

# INTERACTION STRENGTHS FOR THE FOCK-SPACE FORMULATION OF THE NUCLEAR PAIRING PROBLEM\* \*\*

J. DUDEK<sup>a</sup>, K. MAZUREK<sup>a,b</sup> AND B. NERLO-POMORSKA<sup>b</sup>

<sup>a</sup>Institut de Recherches Subatomiques, IN<sub>2</sub>P<sub>3</sub>-CNRS/Université Louis Pasteur  
F-67037 Strasbourg Cedex 2, France

<sup>b</sup>Institute of Physics, University of M. Curie-Skłodowska  
pl. M. Curie-Skłodowskiej 1, 20-031 Lublin, Poland

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A realistic nuclear mean-field Hamiltonian with pairing has been diagonalized using Fock space representation that allows for nearly exact treatment of the problem. Calculations were performed for all the even-even nuclei with  $Z \in (20, 100)$ , whose pairing gaps were possible to extract out of the experimental masses. The optimal values of the pairing strength constants for the protons and neutrons have been found.

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In the large scale microscopic calculations of the nuclear total-energy surfaces the method of Strutinsky plays an important role allowing for fast and fully automatic calculations related to the equilibrium deformations, shape coexistence, fission probabilities and many other mechanisms and phenomena. In the related formalism the calculation of shell- and pairing-energies plays a decisive role, the latter obtained so far with the help of the Bogolyubov transformation and the associated Bardeen-Cooper-Schrieffer approximate method. The new method proposed in Ref. [1] allows to obtain the exact (in some cases nearly exact) solutions of the pairing problem using realistic Hamiltonians — in particular those with the state-dependent pairing Hamiltonian. The new method is based on the direct solution of

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the many-body problem in Fock space; it employs techniques similar to those used in the nuclear shell-model, including the Lanczos diagonalisation scheme. The Hamiltonian in question has the form:

$$\hat{H} = \hat{H}_{\text{mf}} + \sum_{\alpha,\beta} G_{\alpha,\beta} c_{\alpha}^{\dagger} c_{\bar{\alpha}}^{\dagger} c_{\bar{\beta}} c_{\beta}, \quad (1)$$

where  $\hat{H}_{\text{mf}}$  denotes any mean-field Hamiltonian *e.g.* the one with deformed Woods–Saxon potential and matrix  $G_{\alpha,\beta}$ , in general *non-diagonal* in its indices, defines the state-dependent pairing-Hamiltonian. Owing to three exact symmetries obeyed by Hamiltonians of the above general form, *cf.* Ref. [1] for details, the corresponding matrix written down using a Fock space basis can be block-diagonalized analytically, thus reducing the problem of diagonalisation of huge-size matrices to much smaller ones that can be treated easily with the help of the Lanczos methods.

In this article we report on the results of an introductory large-scale test that consists in fitting the coupling constants of the simplest (monopole-pairing) version of pairing Hamiltonian in (1), *i.e.*  $G_{\alpha,\beta} = G \delta_{\alpha,\beta}$ . We do not aim at any extension of the method of Ref. [1] but rather at supplementing it in an important manner: by providing the constants of the Hamiltonian that allow those interested to apply the nearly exact method presented in [1] in the realistic calculations of the total nuclear energies. Pairing calculations were performed within Fock space defined by 24 particles placed on 24 double-degenerate single-particle energy levels in the ‘pairing window’. The final pairing-energy results do not depend very much on such a limiting assumption since the increase/decrease in the pairing constant compensates significantly for an increase/decrease of the basis size. The corresponding full Hamiltonian-matrix has the size of  $N_{\text{Ham}} = 32\,247\,603\,683\,100$ , while the sizes of Hamiltonian-blocks, after applying the formalism of Ref. [1], are:  $N_{s=0} = 2\,704\,156$  in the seniority-zero and  $N_{s=2} = 705\,432$  in seniority-two blocks. In both cases we have applied a basis cut-off reducing the sizes of the effectively diagonalized matrices to  $N'_{s=0} = 27\,703$  and  $N'_{s=2} = 26\,263$ . We have verified by comparison with the results of the exactly soluble method of Richardson that the basis cut-off that reduces the matrix sizes by roughly two orders of magnitude introduces errors of  $\sim 2\%$  only.

As in the BCS-pairing studies by other authors, the fit of the  $G$ -constants is always relative to the energy-window used; however starting from a sufficiently stable global fit-parameters one can often easily rescale the  $G$ -values for a larger energy window. This aspect will not be further discussed here.

We have diagonalized the Fock-space Hamiltonian-matrix corresponding to (1) and we obtained the ground-state energy (seniority-zero) and the lowest-energy seniority-two states. The difference between the two energies

has been interpreted as corresponding approximately to twice the ‘pairing gap’, the latter obtained from the mass-difference expression

$$\Delta_n^{(3)}(N) = \frac{\pi_N}{2}(B(N) + B(N + 2) - 2B(N + 1)), \quad (2)$$

as discussed recently in detail in Ref. [2]. Above,  $\pi_N = (-1)^N$ ,  $B(N)$  are (negative) binding energies of nuclei, for the fixed  $Z$ -number. To obtain the proton ‘pairing gap’ one has to replace  $N$  with  $Z$  and fix the neutron number  $N$ . The resulting proton and neutron pairing gaps for even–even nuclei are illustrated in Fig. 1, top, in function of the mass number  $A$ ; the  $\Delta^{(3)}$ -values with the experimental errors exceeding 250 keV were not taken into account.

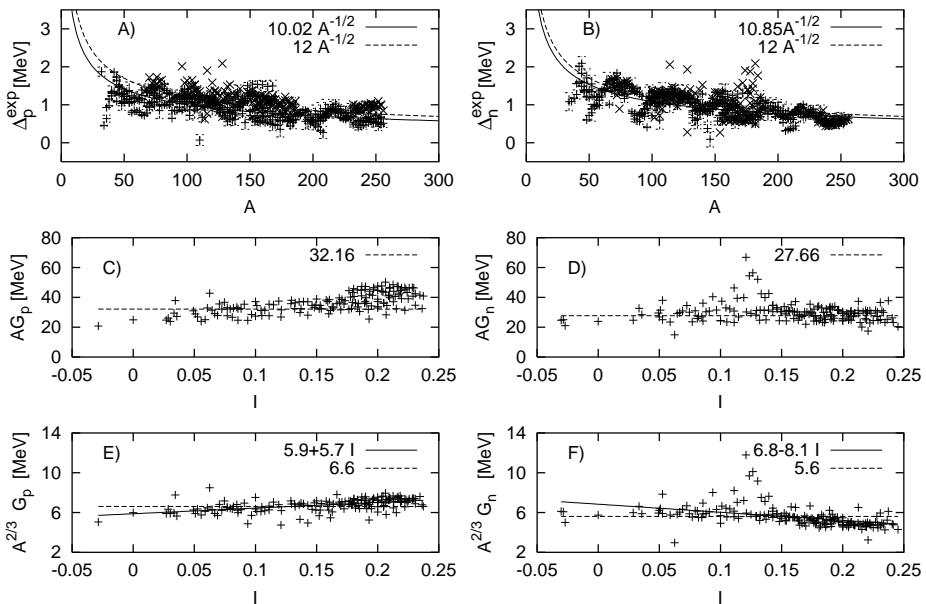


Fig. 1. Experimental  $\Delta_p^{(3)}$ , protons, frame (A), and  $\Delta_n^{(3)}$ , neutrons, frame (B), together with the average-fit curves of the form  $\alpha/\sqrt{A}$  (top). The usually quoted average behavior  $12/\sqrt{A}$  [4] is also illustrated. Frames (C) to (F) represent the results of the pairing constant- $G$  fitting procedure (see the text and Table I). Experimental masses are from Ref. [6].

The best average fit in terms of the  $\alpha/\sqrt{A}$  one-parameter dependence for all the nuclei is  $\Delta_n^{(3)} = 10.85/\sqrt{A}$  and  $\Delta_p^{(3)} = 10.02/\sqrt{A}$  (solid lines in Fig. 1, top). The fitted  $\alpha$ -values are smaller than those obtained in [3]

TABLE I

Pairing strength constants in terms of the approximating expressions for neutrons and protons for 5 regions considered.

$G_{p(n)}$	$a/A + bI$		$(c + dI)/A^{2/3}$		$(c' + d'I + eI^2)/A^{2/3}$		
Region	$a$	$b$	$c$	$d$	$c'$	$d'$	$e$
all <sub>n</sub>	24.872	0.147	6.517	-5.921	5.657	10.96	-64.18
I	24.230	-0.001	5.887	-1.438	5.657	7.43	-65.63
II	27.177	-0.165	6.230	-6.453	5.657	0.67	-21.39
III	33.129	-0.214	7.097	-10.298	5.657	19.60	-152.21
IV	56.170	-0.849	11.457	-33.690	5.657	36.46	-206.93
V	38.548	-0.144	6.315	-6.408	5.657	-0.29	-13.35
all <sub>p</sub>	26.861	0.331	7.100	-2.905	6.529	2.65	-5.51
I	24.548	0.018	5.934	4.493	6.529	-9.13	103.58
II	45.893	-0.750	7.249	-2.892	6.529	1.74	-3.91
III	39.296	-0.481	7.178	-3.700	6.529	4.55	-7.60
IV	54.546	-0.608	7.289	-3.327	6.529	1.88	-4.47
V	78.763	-0.635	8.333	-4.289	6.529	6.18	-8.67

from the liquid drop model formula, the corresponding curves denoted in Fig. 1 with the dashed lines. One can also use alternative one-parameter expressions similar to those in Ref. [5]; the results of the fit are

$$\Delta_n = 23/(\sqrt{A})^3 \quad \text{and} \quad \Delta_p = 21/(\sqrt{A})^3, \quad (3)$$

or

$$\Delta_n = 4.66/A^{1/3} \quad \text{and} \quad \Delta_p = 4.18/A^{1/3}, \quad (4)$$

but the fit precision remains similar to that with the  $\alpha/\sqrt{A}$ -type dependence.

To avoid the undesired type of variation in pairing delta, characteristic of spherical (especially doubly magic) nuclei, the nuclei with available experimental masses have been arbitrarily divided into 5 regions: (I) with  $Z \in (32, 38)$  and  $N \in (32, 44)$ ; (II) with  $Z \in (40, 46)$  and  $N \in (56, 72)$ ; (III) with  $Z \in (54, 66)$  and  $N \in (58, 76)$ ; (IV) with  $Z \in (60, 82)$  and  $N \in (88, 104)$  and (V) with  $Z \in (90, 100)$  and  $N \in (142, 156)$ ; there the doubly-magic nuclei have been eliminated. Within those regions the  $\Delta$ -values were extracted and, as the next step, the  $G$ -constants found that reproduce the extracted  $\Delta$  values exactly for each of the studied nuclei at the calculated in advance equilibrium deformations. The mean field Hamiltonian used is the same as in Ref. [7] (see also references there). The irregular behavior of  $\Delta^{(3)}$  in function of  $A$  suggests that the resulting  $G$  values will also vary in a relatively irregular fashion and as a consequence we have tried 'a few parameter' fits in terms of  $A$  and  $I \equiv (N - Z)/(N + Z)$  and possibly some powers of those

variables:

$$G_{p(n)} = \frac{\rho_{p(n)}}{A^s} (\rho_0 + \rho_1 I + \rho_2 I^2) \quad \text{where } s = 1, 1/2, 1/3, 2/3 . \quad (5)$$

The isospin dependence of pairing strength  $G$  was tested in various regions of nuclei; it was found that the coefficients in front of  $I^2$  may vary as one can see in Table I, from one fit to another, signifying indeed only a secondary relevance of such a term. The  $s=1/2, 1/3$  values were not better than the values  $s=1, 2/3$ ; all four have been introduced for the sake of numerical tests rather than to model some theoretical result.

In Fig. 1, frames (C) and (D), the products  $G_{p(n)} \cdot A$  are shown in function of the isospin factor  $I$  while the corresponding average behavior is shown with the help of the solid lines. In frames (E) and (F) the quantities  $G_{p(n)} \cdot A^{2/3}$  are illustrated. The dependence in terms of  $I$  obtained here is nearly constant.

Results in Table I give the overall r.m.s. deviations that are rather small. The variations obtained, based on the experimental data concerning a broad range of nuclei, show more structure than the simple parametrisations tested can take care of. In particular, strong variations in parameter  $e$  from one nuclear range to another, *cf.* column 8 in the Table, indicates that the  $I^2$  fluctuations are too rapid to allow deducing any systematic trends in this context. In other words, the fits show no good reason to search for the optimisation in terms of  $I^2$ -dependence: there seems to be no such type of a correlation in the experimental data.

The parametrisations summarized in the Table are ‘ready to use’ in conjunction with the Fock-space diagonalisation method of Ref. [1].

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