Vol. 34 (2003) *ACTA PHYSICA POLONICA B* No 4

NEUTRON SUPERFLUIDITY AND THE COOLING OF NEUTRON STARS*

E. Vigezzi[a], F. Barranco[b], R.A. Broglia[a,c,d]
and P.M. Pizzochero[a,c]

[a]INFN, Sezione di Milano, Milano, Italy
[b]Departamento de Fisica Aplicada III, Escuela Superior de Ingenierios
Sevilla, Spain
[c]Dipartimento di Fisica, Università degli Studi di Milano, Italy
[d]The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

(Received December 16, 2002)

We present a mean field quantum calculation of the superfluidity in the inner crust of neutron stars, taking into account the inhomogeneous character of the system, in which a lattice of neutron-rich nuclei coexists with a gas of unbound neutrons. We compare the resulting thermal properties of the star with those obtained neglecting the nuclear impurities.

PACS numbers: 21.60–n, 26.60.+c, 97.60.Jd

1. Introduction

Neutron stars are frequently referred to as unique laboratories for studying the properties of cold dense matter. Indeed, their radial profile presents densities ranging from zero to several times the value for standard symmetric nuclear matter ($\rho_0 = 0.16$ neutron fm^{-3}, corresponding to 2.4×10^{14}g cm^{-3}). In particular, observations of the thermal (not pulsed) emission from the surface of a neutron star can both give information about the state of matter inside the star and provide constraints on its global structure. The potential of this approach is closely related to the growing database of observed neutron star surface temperatures.

Neutron stars can be schematically described as consisting of four regions: the *core* ($\rho > \rho_{\rm c}$ with $\rho_{\rm c}$ of the order of nuclear matter density, typically $\rho_{\rm c} \sim 0.6\rho_0$), composed of uniform dense neutron matter; the *inner crust* ($\rho_{\rm d} < \rho < \rho_{\rm c}$ with $\rho_{\rm d} = 4 \times 10^{11}$ g cm^{-3}, the neutron drip density)

* Presented at the XXXVII Zakopane School of Physics "Trends in Nuclear Physics", Zakopane, Poland, September 3–10, 2002.

made of a Coulomb lattice of neutron-rich nuclei permeated by a gas of unbound degenerate fermions, namely electrons and neutrons dripped out of the nuclei; the *outer crust* ($\rho_{\rm s} < \rho < \rho_{\rm d}$ with $\rho_{\rm s} \simeq 10^6$ g cm^{-3}), where the lattice of nuclei is permeated by relativistic degenerate electrons; and, extending up to the surface of the star, the *skin* ($0 < \rho < \rho_{\rm s}$), where the electrons permeating the nuclear lattice are non-degenerate. The thin skin has a small heat capacity, so that the surface temperature responds almost instantaneously to temperature variations at the interface $\rho_{\rm s}$.

Recent studies have shown that the high-density core is likely to cool very rapidly by different neutrino emission processes. Therefore, a temperature inversion is formed between the core and the crust: heat flows from the crust to the core. Before the crust's heat reservoir is consumed, the surface temperature is of the order of 10^6 K or more, and the thermal emission can be observed in the X-ray or UV bands. When the cooling wave reaches the surface, its temperature plummets abruptly to values below 5×10^5 K, which are likely to be unobservably low. The diffusion time, that is the time t_w between the formation of the neutron star and the drop of its surface temperature, is expected to depend on the physical conditions in the core and along the crust. It has been argued that the details of the cooling mechanism of the core and the fraction of the core undergoing it have little effect on the diffusion time, so that t_w can be related to the radius and the mass of the star, thus constraining its structure and thence the underlying equation of state of dense matter [1]. However, the relationship one obtains between t_w, R and M depends in a crucial way on the physical properties of the crust, namely its thickness, thermal conductivity and specific heat.

According to calculations performed with various two-body interactions, neutron matter is expected to be superfluid at the densities relevant for the inner crust. This can affect the thermal properties of the crust in an important way, since the specific heat depends exponentially on the pairing gap. For a quantitative study, one must take into account the spatial inhomogeneity inside any given elementary cell of the Coulomb lattice, where a finite nucleus coexists with the gas of unbound neutrons. This is done in the present work, where we shall calculate the pairing gap and the specific heat of the system, performing detailed mean-field quantum calculations which take the interplay of bound and unbound orbitals into account. We shall then estimate the cooling times of neutron stars, comparing our results with those obtained assuming homogeneous neutron matter, that is, neglecting the presence of the nucleus at the center of the cell.

2. Pairing gaps in a Wigner–Seitz cell

One of the most detailed studies of the structure of the inner crust of neutron stars is the Hartree–Fock calculation of Negele and Vautherin [2], who determined the numbers of protons and neutrons which are energetically favored at the different densities. Their studies can be taken as a good starting point to study the superfluidity in the inner crust. Following the work of Ref. [2], we have subdivided the inner crust in ten zones, which correspond to different values of the baryon density ρ, going from the deepest zone, $N_{\text{zone}} = 1$, corresponding to $\rho = 1.3 \times 10^{14}$ g cm^{-3} (or 0.09 neutron fm^{-3}), to N_{zone}=10, corresponding to 4.7×10^{11} g cm^{-3} (or 3×10^{-4} neutron fm^{-3}). Associated to each zone, there is a typical value for the radius R_{WS} of the Wigner–Seitz cell, namely the elementary cell of the Coulomb lattice; the value of R_{WS} decreases going from the surface towards the core. In each cell, part of the neutrons are bound to the nucleus placed at the center of the cell, while the remaining neutrons occupy orbitals at positive energies and their wavefunctions extend throughout the cell. The radial wavefunctions $\phi_{nlj}(R)$ in a given cell are obtained by solving the Schrödinger equation associated with a spherically symmetric Saxon–Woods potential $V(R)$, parametrized in such a way as to reproduce the density profiles calculated in Ref. [2]. We diagonalize the $^1S_0$ component of a two-body interaction in the (generalized) BCS approximation, in a basis composed of pairs of neutrons coupled to zero angular momentum and moving in states with n and n' number of nodes, taking into account the interplay of bound and unbound orbitals [1, 4]. In this way one obtains the pairing gap $\Delta_{nn'lj}$. We shall present results obtained with two interactions: an effective potential, namely, the Gogny force, and a realistic interaction, namely, the Argonne v_{14} [5]. The present study should be considered as a first step towards a more complete investigation, which should take into account also the induced interaction arising from polarization effects in the medium (*cf. e.g.* [6]).

It will be convenient to compare our results with those obtained in uniform neutron matter, where, for a given density ρ or a given Fermi momentum $k_{\text{F}}(\rho) = (3\pi^2\rho)^{1/3}$, the pairing gap $\Delta^{\text{unif}}(k)$ depends only on momentum. In Fig. 1 we show the gap $\Delta_{\text{F}} \equiv \Delta^{\text{unif}}(k = k_{\text{F}})$ calculated in neutron matter, as a function of k_{F}, for the two interactions we have adopted. To be noted that here and in the following we do not introduce an effective mass m^*, but we use its bare value. This is in keeping with the fact that the ratio m^*/m in neutron matter is close to one at the densities relevant for the inner crust of neutron stars.

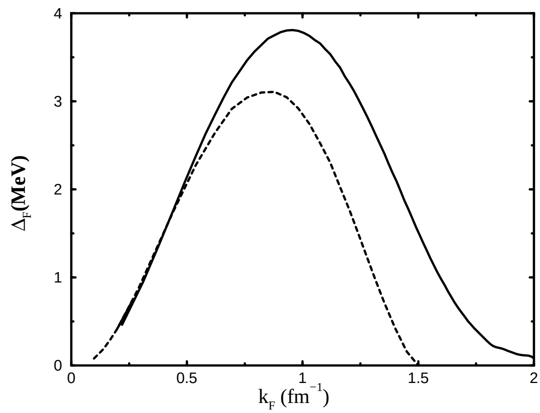

Fig. 1. Pairing gap at the Fermi energy, Δ_F, calculated in neutron matter as a function of Fermi momentum, for the Gogny potential (solid line) and the Argonne potential (dashed line).

We first show some results obtained for a cell ($N_{\rm zone} = 3$) lying deep inside the inner crust, corresponding to a Fermi energy $E_F = 13.5$ MeV. The diagonal elements of the state-dependent pairing gap Δ_{nnlj} calculated with the two interactions are shown in Fig. 2 as a function of the single-particle energies, with and without the nucleus at the center of the Wigner–Seitz cell. It is seen that, in the presence of the nucleus, the pairing gap for levels close to the Fermi energy is lower by a few hundred keV.

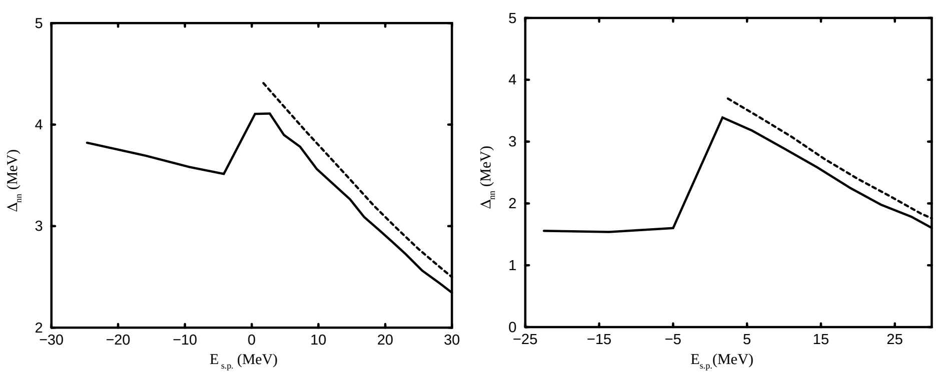

Fig. 2. Left: diagonal matrix elements of the state-dependent pairing gap Δ_{nnlj}, calculated using the Gogny interaction, as a function of the single particle energies with (solid curve) and without (dashed curve) the nucleus at the center of the Wigner–Seitz cell. The gaps are averaged over an interval of 5 MeV. Right: the same, for the Argonne interaction.

This can be understood qualitatively by a local approximation to the quantum results, given by the following integral over the cell:

$$\Delta_{nnlj} \approx \int d^3R\phi^2_{nlj}(R)\Delta^{\mathrm{unif}}[k_{\mathrm{F}}(R)]\,, \tag{1}$$

where $\Delta^{\mathrm{unif}}[k_{\mathrm{F}}(R)]$ denotes the gap calculated in neutron matter (*cf.* Fig. 1) at a Fermi momentum equal to the local Fermi momentum at the point R in the cell: $k_{\mathrm{F}}(R) = \varepsilon_{nlj} - V(R)$. The wavefunction of orbitals nlj close to the Fermi energy are distributed rather uniformly throughout the Wigner–Seitz cell. Inside the nucleus the local Fermi momentum is higher than outside, so that the contribution of this region to the integral (1) is suppressed, the effect being stronger with the Argonne interaction than with the Gogny interaction. Quantitatively, however, one must take into account important proximity effects, which are present in the quantum calculations [3,4].

For large cells ($N_{\mathrm{zone}} > 3$), the main effect of the nucleus, compared to the homogeneous case, is the fact that the neutron Fermi energy is lower in the presence of the nucleus, because part of the neutrons lie in bound orbitals. This change in Fermi energy is especially important at very low densities (large values of N_{zone}).

The specific heat of the neutrons in the inner crust, $C_{V,n}$ is one of the basic quantities which control the thermal behaviour of a neutron star after the early neutrino emission, which is expected to lower the temperature of the crust down to about $T = 0.1$ MeV. The specific heat depends exponentially on the pairing gap, *i.e.* $C_v \propto \exp(-\Delta/T)$ [7]. It is shown in Fig. 3 for the various zones of the inner crust. Its overall behaviour reflects the bell-shape dependence of the pairing gap in neutron matter as a function of density, or Fermi momentum, shown in Fig. 1. The gap reaches its maximum around $k_{\mathrm{F}} \approx 0.9$ fm^{-1}, a density which roughly corresponds to $N_{\mathrm{zone}} = 3$, where the specific heat is minimum (*cf.* Fig. 3). Approaching the surface of the star (for $N_{\mathrm{zone}} > 3$), the specific heat increases, because the Fermi energy becomes smaller and the pairing gap decreases (in the present calculation the phase transition to a normal system takes place for $N_{\mathrm{zone}} = 9$, or $\rho = 7 \times 10^{11}$ g cm^{-3}). The effect of the nucleus is particularly strong in the inner regions, essentially because the radius of the Wigner–Seitz cell is smaller there, and the nucleus occupies a larger fraction of its volume.

3. Calculation of cooling times

The quantity controlling the diffusion of heat is neither the thermal conductivity nor the total specific heat alone, but it is rather their ratio, the thermal diffusivity $D = \kappa/C_V$, which appears in the heat equation. In the case of the inner crust, the thermal conductivity κ is mostly due to electrons

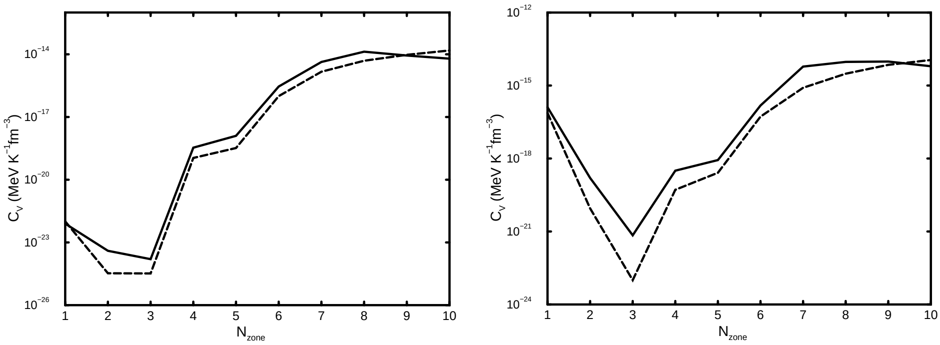

Fig. 3. Left: neutron specific heat for the various zones, calculated at $T = 0.1$ using the Gogny interaction, with (solid curves) and without (dashed curves) the nucleus at the center of the associated Wigner–Seitz cells. Right: the same, for the Argonne interaction.

and therefore it is not affected by the superfluid properties of the neutrons. We take for κ the values reported in [1]. As for the total specific heat of matter C_V, the contributions from the Coulomb lattice and the nuclear protons are negligible when compared to those from the neutrons and the electrons. We thus have $C_V = C_{V,n} + C_{V,e}$, where the neutron contribution is that obtained before for the Wigner–Seitz cells, while the contribution of the relativistic degenerate electrons is calculated with the standard expression. In the inner regions ($N_{\rm zone} \leq 6$), where $C_{V,n}$ is strongly suppressed by the superfluidity (*cf.* Fig. 3), the total specific heat is dominated by $C_{V,e}$, which instead is not affected by the details of neutron superfluidity. This masks the effects due to the presence of the nuclear lattice, obtained in the preceding section. For the outer regions, instead, $C_{V,n}$ plays a more important role and the effect of the nuclear lattice on the thermal diffusivity is appreciable, as can be seen from Fig. 4, where we show D for both non-uniform (with nuclear lattice) and uniform (without nuclei) inner crust matter at $T = 0.1$ MeV. For both interactions, the diffusivity is about two orders of magnitude smaller in the region $\rho_{\rm d} < \rho < 7\,\rho_{\rm d}$ than in the denser parts of the inner crust for both non-uniform (solid line) and uniform (dashed line) neutron matter. Similar results are obtained for temperatures $T = 80$ keV and $T = 120$ keV. Note that the cooling time will depend mostly on the low-density parts of the inner crust, since there the thermal signal will diffuse very slowly. From numerical simulations, these parts are found to cool very quickly to temperatures around $T \approx 0.1$ MeV, due to early neutrino processes. This explains why we have considered as physically relevant to our analysis the temperature range around $T \sim 0.1$ MeV. Our first conclusion is that the bottleneck for the diffusion of heat from the core to the surface is represented by the low-density regions of the inner crust.

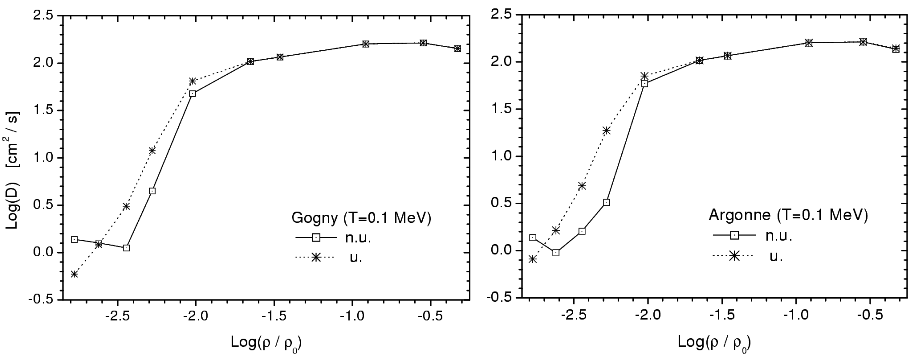

Fig. 4. Thermal diffusivity for inner crust matter calculated at $T = 0.1$ MeV. The solid line represents the case of non-uniform neutron matter with nuclear impurities, while the dashed line is the standard uniform neutron matter. The symbols represent the zones where the calculation has actually been performed.

We shall now estimate the time for heat diffusion along the inner crust, following an initial rapid cooling of the core. In order to do this, one should solve the complete heat equation with the proper temperature and density profiles of the matter encountered by the cooling wave which propagates across the crust from the cold core out to the surface. Several sophisticated computer codes have been developed to that goal, but here we only need a simple yet reasonable estimate in order to assess whether the effect of nuclear impurities is actually relevant to the observations of neutron stars or not. We refer to Ref. [3] for details about the random-walk approach used, and just quote the main results. The diffusion time across an inner crust of thickness $R_{\rm crust}$ is $t_{\rm diff} = \gamma\overline{(1/D)}_{\rho_d} R^2_{\rm crust}$, where γ is a numerical factor of order one, and $\overline{(1/D)}_{\rho_{\rm d}}$ represents an average value of $1/D$ (the thermal "resistance") over the whole inner crust, from $\rho_{\rm c}$ down to $\rho_{\rm d}$. This is the quantity affected by the presence of nuclear impurities, while it does not depend on the EOS of dense matter, *i.e.* on the thickness of the crust or the mass and radius of the neutron star. When multiplied by the proper value of γ and by the square of the inner crust thickness, it yields directly the diffusion time across the crust itself.

In Table I we give results corresponding to the two pairing interactions and to a range of physically relevant temperatures. In addition to the values of $\overline{(1/D)}_{\rho_{\rm d}}$ for non-uniform and uniform neutron matter, we give their percentage difference; this will also represent the percentage difference in diffusion times, $\delta t_{\rm diff} = t_{\rm diff}({\rm n.u.})/t_{\rm diff}({\rm u.}) - 1$, which is the physical quantity we are interested in. From Table I we can draw some important general conclusions:

TABLE I

The values of $\overline{(1/D)}_{\rho_\mathrm{d}}$ for non-uniform (n.u.) and uniform (u.) neutron matter. The columns δt_diff are the percentage differences between the non-uniform and uniform cases, namely $\delta t_\mathrm{diff} = t_\mathrm{diff}(\mathrm{n.u.})/\mathrm{t}_\mathrm{diff}(\mathrm{u.}) - 1$.

T (keV)	Gogny n.u.	Gogny u.	Gogny δt_diff	Argonne n.u.	Argonne u.	Argonne δt_diff
80	0.145	0.096	+51%	0.154	0.062	+148%
90	0.155	0.141	+10%	0.192	0.096	+100%
100	0.190	0.192	–1%	0.204	0.140	+46%
110	0.226	0.248	–9%	0.206	0.192	+7%
120	0.264	0.307	–14%	0.250	0.252	–1%

(i) the effects of the nuclear lattice on the diffusion times are altogether quite significant and therefore cannot be neglected. In particular, with typical cooling times of the order of 10 years [1], our calculations yield time differences between non-uniform and uniform cases up to several years, which are well above observational uncertainties and comparable in magnitude to the cooling times themselves;

(ii) as typical for pairing-related phenomena, the results are quite sensitive to the matter temperature. The general trend is that the percentage difference in diffusion time δt_diff between non-uniform and uniform matter is a decreasing function of temperature. Obviously, a detailed cooling calculation, that goes beyond our simple estimate, is required in order to evaluate the actual diffusion times in realistic models;

(iii) the direction and magnitude of the effect due to the nuclear lattice depend on the pairing interaction used. The Gogny interaction yields values for δt_diff that go from positive to negative with temperature increasing in the range $80 < T < 120$ keV. The Argonne potential, instead, yields values for δt_diff that are mostly positive in the same temperature range and are significantly larger than their Gogny counterparts.

Therefore, the need for further studies, that pin down the correct nucleon–nucleon (renormalized) interaction needed to calculate the pairing properties of the inner crust of neutron stars, appears evident also from the astrophysical point of view.

REFERENCES

[1] J.M. Lattimer, K.A. Van Riper, M. Prakash, M. Prakash, *Astrophys. J.* **425**, 802 (1994).

[2] J. Negele, D. Vautherin, *Nucl. Phys.* **A207**, 298 (1973).

[3] P.M. Pizzochero, F. Barranco, E. Vigezzi, R.A. Broglia, *Astrophys. J.* **569**, 381 (2002).

[4] F. Barranco, R.A. Broglia, H. Esbensen, E. Vigezzi, *Phys. Rev.* **C58**, 1257 (1998).

[5] F. Barranco, R.A. Broglia, H. Esbensen, E. Vigezzi, *Phys. Lett.* **B390**, 13 (1996).

[6] H.-J. Schulze, J. Cugnon, A. Lejeune, M. Baldo, E. Lombardo, *Phys. Lett.* **B375**, 1 (1996).

[7] L.D. Landau, E.M. Lifshitz, *Theory of the Condensed State*, 2nd ed., Pergamon, Oxford 1980.