^{98}Mo AND THE NEUTRON–PROTON MODE IN COLLECTIVE PAIRING VIBRATIONS* **

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The low–lying collective excitations of ^{98}Mo were investigated within microscopic collective Bohr Hamiltonian modified by the coupling with pairing vibrations. Some discrepancies with experimental data concerning the 0_2^+ excitation and the interpretation of this state in terms of isoscalar ($T = 0$) and isovector ($T = 1$) bosons suggest that the collective Hamiltonian should be completed with proton–neutron pairing vibrations.

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The structure of low-lying excitations of ^{98}Mo has been recently investigated in some experiments using the multiply Coulomb excitation [1]. The aim of the present work is to show that in the microscopic description of this isotope one has to look more carefully at possible new collective excitation modes including the proton–neutron pairing degrees of freedom.

The recently developed [2–5] microscopic approach which takes into account the influence of pairing correlations was successfully applied in the wide range of transitional nuclei from neutron-rich Ru to transuranic isotopes. It was obtained from the complete "quadrupole + pairing" Hamiltonian which for a given nucleus with Z protons and A–Z neutrons reads as

$$\begin{aligned}\hat{\mathcal{H}}_{\rm CQP} &= \hat{\mathcal{H}}_{\rm CQ}(\beta,\gamma,\Omega\,;\Delta^p,\Delta^n) + \hat{\mathcal{H}}^{Z}_{\rm CP}(\Delta^p;\,\beta,\gamma) \\ &\quad + \hat{\mathcal{H}}^{A-Z}_{\rm CP}(\Delta^n;\,\beta,\gamma) + \hat{\mathcal{H}}_{\rm int}\,. \end{aligned} \tag{1}$$

The operator (1) acts in the collective space of Bohr deformation parameters β, γ, Euler angles Ω and variables describing the pairing correlations, that is:

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Δ^p and Δ^n — the proton and neutron pairing gap parameters (the projection onto the correct particle numbers eliminates appropriate gauge angles). In our approximation we neglect the term $\hat{\mathcal{H}}_{\rm int}$ mixing the quadrupole and pairing vibrations. Then at each (β,γ) deformation point the eigenproblems of proton $\hat{\mathcal{H}}^Z_{\rm CP}$ and neutron $\hat{\mathcal{H}}^{A-Z}_{\rm CP}$ collective pairing [6] terms are solved and looking at the proton- and the neutron ground-state functions we can find the most probable values of the proton and neutron energy gaps Δ^p_0 and Δ^n_0. Because of the strong anharmonicity of $\hat{\mathcal{H}}^Z_{\rm CP}$ and $\hat{\mathcal{H}}^{A-Z}_{\rm CP}$ the values of Δ^p_0 and Δ^n_0 are shifted towards smaller gaps from the equilibrium points determined by the BCS formalism. We take into account the zero-point pairing vibrations and, therefore, [2, 3, 5] the Hamiltonian (1) is approximated by the operator which takes the form of the usual generalized Bohr Hamiltonian [7, 8]

$$\hat{\mathcal{H}}_{\rm CQ} = \hat{\mathcal{T}}_{\rm vib}(\beta,\gamma;\,\Delta^p_0,\Delta^n_0) + \hat{\mathcal{T}}_{\rm rot}(\beta,\gamma,\Omega;\,\Delta^p_0,\Delta^n_0) + V_{\rm coll}(\beta,\gamma;\,\Delta^p_0,\Delta^n_0)\,. \quad (2)$$

Here the collective potential $V_{\rm coll}$ is corrected by the ground-state energies of $\hat{\mathcal{H}}^Z_{\rm CP}$ and $\hat{\mathcal{H}}^{A-Z}_{\rm CP}$ pairing terms. The effect of zero-point pairing vibrations modifies all inertial functions appearing in vibrational $\hat{\mathcal{T}}_{\rm vib}$ and rotational $\hat{\mathcal{T}}_{\rm rot}$ terms as well as in the collective potential — they are calculated using the most probable values of proton and neutron energy gaps Δ^p_0 and Δ^n_0 instead of the equilibrium (BCS) ones [2]. In this way the average effect of the coupling with pairing vibrations is included into the microscopic description of quadrupole nuclear excitations and no adjustable parameters are introduced. The inertial functions and the potential are obtained using the cranking and the microscopic–macroscopic method with the standard single-particle potential (Seo–Nilsson) and the standard strengths of monopole pairing interaction.

The model sketched above was successfully used in the $A = 100$ nuclear region. However, applying it to low-lying excitations of ^{98}Mo one can find some characteristic disagreements with the experimental data. The comparison of the data obtained with the Hamiltonian (2) and the observed ones is presented in Fig. 1. The experimental data are well reproduced except the 0^+_2 band (marked out by the dashed frame). In the experiment the 0^+_2 excitation is the lowest one and moreover, it corresponds to the prolate shape [1] of the nucleus. Such a shape coexistence does not appear in our calculations (see the right side of Fig. 2). In the same figure (left side) some matrix elements of $E2$ operator (grey points) are compared to the experimental values (black ones) and to the results of a pure phenomenological IBM-1 approximation (contained also in the Ref. [1]) assuming two different kinds of "s-bosons" connected with two possible configurations of valence protons.

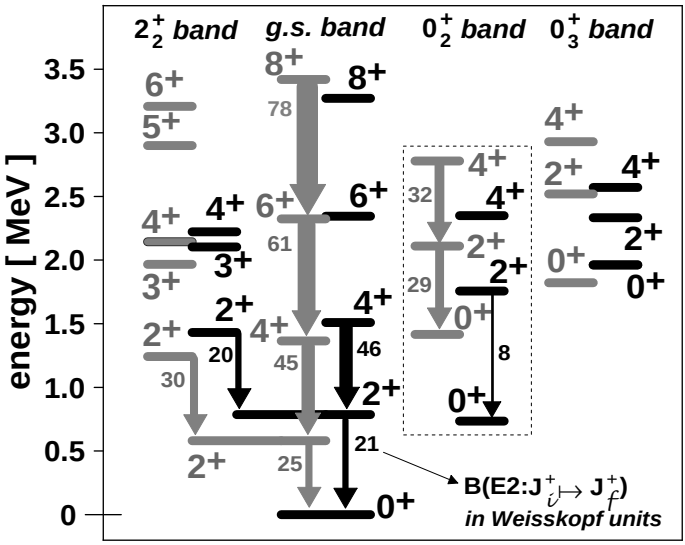

Fig. 1. The experimental (black) [9] and theoretical (grey) band structure of ^{98}Mo isotope.

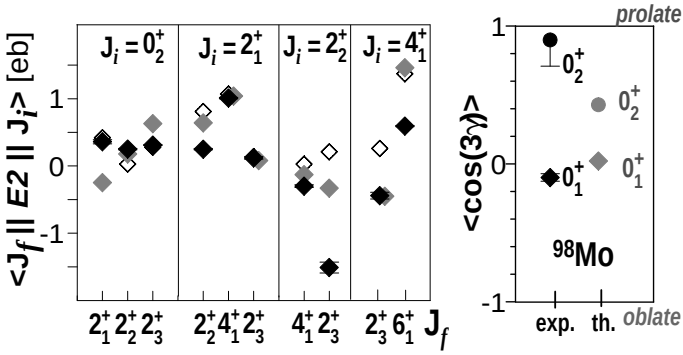

Fig. 2. On the left side: the experimental [1] $E2$ matrix elements (black) and theoretical ones resulting from the presented model (grey) and IBM [1] (empty points). On the right side: experimental [1] (black) and theoretical (grey) triaxiality measure of the ^{98}Mo shape in ground and 0^+_2 states.

In the frame of the shell model of ^{98}Mo the valence neutrons occupy the $g_{9/2}$ subshell while the valence protons — the $g_{7/2}$ one, so the "deuteron-like" pair with the orbital momentum $L = 0$, spin $S = 1$ and the isospin $T = 0$ is easily formed. In order to take into account this possibility a simplified version of the IBM-4 approximation [10] can be applied and we believe that it can point out some hidden symmetries. Let us assume that collective 0^+ and

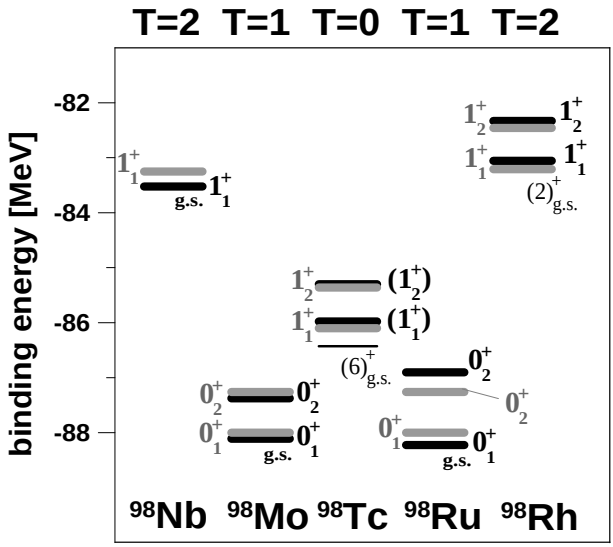

Fig. 3. Experimental [9] (black) binding energies of two first 0^+ and 1^+ states of $A = 98$ isobars compared with values calculated (grey) within isospin boson model (T means the isospin of boson system).

1^+ excitations of an $A = 98$ isobar can be described in terms of the system of $N =$ interacting isoscalar $L = 0$, $S = 1$, $T = 0$ bosons corresponding to the "deuteron-like" nucleon pairs and isovector $L = 0$, $S = 0$, $T = 1$ bosons representing pairs of identical particles. The total number of bosons is connected with the number of nucleons outside the core (including the f–p proton shell).

In so called "vibrational limit" (the numbers of isoscalar and isovector bosons are preserved) the Hamiltonian of the boson system consists of the Casimir operators following the group chain [10]

$$\mathrm{U}(6) \supset \mathrm{U}_S(3) \otimes \mathrm{U}_T(3) \supset \mathrm{SO}_S(3) \otimes \mathrm{SO}_T(3)\,, \tag{3}$$

and the simplest formula describing its eigenenergies reads as

$$E(n, S, T) = un + vS(S+1) + wT(T+1) + E_0\,, \tag{4}$$

where n is the number of isoscalar bosons, S means the total angular momentum (spin) and T the total isospin of the boson system. This formula with parameters $u = 0.370$ MeV, $v = 1.433$ MeV, $w = 0.483$ MeV and $E_0 = -88.966$ MeV reproduces unexpectedly well (Fig. 3) the binding energies of 0^+ and 1^+ levels of $A = 98$ isobars corresponding to the third components of the isospin. The 0_2^+ and 1_2^+ states according to boson description are built from 0_1^+ and 1_1^+ states by changing two isovector bosons

into isoscalar ones. It means that the energy distance between the first and the second 0^+ as well as between the first and the second 1^+ level is due to the coupling of two proton–neutron pairs and, therefore, it should be constant for all $A = 98$ isobars. The experimental data seems to confirm such a statement in spite of discrepancies (^{98}Ru) caused by other (quadrupole) excitation modes.

It is rather hard to believe that the agreement of observed and calculated binding energies presented in Fig. 3 is completely accidental, so it seems that the proton–neutron pairing interaction influences the ^{98}Mo band structure and it should be included in the Hamiltonian (1). In conclusion, the competition between isoscalar and isovector mode of pairing vibrations could play a non negligible role in the description of the collective excitations in different regions of nuclei.

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