BINDING ENERGY FOR NUCLEAR SUPERMULTIPLETS IN LIGHT NUCLEI* **

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A simple three-parameter supersymmetric mass formula has been applied to binding energy calculations of nuclei in the s–d shell grouped into supermultiplets. The earlier suggestion of instability of ^{26}O has been confirmed and binding energies of other exotic nuclei have been predicted.

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1. Introduction

According to the Interacting Boson Model [1] (IBM) valence nucleons of nuclei are coupled to pairs with $L = 0$ and $L = 2$ to form approximate bosons. Besides, there are one or two unpaired nucleons left which, together with bosons, form a boson–fermion system with an assumed supersymmetry. The unitary–unitary supersymmetry group, $U(m|n)$, has been applied, where $m(n)$ is a number of single particle states in the boson (fermion) space.

Details of our model and of adopted notation have been given in [2, 3]. Ref. [3] contains also first results in evaluations of binding energies of even–even nuclei belonging to a given supermultiplet N, where N is a total number of valence bosons and fermions. For even–even nuclei $N = 1/2(A - 16)$, where A is an atomic number and ^{16}O is a core.

We will exploit, in what follows, the fundamental property of the supersymmetric model in which even–even as well as even–odd and odd–odd nuclei belong to the same supermultiplet with the same supersymmetric parameters.

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2. The supersymmetric model for binding energies

Following the previous considerations [2,3] we have constructed the binding energy formula

$$E_{\rm sup} = E_0' + aN_f + bN_b + \varepsilon T(T+1)\,, \tag{1}$$

where E_0' is given from considerations of ground and excited states of the s–d shell nuclei. It reads

$$E_0' = E_0 + \beta J(J+1) + \gamma T_f(T_f+1)\,, \tag{2}$$

where $E_0 \equiv E_{\rm exp}(^{16}0) = -127.62$ MeV, J is the total nuclear spin of the ground state, T_f is the isospin of unpaired nucleons. ($T_f = 0; 1/2; 1$) and $\beta = 0.08$ MeV, $\gamma = -0.82$ MeV for the supermultiplet $N = 5$. However, we will adopt the same parameters for $N = 3$ and 4 which can cause a negligible error 0.1–0.2 MeV.

The new binding energy parameters a, b and c are given in Table I.

TABLE I

The binding energy formula parameters (in MeV).

N	a	b	ε
3	−7.00	−20.94	2.35
4	−7.60	−21.54	2.27
5	−8.20	−22.14	2.41

It is interesting to note very regular changes of the parameters a and b. The parameter ε has been taken from [3]. At last, $N_f(N_b)$ is the number of valence fermions (bosons) and T is the total isospin of a nucleus. For the supersymmetric binding energy the Coulomb energy must be added to compare with experimental data:

$$E_{\rm th} = E_{\rm sup} + E_{\rm C}\,, \tag{3}$$

where we have adopted $E_{\rm C}$ according to [4] (Table II).

TABLE II

The Coulomb energies (in MeV).

nucleus	$_8$O	$_9$F	$_{10}$Ne	$_{11}$Na	$_{12}$Mg	$_{13}$Al	$_{14}$Si	$_{15}$P	$_{16}$S
$E_{\rm C}$	18.29	21.83	25.91	30.24	35.10	40.18	45.77	51.28	57.33

3. Results

The theoretical energies are compared with the experimental data in Table III(a), III(b), III(c) for the supermultiplets $N = 3$, 4 and 5 respectively. In Table IV we give the predicted binding energies for exotic nuclei from these supermultiplets. In the present calculations, improved and much more complete than in [3], we come once again to the conclusion that ^{24}O is the last stable oxygen isotope. The conclusion is visible from the binding energies of oxygen isotopes (Tables III, IV).

TABLE III(a)

Comparison of experimental [5] and supersymmetric binding energies (in MeV) for supermultiplets $N = 3$.

	$_8$O$_{14}$	$_{10}$Ne$_{12}$	$_{12}$Mg$_{10}$	$_9$F$_{11}$	$_{11}$Na$_9$
$-E_{\rm exp}$	162.03	177.77	168.58	154.40	145.98
$-E_{\rm th}$	162.24	178.12	168.93	153.84	145.43
Δ	0.21	0.35	0.35	−0.56	−0.55
	$_8$O$_{13}$	$_9$F$_{12}$	$_{10}$Ne$_{11}$	$_{11}$Na$_{10}$	$_{12}$Mg$_9$
$-E_{\rm exp}$	155.18	162.50	167.41	163.08	149.20
$-E_{\rm th}$	155.85	164.06	167.43	163.10	150.79
Δ	0.67	1.56	0.02	0.02	1.59

TABLE III(b)

Comparison of experimental [5] and supersymmetric binding energies (in MeV) for supermultiplets $N = 4$.

	$_8$O$_{16}$	$_{10}$Ne$_{14}$	$_{12}$Mg$_{12}$	$_{14}$Si$_{10}$	$_9$F$_{13}$	$_{11}$Na$_{11}$
$-E_{\rm exp}$	168.48	191.84	198.26	172.00	167.73	174.15
$-E_{\rm th}$	168.38	192.54	196.97	172.68	168.78	174.63
Δ	−0.10	0.70	−1.29	0.68	1.05	0.48
	$_8$O$_{15}$	$_9$F$_{14}$	$_{10}$Ne$_{13}$	$_{11}$Na$_{12}$	$_{12}$Mg$_{11}$	$_{13}$Al$_{10}$
$-E_{\rm exp}$	164.77	175.27	182.97	186.56	181.72	168.70
$-E_{\rm th}$	164.64	176.35	183.62	186.50	181.64	169.75
Δ	−0.13	1.08	0.65	−0.06	−0.08	1.05

TABLE III(c)

Comparison of experimental [5] and supersymmetric binding energies (in MeV) for supermultiplets $N = 5$.

	${}_{10}\mathrm{Ne}_{16}$	${}_{12}\mathrm{Mg}_{14}$	${}_{14}\mathrm{Si}_{12}$	${}_{9}\mathrm{F}_{15}$	${}_{11}\mathrm{Na}_{13}$	${}_{13}\mathrm{Al}_{11}$
$-E_{\mathrm{exp}}$	201.60	216.68	206.05	179.13	193.52	183.60
$-E_{\mathrm{th}}$	201.78	216.69	206.02	179.14	193.71	183.77
Δ	0.18	0.01	−0.03	0.33	0.01	0.17
	${}_{9}\mathrm{F}_{16}$	${}_{10}\mathrm{Ne}_{15}$	${}_{11}\mathrm{Na}_{14}$	${}_{12}\mathrm{Mg}_{13}$	${}_{13}\mathrm{Al}_{12}$	${}_{14}\mathrm{Si}_{11}$
$-E_{\mathrm{exp}}$	183.48	196.02	202.53	205.59	200.53	187.00
$-E_{\mathrm{th}}$	182.80	195.99	203.31	205.68	200.60	187.78
Δ	−0.68	−0.03	0.78	0.09	0.07	0.78

TABLE IV

The predicted binding energies (in MeV) of exotic nuclei from the supermultiplets $N = 3, 4, 5$.

	${}_{8}\mathrm{O}_{17}$	${}_{8}\mathrm{O}_{18}$	${}_{13}\mathrm{Al}_{8}$	${}_{13}\mathrm{Al}_{9}$	${}_{14}\mathrm{Si}_{8}$	${}_{14}\mathrm{Si}_{9}$	${}_{15}\mathrm{P}_{9}$	${}_{15}\mathrm{P}_{10}$	${}_{16}\mathrm{S}_{10}$
$-E_{\mathrm{th}}$	165.05	166.02	134.35	149.43	134.76	152.42	149.69	170.86	170.37

The problem of an oxygen stability (^{26}O and ^{28}O) had been very often discussed. However, the majority of other theoretical predictions showed the stability of ^{26}O and even ^{28}O. In Table V we give some recent theoretical results for oxygen binding energies.

It is clear from Table V that only in few cases the instability of ^{26}O was predicted which is also our previous [3] and present result. It is also crucial to note that in the two experimental publications [10,11] the particle instability of ^{26}O has been concluded from the experimental observations that the lifetime of ^{26}O must be much shorter than 188 ns in the first work and much shorter than 140 ns in the second one.

TABLE V

Comparison of present and other theoretical calculations of ^{24}O, ^{26}O, ^{28}O isotopes.

	$E_{\rm exp}$ [5] 1995	[6] 1990	[7] 1995	[8] 1997	[9] 1999			
^{24}O	−168.48	−168.48	−170.46	−165.31	−171.9	−171.8	−172.4	−173.5
^{26}O		−169.66	−172.94	−166.85	−167.9	−170.9	−172.3	−174.0
^{28}O		−168.88	−177.40	−166.01	−164.6	−171.4	−173.8	−175.1

	[3] 1998	[10] 1996	present work
^{24}O	−168.82	−168.669	−168.38
^{26}O	−166.02	−169.664	−166.02
^{28}O	−160.38	−168.879	

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