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RESIDUAL INTERACTIONS AND HIGH SPIN STATES IN $A = 211$ ISOBARS*

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The shell model has been particularly successful in describing the structure of states near the doubly magic ^{208}Pb core. In particular semi-empirical calculations are able to reproduce the yrast states in the three valence proton nucleus, ^{211}At to a high degree of accuracy. Recent spectroscopic measurements using a variety of novel reaction mechanisms have provided information on the related $A = 211$ nuclei ^{211}At, ^{211}Po ^{211}Bi and ^{211}Pb allowing a comparison between the data and the results of semi-empirical calculations. While good agreement is observed for the lower spin yrast states, the calculations predict the presence of isomeric states not yet completely characterized in the current experiments.

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1. Introduction

The nuclei around the doubly magic ^{208}Pb core provide some of the best examples of cases where the shell model can be used to explain the structure of yrast states formed by the excitation of several nucleons. A detailed description of the highest spin states known in the region, in the radon and francium isotopes, can be provided by the semi-empirical shell model [1, 2]. These nuclei explore the valence proton, neutron and neutron-hole model space, since the high spin states involve excitation across the neutron shell gap. Much is also known about the nuclei with valence proton holes and neutron holes, however a region not greatly explored is that which involves only valence protons and neutrons added to ^{208}Pb, primarily because of

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the difficulty in producing these nuclei with conventional heavy-ion fusion evaporation reactions using stable beams and targets. Results from recent spectroscopic measurements in progress [3–6], which use a variety of alternative techniques are now beginning to provide information on the nuclei in this neutron rich corner. Here we report some of the latest results, focusing on the behaviour of the $A = 211$ isobars and we demonstrate the success of the semi-empirical shell model in explaining the structure of the yrast states.

2. Semi-empirical calculations

A variety of shell model approaches has been applied to nuclei in the lead region. The most enduring of these has been the use of a realistic interaction from the work of Kuo and Herling [7] and subsequent modifications of this by Warburton and Brown [8]. Other approaches have been to use phenomenological interactions, such as the surface delta interaction by Zwarts and Glaudemanns [9] and more recent work by Alexa *et al.* [10]. Current calculations also include the work of the Napoli group [11] and Caurier *et al.* [12]. The disadvantage of most of these calculations is that the restricted model spaces considered do not readily allow the determination of the structure of states which involve excitation of both protons and neutrons. Additionally, few of the calculations address the many-particle octupole vibrational coupling which plays such a dominant role in determining the yrast structure in this region. Consequently the empirical approach, as pioneered by Blomqvist [13] and co-workers, has become perhaps the most successful way of calculating the properties of the high spin yrast states in the region.

For the calculation of the yrast states within the empirical model, a suitable truncation is to consider only the lowest three active orbits in both the proton and neutron spaces, namely $h_{9/2}$, $i_{13/2}$, and $f_{7/2}$ for the protons; and $g_{9/2}$, $i_{11/2}$, $j_{15/2}$ for the neutrons. The dominant neutron hole orbits, $p^{-1}_{1/2}$ and $f^{-1}_{5/2}$ must also be included in the calculation of most high spin states as they are activated through core excitation. Calculations for nuclei on the neutron deficient side of the line of stability must also include the neutron $i^{-1}_{13/2}$ and $p^{-1}_{3/2}$ orbits.

A large number of the residual two body interactions required for this model space can be derived directly from the states known in the two valence nucleon systems about the ^{208}Pb core. For example, the two proton interaction energies can be determined from ^{210}Po, the two neutron interactions from ^{210}Pb and the proton-neutron interactions from ^{210}Bi. These interactions have been tabulated by Lönnroth [14] with a more up-to-date list given in our own work [15, 16]. Where interaction energies are not known we have tended to take the values given by Kuo and Herling [7]. Further discussion on residual interactions is found below in the context of the calculations for ^{211}Bi. It should be emphasized that while our calculations do take into account the mixing between states that can occur when more than one state of the same spin and parity is able to be formed by the same configuration, mixing between states of different configurations is not included explicitly. To some degree this mixing may often be indirectly included via the use of empirical interactions which are themselves derived from states with mixed configurations.

3. The three valence proton system, ^{211}At

Excited states in ^{211}At have been produced using the ^{208}Pb(^{7}Li,4n) reaction with beams provided by the ANU 14UD Pelletron accelerator. This reaction is highly selective and of high yield, making the study relatively straight forward. Indeed, the early work of Bergström *et al.* [18, 19] and Maier *et al.* [20] on ^{211}At were among some of the defining experiments in the region. Our work has extensively expanded the original level scheme and details of the measurements and results can be found in the papers by Bayer *et al.* [16, 17]

A comparison of the calculated states with the observed yrast states is shown in Fig. 1. This figure includes the complete set of valence states for spins $J \geq \frac{9}{2}$ available using the $h_{9/2}$, $i_{13/2}$, and $f_{7/2}$ orbits (open and closed circles). For core-excited states (triangles), at most the lowest three states for any given spin in a configuration are shown. For the yrast states formed only from the three valence protons the agreement between experiment and theory is exceptional with the largest deviations being for the ground state (33 keV) the $27/2^-$ state (33 keV) and the $29/2^+$ state (27 keV). The deviation in the ground state can be understood since the model does not include mixing between the other seniority one, $9/2^-$ states while the other two states are affected by octupole correlations [21]. The agreement for states involving core excitation is also very good, with most states observed within 100 keV of the calculated value. One notable exception is the $39/2^-$ state from the $\pi[h^2_{9/2}i_{13/2}]\nu g_{9/2}p^{-1}_{1/2}$configuration which in the simple empirical model is calculated 142 keV too low in excitation energy. Again, like the $29/2^+$ state, this is strongly affected by octupole mixing and a more sophisticated approach is needed to reproduce its energy. These states can be understood in terms of a model which includes a specific coupling to the octupole vibration. This model can be used to make definite predictions about the structure of mixed states and about possible changes in the nature of the microscopic structure of the octupole vibration itself [21]. We have recently extended the work on the systematics of the octupole mixed states in the astatine isotopes with a measurement of states in ^{213}At [6]. The result from that work bears out the predicted reduction in E3 transition strength which occurs in the decay of specific configurations as one moves to more neutron rich systems.

The success of the semi-empirical model is not restricted to states formed only from the three valence protons. The structure of the high spin, core-excited states in ^{211}At above the previously known isomers can also be successfully understood in terms of relatively simple configurations. The approach can also be used to highlight specific deviations from the simple model, leading Bayer *et al.* [16] to examine the need for core-polarization effects in reproducing state energies.

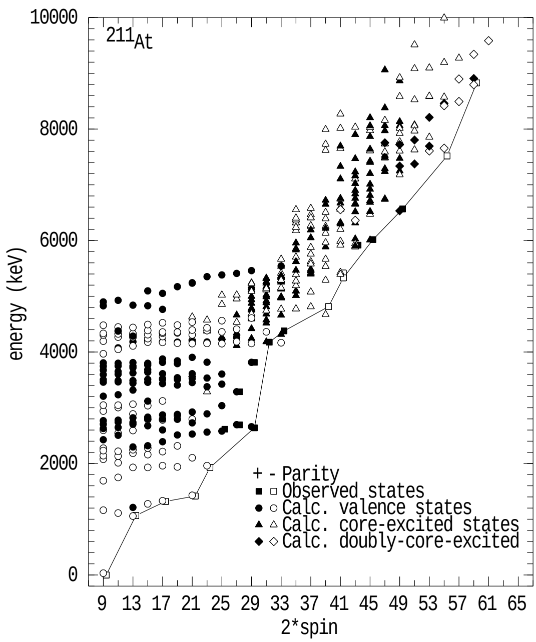

Fig. 1. Empirical shell model calculations for ^{211}At. The filled and unfilled symbols show the calculated energy levels for states of positive and negative parity respectively. From [16].

4. The two proton–one neutron system, ^{211}Po

While ^{211}At is readily produced using heavy-ion, neutron evaporation reactions, no stable beam and target combination is able to produce ^{211}Po. Instead, states in ^{211}Po have been populated using both deep inelastic reactions in the work of Fornal *et al.* [22] and incomplete fusion/breakup reactions [23]. The independent results from the two groups are in good accord, with both identifying a high spin isomer at an excitation energy of 4874 keV, although the incomplete fusion reactions allow the identification of considerably more non-yrast states. Again, the empirical shell model calculations, shown in Table I, reproduce the observed yrast decay scheme rather well. The highest lying isomer is attributed to the $43/2^+$ state from

TABLE I

Configurations assigned to valence states in ^{211}Po (From [23].)

Configuration	J^π	Energy	Theory	Δ
$\pi[h^2_{9/2}]\nu g_{9/2}$	$9/2^+$	0	48	48
	$13/2^+$	1181	1263	82
	$15/2^+$	1459	1440	−19
	$17/2^+$	1428	1477	49
	$21/2^+$	1428+Δ	1516	<88
	$23/2^+$		1649	
	$25/2^+$	1463	1530	67
$\pi[h^2_{9/2}]\nu i_{11/2}$	$11/2^+$	687	587	100
	$27/2^+$	1820	1837	13
$\pi[h^2_{9/2}]\nu j_{15/2}$	$15/2^-$	1065	1079	14
$\pi[h_{9/2}i_{13/2}]\nu g_{9/2}$	$31/2^-$	2136	2105	−31
$\pi[h_{9/2}i_{13/2}]\nu i_{11/2}$	$33/2^-$	2867	2850	−17
$\pi[h_{9/2}i_{13/2}]\nu j_{15/2}$	$37/2^+$	3443	3408	35
$\pi[h^2_{9/2}]\nu g_{9/2}i_{11/2}p^{-1}_{1/2}$	$37/2^-$	4365	4492	127
$\pi[\mathrm{h}_{9/2}\mathrm{i}_{13/2}]\nu\mathrm{g}_{9/2}\mathrm{i}_{11/2}\mathrm{p}^{-1}_{1/2}$	$43/2^+$	4874	4802	−72

the $\pi[h_{9/2}i_{13/2}]\nu g_{9/2}i_{11/2}p^{-1}_{1/2}$ configuration. It decays via an enhanced E3 transition with a strength of 17(5) single particle units to the $37/2^-$ state from the $\pi[h^2_{9/2}]\nu g_{9/2}i_{11/2}p^{-1}_{1/2}$ configuration. While most enhanced E3 decays in the astatine isotopes are well understood within the framework of the particle-vibration coupling model, this particular decay, and the corresponding decay from the $31/2^-$ state, involve spin flip $\pi i_{13/2} \rightarrow \pi h_{9/2}$ transitions and are predicted to have strengths less than half their observed values [23].

Further insight into the anomalous nature of these decays may be obtained by studying states with related configurations in ^{212}Po. To date, population of this nucleus has been difficult with only states up to the α-decaying isomer at 2.9 MeV being observed [24]. One option now available is the use of neutron rich radioactive beams such as those at the SPIRAL facility [25] at GANIL. In April 2002, the first approved experiment was performed with the EXOGAM array [26] using a radioactive beam from SPIRAL [3]. The ^{208}Pb(^{8}He,4n) reaction was used with radioactive ^{8}He produced through fragmentation of a 75 MeV/A ^{13}C beam on a carbon target. After extraction, the ^{8}He beam was re-accelerated to 28 MeV and bombarded a 30 mg/cm^2 ^{208}Pb target. Over the course of the measurement, approximately 58 hours of beam was delivered on target with an average beam intensity in the range 3×10^4–3×10^5 ions per second. The transitions from levels up to the 11^- state in ^{212}Po were clearly visible [3] and the measurement represents a significant milestone, opening the door to the exploration of a number of different neutron rich systems in this region.

5. The one proton-two neutron system, ^{211}Bi

Until recently only a few low-lying states from the $\pi h_{9/2}\nu g_{9/2}^2$ configuration were known in ^{211}Bi. States up to the $[\pi h_{9/2}\nu g_{9/2}^2]_{21/2^-}$ isomeric level at 1227 keV (τ=101ns) were identified by Maier *et al.* [29] using the ^{209}Bi(t,p) reaction and γ-ray spectroscopy. A level at 1257(10) keV was observed using a magnetic spectrometer and the ^{210}Bi^m(d,p) reaction [28] and tentatively assigned as the $25/2^-$ state from the maximum spin coupling of the $\pi h_{9/2}\nu g_{9/2}^2$ configuration. An isomer with $\tau = 2.1$ μs was observed by Pfützner *et al.* [30] in a relativistic fragmentation experiment but they could not determine whether the lifetime was associated with the $[\pi h_{9/2}\nu g_{9/2}^2]_{25/2^-}$ or $[\pi h_{9/2}\nu g_{9/2} i_{11/2}]_{29/2^-}$ configuration.

An experiment has been performed [4, 5] using the Gammasphere array at the Argonne National Laboratory to study excited states in neutron-rich isotopes near and above ^{208}Pb populated in deep-inelastic collisions. Details of the experiment have been presented in Refs [4,5]. Excited states in ^{211}Bi were identified by projecting a matrix of all pairs of γ-rays which preceded in time, the known γ-rays below the 101 ns and 2.1 μs isomers. A preliminary level scheme from that work is shown in the right-hand side of Fig. 2. The left-hand side shows the results of empirical shell model calculations using the same two-body interactions as used in the calculation of the ^{211}At and ^{211}Po level schemes discussed earlier. Above the 4835 keV state a fragmentation of the level scheme is observed, indicating a change in structure beyond this point. It is likely that the state at 4835 keV corresponds to the maximum spin possible from the three valence nucleons, the calculated excitation energy of the $41/2^+$ state of the $[\pi i_{13/2}\nu j_{15/2}^2]$ configuration, 4865 keV, being in excellent agreement with the observed energy. States of higher angular momentum must be generated by core excitation. Unfortunately it is not clear from the current data whether the transitions observed feeding this level are from yrast states. Indeed, the paucity of firm spectroscopic information for the transitions deexciting this level allows several alternative interpretations of the level scheme. If the empirical shell model calculations

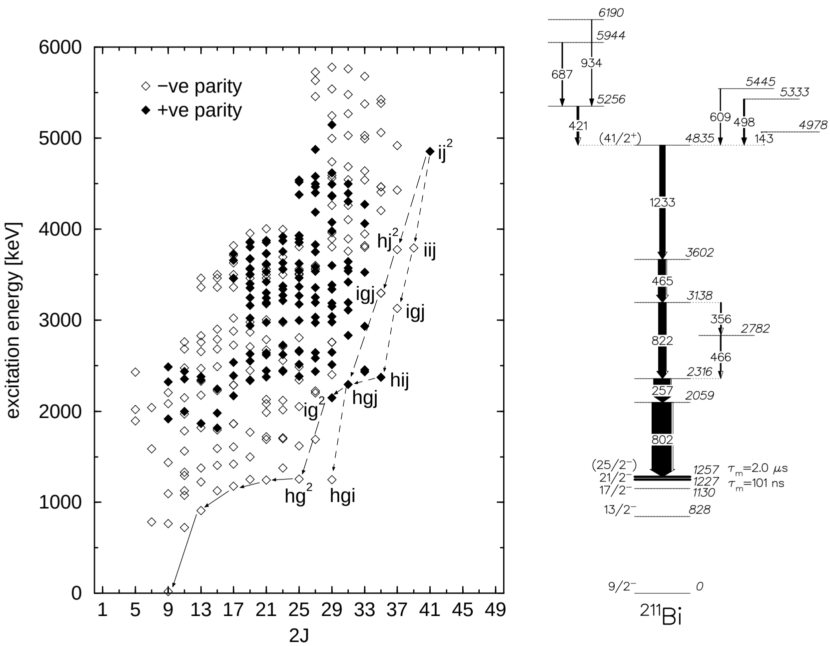

Fig. 2. Empirical shell model calculations for ^{211}Bi. The filled and unfilled symbols show the calculated energy levels for states of positive and negative parity respectively. From [6].

are to be believed, (and support for this is given by the success of the calculations for ^{211}At and ^{211}Po discussed above), the yrast cascade would occur through the states listed as Alternative 1 in Table II. The difficulty with this interpretation is the lack of a direct E3 decay from the state at 2316 keV to that at 1257 keV. Alternative 1 suggests an E2 transition deexciting the 2316 keV state which will be faster, but it will have a decay width only a little over twice that of the E3 transition. Also there is no clear candidate for the branch which includes the state at 2782 keV. An alternative pathway (Alternative 2) takes the decay through non-yrast states and directly feeds the $25/2^-$ state. While at first sight this may seem unlikely, the calculations predict that the $29/2^-$ state will fall below the $25/2^-$ state resulting in a state which is likely to α-decay in a manner similar to the related states in ^{212}Po and ^{211}Po. The present way of identifying states as belonging to ^{211}Bi by using time correlated coincidences associated with transitions below the 2 μs isomer may have selected a non-yrast sequence, bypassing a possible very long lived isomeric state. Maier *et al.* searched specifically for an α-decaying isomer in their ^{209}Bi$(t,p)^{211}$Bi measurement [29] but did not observe any α-decay with a half-life between 1 s and 10 hr, with greater than

0.5% of the ground state intensity. The lifetime measurement of Pfützner *et al.* [30] could not assign the measured 2.1 μs lifetime to a particular state. However, assuming that the states arise from the same multiplet, the observed $B(\mathrm{E2}, 21/2^- \rightarrow 17/2^-)$ transition strength can be used to obtain the transition strength for the $25/2^- \rightarrow 21/2^-$ decay. Assuming a transition energy of 30(10) keV this gives a lifetime for the $25/2^-$ state of about 150 ns, a value relatively constant for any transition energy between 80 and 22 keV. A 2.1 μs lifetime is consistent with a $25/2^- \rightarrow 21/2^-$ decay only if the energy of this transition is between 2.5 and 4.5 keV. Higher transition energies imply that the 2.1 μs lifetime must be attributed to another state, most likely the $29/2^-$ state.

TABLE II

Alternative Configurations assigned to states in ^{211}Bi

State (keV)	Alternative 1 Configuration	J^π	Theory (keV)	Δ (keV)	Alternative 2 Configuration	J^π	Theory (keV)	Δ (keV)
4835	$\pi[i_{13/2}]\nu j_{15/2}^2$	$41/2^+$	4856	35	$\pi[i_{13/2}]\nu j_{15/2}^2$	$41/2^+$	4856	35
3602	$\pi[i_{13/2}]\nu i_{11/2} j_{15/2}$	$39/2^-$	3794	192	$\pi[h_{9/2}]\nu j_{15/2}^2$	$37/2^-$	3778	176
3138	$\pi[i_{13/2}]\nu g_{9/2} j_{15/2}$	$37/2^-$	3129	-11	$\pi[i_{13/2}]\nu g_{9/2} j_{15/2}$	$35/2^-$	3298	160
2782	?				$\pi[h_{9/2}]\nu i_{11/2} j_{15/2}$	$33/2^+$	2934	152
2316	$\pi[h_{9/2}]\nu i_{11/2} j_{15/2}$	$35/2^+$	2374	58	$\pi[h_{9/2}]\nu g_{9/2} j_{15/2}$	$31/2^+$	2293	-23
2059	$\pi[h_{9/2}]\nu g_{9/2} j_{15/2}$	$31/2^+$	2293	234	$\pi[i_{13/2}]\nu g_{9/2}^2$	$29/2^+$	2147	88
1257+Δ	$\pi[h_{9/2}]\nu g_{9/2} i_{11/2}$	$29/2^-$	1247					
1257					$\pi[h_{9/2}]\nu g_{9/2}^2$	$25/2^-$	1257	0

Of key importance are the energies. The states involved have very simple configurations, and as indicated with the calculations for ^{211}At and ^{211}Po described above, the energies can be calculated to a high degree of accuracy. All of the states in ^{211}Po with configurations related to the states in ^{211}Bi can be calculated with an accuracy of better than 100 keV as shown in Table I. In ^{211}Bi the same interactions are involved yet the simplest interpretation incorrectly calculates the energy of the $\pi[h_{9/2}]\nu g_{9/2} j_{15/2}$ state by over 234 keV. Also of importance is whether the $29/2^-$ state actually falls below the $25/2^-$. Again, all the residual interactions are well determined empirically from states in ^{210}Pb and ^{210}Bi. The main reason the states are close in energy at all, is that the observed residual interaction for $[\pi h_{9/2} \nu i_{11/2}]_{10^-}$ coupling is abnormally attractive compared to the Kuo and Herling interactions [7]. It is highly unlikely that the 10^- state in ^{210}Bi has been mis-assigned and Bergström *et al.* [27] have pointed out that the discrepancy occurs because of the large spatial overlap of the two orbits involved the calculation of which is sensitive to the form of potential chosen. Similar behaviour is also observed for the spin orbit partners $[\pi h^{-1}_{11/2}\nu i^{-1}_{13/2}]_{12^-}$ in ^{206}Tl. Of interest is whether modern realistic interactions are able to reproduce these energies correctly.

From an experimental point of view what is needed is better time information. The alternative decay paths above the known isomers predict different nanosecond isomers due to either E3 or M2 transitions. A more extensive measurement of the 2.1 μs isomer is also required to establish whether it arises from either of the $25/2^-$ or $29/2^-$ states.

6. The three neutron system ^{211}Pb

Low spin states in ^{211}Pb are known from the alpha decay of ^{215}At [31]. High spin states have been populated in the same deep inelastic measurement described above for ^{211}Bi [4, 5]. The decay scheme deduced from that work has identified levels up to an excitation energy of 4400 keV, with two isomeric states present. For ^{211}Pb the residual interactions within the $\nu g^2_{9/2} i_{11/2}$ configuration predict an inversion of the ordering of the highest spin levels (in contrast to the $\nu h^2_{9/2} f_{7/2}$ configuration in ^{211}At). Again, as in ^{211}Bi, the inability to associate the lifetimes to particular states prevents definite assignment to states above this point.

7. Conclusions

The systems with a few valence nucleons outside the lead core provide a rich testing ground for the nuclear shell model. New measurements, using exotic reactions and powerful arrays are beginning to unravel the structure of the neutron rich systems. Empirical shell calculations predict isomeric states including some which are expected to decay by alpha emission. The very simple structure of the states allows a detailed examination of the residual interactions used in the calculations, with the measurements hinting that changes from the set of interactions currently used may be required.

The results of several experiments have been used here. Our thanks go to a large number of colleagues and, in particular, S. Bayer, T. McGoram, T. Kibédi and A.M. Baxter and staff of the Heavy-Ion Facility at the Australian National University, P. Walker and H. Mach as spokespersons for the SPIRAL experiment, and R. Broda and B. Fornal as co-spokesperson and principal collaborator for the deep-inelastic experiment at Argonne National Laboratory. The authors acknowledge assistance from the Australian Government, Access to Major Research Facilities Program for their support.

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