the bare nucleon–nucleon interaction [19]. In practice, there are many indications that they include the short-range correlations and not the long-range ones. It does make sense, therefore, to apply the particle-vibration coupling theory using effective forces.

This is part of a wide field of research. As far as the single-particle states are concerned, the theory is well established and many calculations have been performed [20]. However, essentially only the nuclei around the ^{208}Pb core have been studied. Since we are now interested in the case of exotic nuclei, probably some attention has be paid to the proper treatment of the neutron and proton degrees of freedom, without making simple approximations based on the isospin quantum number. The expressions of the second-order particle-vibration coupling diagrams in the full neutron–proton scheme can be found in [21]. Results around the ^{132}Sn core will be discussed in Sec. 3.1.

Also in the case of the collective multipole states, models based on the particle-vibration coupling have been applied for many years to the study, *e.g.*, of giant resonances in stable nuclei. In particular, within a Milano–Orsay collaboration, in the last decade we have developed a microscopic model aiming at a detailed description of the giant resonances excitation and decay [22]. Recently we have extended the model to include pairing correlations [7]. Essentially, we start from a HF–BCS plus QRPA calculation of a giant mode, like the isovector dipole, as well as of the other nuclear vibrations. The effective Hamiltonian including the unperturbed quasi-particle states and their residual interaction (written using the Landau prescription), is then diagonalized in a large space which includes not only the two quasi-particle states (this would amount to solve QRPA) but more complicated states made up with two quasi-particles and one low-lying vibration. These complicated configurations give rise to the spreading width of the giant resonance, but also affect the position of the QRPA peaks. A few results concerning the O isotopes are reported in Sec. 3.2.

3. Results

3.1. Coupling of the single-particle states with vibrations: the ^{132}Sn case

As already stressed, there is nowadays much interest in the evolution of the mean field far from stability. There have been claims of peculiar features, namely the disappearance of well known magic numbers and the appearance of a new shell structure in neutron-rich nuclei. In the light isotopes (especially in the Be and O chains) there is some experimental evidence of the breakdown of the N=8 neutron shell closure and of a possible N=16 magic number [23]. In the case of heavier system, measurements are not available. On the other hand, some authors have claimed that in neutron-rich nuclei the shell structure can undergo sizeable changes due to the large surface diffuseness, to pairing effects and to the decrease of the self-consistent spin–orbit potential.

Within the mean-field framework, the rationale for the decrease of the spin–orbit potential $U_{\rm s.o.}$ is the following. If the neutron density extends further away from the nucleus than the proton density, on one hand the gradient of the neutron density is smaller (in absolute value) than in the case of a well bound system, and on another hand the peaks of the two gradients of the neutron and proton density do not coincide. Since the spin–orbit potential is proportional, within the framework of Skyrme or Gogny functionals, to the gradient of the density, it turns out that the two mentioned facts have the effect of quenching $U_{\rm s.o.}$.

One must stress that (a) it is important to check whether this theoretical expectation is actually realized in nuclei which are still bound, and (b) one cannot forget the role played by the particle-vibration coupling. The studies of this coupling around the ^{208}Pb core have shown that particle states just above the Fermi surface are mainly coupled to the particle states above [this coupling is diagrammatically shown in Fig. 1(a); the coupling with the holes, *cf.* Fig. 1(b), is as a rule smaller], and therefore they are pushed downwards; for the holes, the same argument leads to upwards shifts. Thus, the particle-vibration coupling decreases the energy gap and increases the level density near the Fermi surface, or, in other words, the effective mass m^*. One has to investigate whether this pattern is still valid in the case of neutron-rich nuclei.

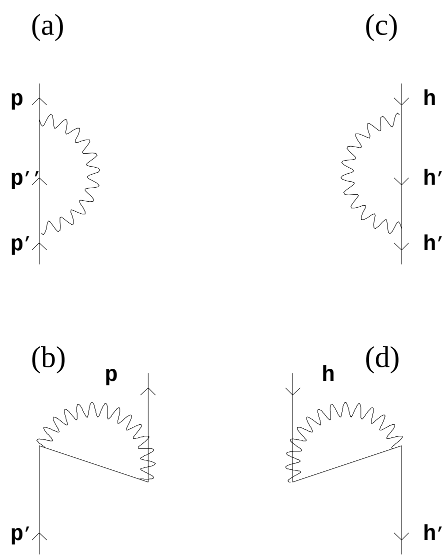

Fig. 1. Diagrams associated with the single-particle [(a) and (b)] or single-hole [(c) and (d)] self-energy coming from the coupling with phonons (wavy lines). The real part of these self-energy terms provides the shift of the particle or hole states.

In any case, the coupling with vibrations *should* be taken into account in a complete theoretical calculation, *before* comparing with the experimental data.

We have performed HF calculations using the Skyrme force SGII [24]. With the same interaction, we have calculated the multipole response associated with the multipolarities 2^+, 3^- and 4^+, and we have calculated the energy shifts of the single-particle levels induced by the phonon coupling. The results are displayed in Fig. 2. One can see that the findings are similar to those described for the ^{208}Pb, and the single-particle shifts are indeed of the same sign and order of magnitude. With some exception, one can claim an overall agreement with the experimental data which confirms the solidity of the dynamical shell-model picture around the neutron-rich ^{132}Sn core. Coming to the spin–orbit splitting, we have obtained for the energy difference between the $1h_{9/2}$ and $1h_{11/2}$ levels a value of 7.44 MeV at the HF level, which is not far from the values obtained for all the stable Sn isotopes. Coupling with the phonons, this value is reduced to 6.78 MeV, in good agreement with the experimental value of 6.75 MeV. One can then conclude that no special adjustments of the theory are needed.

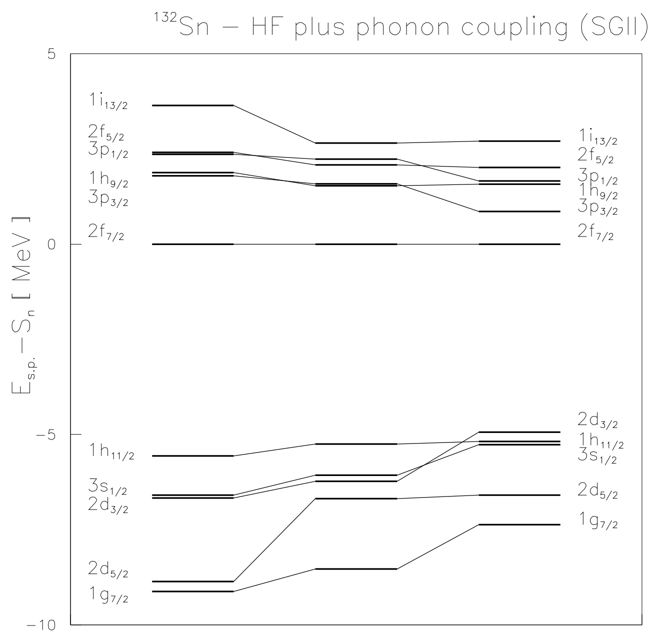

Fig. 2. Results for the single-particle states around the ^{132}Sn core. In the first column are shown the HF results, in the second the results after coupling with phonons, and in the last the experimental data from [25]. The energy scale is fixed on the neutron separation energy of ^{133}Sn.

3.2. Isovector dipole states

The dipole states have the property of being efficiently probed by means of Coulomb excitation experiments at relativistic energies. Recent measurements of this type have been performed at GSI [26], and have shown that the low-lying dipole strength is increasing along the oxygen isotope chain as a function of the neutron number from ^{18}O to ^{22}O. Experiments for the ^{68}Ni and ^{132}Sn nuclei are planned or already under way, and will hopefully provide us with an answer to the question whether there is some systematic evidence of low-lying dipole strength with collective character (to be interpreted as a coherent vibration of the valence neutrons against the other particles), or not.

Also in this context, the role of the correlations beyond mean-field, namely of the particle-vibration coupling, has to be taken into account. In Ref. [7] we have studied the dipole response of the neutron-rich oxygen isotopes, and we have found that the effect of the particle-vibration coupling is similar or even somehow stronger than in the case of well bound nuclei. We show this in Fig. 3, for the nucleus ^{22}O. It has to be remembered that in the case of the giant resonances in stable nuclei, energy shifts of the order of 1 MeV can be expected due to the vibrational couplings.

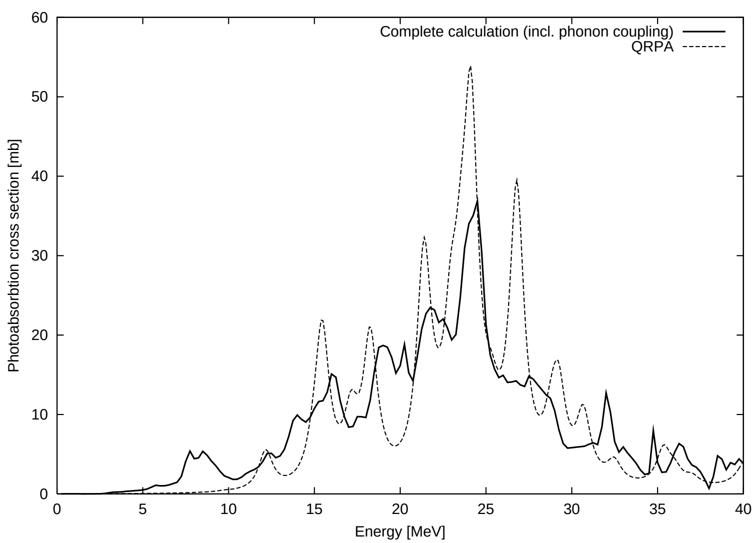

Fig. 3. Effect of particle-vibration coupling on the photoabsorption cross section of the neutron-rich nucleus ^{22}O.

3.3. Low-lying quadrupole and octupole states

A number of experiments which have been done or are going to be done with unstable nuclei, concern the low-lying vibrational states that may be excited through inelastic scattering. The systematics of the 2^+ energies is expected to shed light on basic properties like shell closure or deformation, as it has been the case for the stable isotopes. In this respect, it is necessary to assess the validity of nuclear models on the known nuclei, and to make predictions for the still unknown cases.

We have performed QRPA calculations of the 2^+ and 3^- excitations in the Sn isotopic chain, using either the Gogny interaction, or the Skyrme interactions SIII [2], SGII [24], SLy4 [27] with the force (1) in the particle–particle channel. Typical results are displayed in Fig. 4. The main outcomes are: (a) the energy of the 2^+ is reasonably well reproduced in all the isotopic range, especially we account for its remarkable constancy between ^{120}Sn and ^{130}Sn and for the changes occurring around the shell closure, (b) the B(E2) is systematically underestimated by large factors in the open-shell isotopes, (c) this problem is much less severe as far as the B(E3) is concerned.

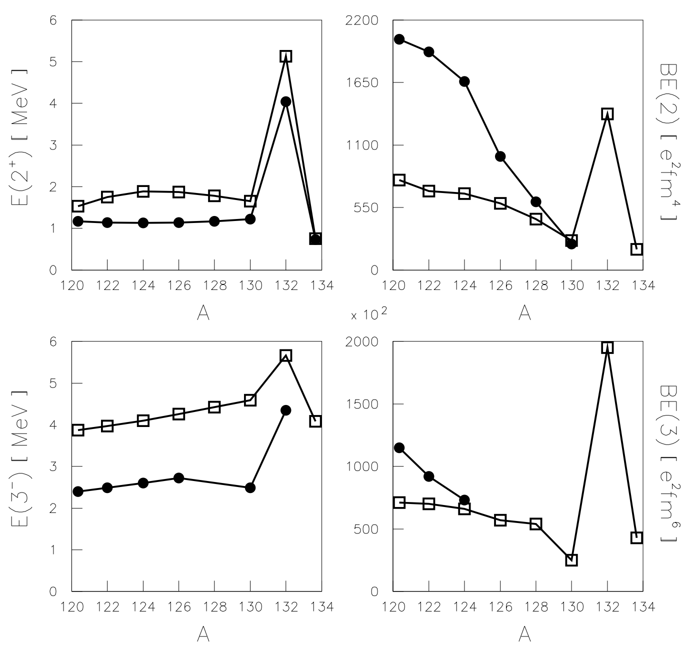

Fig. 4. Results of the QRPA calculations for the even Sn isotopes, performed using the Skyrme interaction SLy4 and the force (1) in the particle–particle channel. The full dots correspond to the experimental data [28] (the values of the B(E2) for the isotopes ^{126,128,130}Sn are preliminary results from [29]), and the empty squares to our results. See the text for a discussion.

We have checked that these qualitative conclusions remain true when we use a different Skyrme parametrization. In the case of the 2^+ we can compare with calculations made using consistently the Gogny force [16], and they give B(E2) values which are similar (or somewhat smaller) than those reported here, and energies which are larger (above 2 MeV). We conclude that effects beyond mean field are crucial to reproduce the transition probabilities of the low-lying quadrupole states in these isotopes [30].

4. Conclusion

In conclusion, we have shown that new self-consistent mean field calculations of the excited states, based on effective forces, are nowadays available and may provide systematic results for the unstable nuclei. The effect of correlations beyond mean field (particle-vibration coupling) may be studied on top of those calculations, and appears crucial in some cases.

For the single-particle states, we have shown, by comparing calculations around the ^{208}Pb and ^{132}Sn core, that their sensitivity to the effective force employed may be around 1 or 2 MeV, while the corrections due to the coupling with the collective vibrations may range from few hundreds of keV to about 1 MeV. It is to be noted that the effect might be much stronger in the light nuclei [21].

For the dipole states, we know from our experience in stable nuclei that the mean field results reproduce as a rule the main location of the strength. Coupling with low-lying vibrations may shift this strength downwards by 1 MeV or less. Calculations for the neutron-rich O isotopes have been completed, while some study of the heavier nuclei is in progress, with the purpose of clarifying the question whether a low-lying collective dipole may exist in these system (and whether it can be important for the calculation of the neutron capture processes).

Finally, the properties of the low-lying 2^+ and 3^- states are expected to be at the forefront of nuclear structure research in the coming years. We have done calculations using our QRPA model, and we expect that inclusion of correlations beyond mean field can help to elucidate the physical problems.

REFERENCES

[1] P. Ring, P. Schuck, *The Nuclear Many-Body Problem*, Springer-Verlag, New York 1980; M. Waroquier, J. Ryckebusch, J. Moreau, K. Heyde, N. Blasi, S.Y. van der Werf, G. Wenes, *Phys. Rep.* **148**, 249 (1987).

[2] M. Beiner, H. Flocard, N. Van Giai, P. Quentin, *Nucl. Phys* **A238**, 23 (1975).

[3] D. Gogny, in: *Nuclear Self-Consistent Fields*, eds G. Ripka, M. Porneuf, North-Holland, Amsterdam 1975, p. 333; J. Dechargé, D. Gogny, *Phys. Rev.* **C21**, 1568 (1980).

[4] G.F. Bertsch, P.F. Bortignon, R.A. Broglia, *Rev. Mod. Phys.* **55**, 287 (1983).
[5] J. Engel, M. Bender, J. Dobaczewski, W. Nazarewicz, R. Surman, *Phys. Rev.* **C60**, 014302 (1999).
[6] E. Khan, N. Van Giai, *Phys. Lett.* **B472**, 253 (2000).
[7] G. Colò and P.F. Bortignon, *Nucl. Phys.* **A696**, 427 (2001).
[8] K. Hagino and H. Sagawa, *Nucl. Phys.* **A695**, 82 (2001).
[9] M. Matsuo, *Nucl. Phys.* **A696**, 371 (2001).
[10] E. Khan, N. Sandulescu, M. Grasso, N. Van Giai, *Phys. Rev.* **C66**, 024309 (2002).
[11] D. Sarchi *et al.*, to be published.
[12] M. Grasso, N. Sandulescu, N. Van Giai, R.J. Liotta, *Phys. Rev.* **C64**, 064321 (2001).
[13] G.F. Bertsch, H. Esbensen, *Ann. Phys. (N.Y.)* **209**, 327 (1991).
[14] J. Dobaczewski, W. Nazarewicz, T.R. Werner, J.F. Berger, C.R. Chinn, J. Dechargé, *Phys. Rev.* **C53**, 2809 (1996).
[15] A. Bulgac, Y. Yu, *Phys. Rev. Lett.* **88**, 042504 (2002).
[16] G. Giambrone *et al.*, to be published.
[17] D.R. Bes *et al.*, *Nucl. Phys.* **A260**, 1, 27, 77 (1976); P.F. Bortignon *et al.*, *Phys. Rep.* **30**, 305 (1977).
[18] V.G. Soloviev, *Theory of atomic nuclei: quasi-particles and phonons*, IOP, Bristol 1992.
[19] J.W. Negele, D. Vautherin, *Phys. Rev.* **C5**, 1472 (1972).
[20] C. Mahaux, P.F. Bortignon, R.A. Broglia, C.H. Dasso, *Phys. Rep.* **120**, 1 (1985).
[21] G. Colò, T. Suzuki, H. Sagawa, *Nucl. Phys.* **A695**, 167 (2001).
[22] G. Colò *et al.*, *Phys. Rev.* **C50**, 1496 (1994).
[23] H. Simon *et al.*, *Phys. Rev. Lett.* **83**, 496 (1999); H. Keller *et al.*, *Z. Phys.* **A348**, 61 (1994); H. Iwasaki *et al.*, *Phys. Lett.* **B491**, 8 (2000); H. Iwasaki *et al.*, *Phys. Lett.* **B481**, 7 (2000); D. Guillemaud-Möller *et al.*, *Nucl. Phys.* **A426**, 37 (1984); T. Motobayashi *et al.*, *Phys. Lett.* **B346**, 9 (1995); A. Ozawa *et al.*, *Phys. Rev. Lett.* **84**, 5493 (2000).
[24] N. Van Giai, H. Sagawa, *Phys. Lett.* **106**, 379 (1981).
[25] H. Grawe, M. Lewitowicz, *Nucl. Phys* **A693**, 116 (2001).
[26] A. Leistenschneider *et al.*, *Phys. Rev. Lett.* **86**, 5442 (2001).
[27] E. Chabanat, P. Bonche, P.Haensel, J. Meyer, R. Schaeffer, *Nucl. Phys.* **A635**, 231 (1998).
[28] S. Raman, C.W. Nestor Jr., P. Tikkanen, *Atom. Data Nucl. Data Tables* **78**, 1 (2001); T. Kibédi, R.H. Spear, *Atom. Data Nucl. Data Tables* **80**, 35 (2002); A. Holt *et al.*, *Nucl. Phys.* **A634**, 41 (1998).
[29] D.C. Radford, private communication; see also D.C. Radford *et al.*, *Phys. Rev. Lett.* **88**, 222501 (2002).
[30] F. Barranco *et al.*, to be published.