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THE SIGNIFICANCE OF M. SMOLUCHOWSKI'S WORK IN SUBATOMIC PHYSICS*

Andrzej Budzanowski

H. Niewodniczański Institute of Nuclear Physics
Radzikowskiego 152, 31-342 Kraków, Poland

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Marian Smoluchowski is considered to be the founder of the physics of stochastic processes. In his studies of the Brownian motion he showed how the underlying thermal motion of the optically invisible molecules can be inferred from the observation of the chaotic motion of suspended colloidal particles discernible by the microscope. Smoluchowski was first to introduce randomness into physical equations. We show how the basic concepts and equations derived by Smoluchowski can be used to study the various forms of nucleonic matter excited in collisions of heavy ions, the liquid-to-gas phase transition in nuclei, multifragmentation phenomena and the possible transition to the quark–gluon plasma, the ultimate state of hadronic matter. The partonic structure of baryons can be studied from the energy and angular distributions as well as correlations of emitted hadrons even if Nature does not allow us to see free quarks and gluons. The various achievements of Smoluchowski's work, like the diffusion equation, fluctuation analysis, critical phenomena close to phase transitions, and foundations of the coalescence model as applied to contemporary problems are discussed, and their universality is stressed.

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1. Introduction

Marian Smoluchowski (1872–1917) the greatest Polish theoretical physicist ever was first to "see atoms" through the Brownian motion of heavy colloidal particles visible in optical microscope. He explained at the turn of centuries 1800/1900 the Brownian motion as the clean example of the stochastic process [1]. It is amazing that after hundred years many of his theoretical investigations can be applied successfully to the contemporary physics of nuclear and subnuclear processes.

The history of the Universe after the "Big Bang" is marked by subsequent phase transitions. In the laboratory we can study these transitions in a

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reversed order *i.e.* from the low temperatures of the order of mK up to the highest ones around 10^{14} K (see Fig. 1). At the lowest temperature we usually have to deal with the crystal structure in which atoms are bound

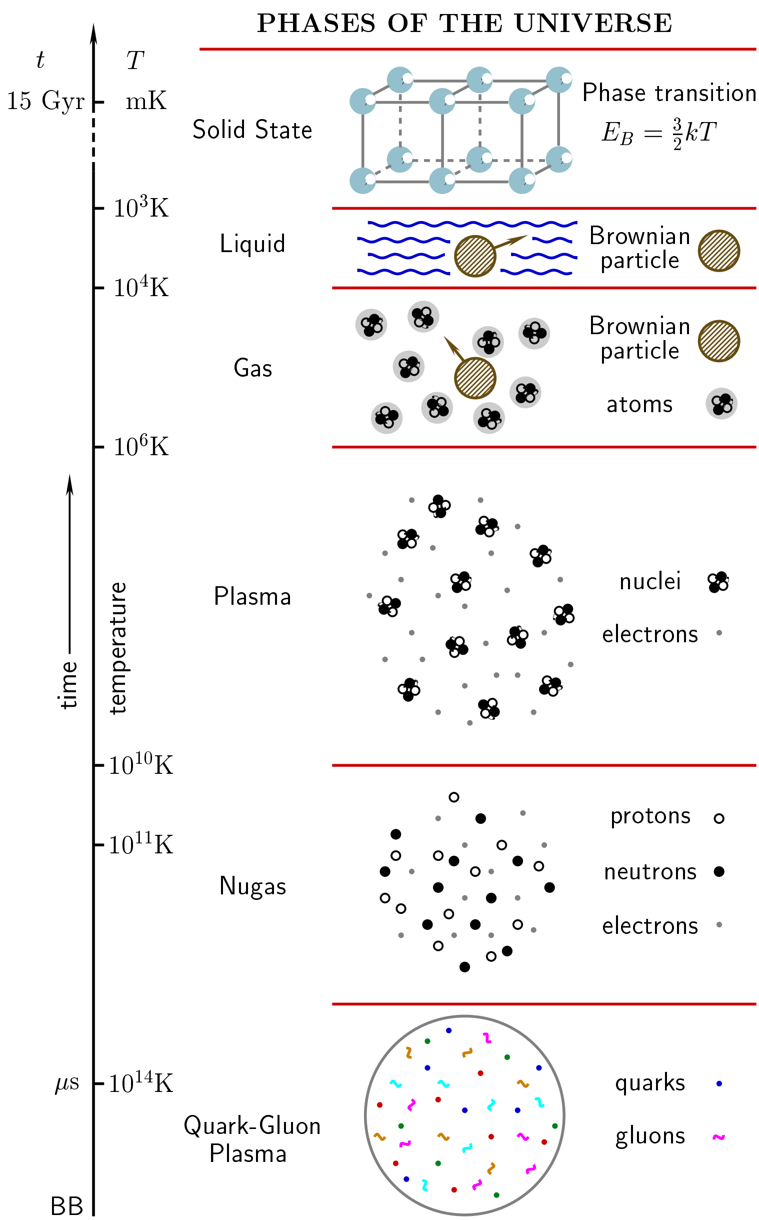

Fig. 1. Scheme of the "thermal" history of the Universe.

to the fixed positions in the lattice. As the temperature increases the solid phase melts and transition to the liquid one occurs. The atoms or molecules are moving more freely through a resistive viscous medium although they are bound within the liquid phase. Further, the liquid to gas phase transition takes place. Generally, each phase transition occurs at the temperature at which the thermal energy of the moving molecules is equal to the binding energy in the appropriate phase *i.e.* $3/2kT \approx E_{\mathrm{B}}$, where k is the Boltzmann constant, T stands for absolute temperature, and E_{B} denotes the binding energy of the considered phase.

The important point is that atoms have internal structure *i.e.* nuclei and electrons. If the energy of the thermal motion of the molecules in the gas phase exceeds the binding energy of electrons in atoms, a new phase of matter, the so called plasma, will appear. It consists of positively charged heavy ions and electrons. Total average electric charge of plasma is zero. Over 90% of matter in the Universe is in the plasma phase. Hot plasma in the interior of stars consisting of fully stripped nuclei and electrons is the principal source of energy enabling the birth and endurance of life on the nearby planets. Electrons are elementary particles but nuclei not. Therefore, further increase of the temperature will lead to the next phase of matter which is sometimes called "nugas" *i.e.* gas of nucleons and electrons. This last transition is called a liquid to gas phase transition since nuclei in its ground state have properties of a liquid drop. To effectuate this last phase transition, temperatures of the order of 10^{11}K are necessary since the average binding energy of nucleons in nuclei is ≈ 8 MeV.

It is quite natural to think about the next phase transition. However, this requires that nucleons should have internal structure. The first sign that the proton may not be an elementary particle can be traced back to the late thirties when its dipole magnetic moment was measured. The experimental value was found to be almost three times higher than that expected for the Dirac point particle. This strongly suggested that proton might have internal structure. Careful studies of this structure by scattering of electrons of the energy of tens of GeV revealed three point particles called quarks with electrical charges $+2/3e$, $+2/3e$, and $-1/3e$ for protons and $+2/3e$, $-1/3e$, and $-1/3e$ for neutrons. Quarks are bounded by strong forces. The strong field quanta gluons were soon discovered too. The way to the new phase of matter the so called quark–gluon plasma (QGP) seemed to be open. However, some unexpected difficulty appeared. Quarks and gluons cannot exist as free particles. After many unsuccessful attempts to find particles with electrical charges being the multiplicity of $1/3e$, physicists had simply to built this fact into the theory. This led to the birth of Quantum Chromodynamics, called shortly QCD. At this point we should quote the famous Einstein's saying "Komplizierter ist der Herr Gott, aber boshaft ist Er nicht". QCD explains the fact that all strongly interacting particles — hadrons — are built up from elementary point particles — quarks and antiquarks — having finite masses. Three quarks form baryons and pairs of quark–antiquark–mesons. Quarks interact by exchange of massless gluons. Gluons and quarks are carrying strong field charges called colours. The colours are strong field sources. All existing hadrons must have white colours therefore the minimal satisfactory number of different strong charges–colours is three by some loose analogy to optics where three colours form a white light. Since gluons as quanta of colour forces have to operate between three colours they must be mathematically represented by matrices of the order of three. Experiments carried out in the second half of the twentieth century led to the conclusion that QGP comprises six types (flavours) of quarks, three colours and eight gluons. Taking into account that the low energy quark–gluon plasma (QGP) should contain two types of light quarks contained in nucleons namely u and d we have for the statistical weight of QGP $w_{\rm QGP} = 8\times 2 + 7/8\times 2\times 2\times 2\times 3 = 37$, where the first term contains the contribution from gluons, 2 for the spin polarization, 8 for colours. The second term represents the contribution due to quarks (in the massless approximation) where 3 factors of 2 represent respectively particle-antiparticle, spin and flavour degrees (u, d) of freedom and the factor 3 is for colours. Factor $7/8$ comes from Fermi Dirac statistics. This simple estimation shows us that QGP phase is a very rich structure with high value of the statistical weight.

Smoluchowski's physics was confinned to three phases shown in the upper part of Fig. 1. However, his methods of analysis of stochastic motion of Brownian particles are fully applicable to processes connected with rich structures existing in the subatomic world shown in lower part of Fig. 1.

In order to see this we have to repeat briefly some basic informations on the motion of the Brown's particles within the so called random-walk model.

2. Brownian motion as fluctuation

The ratio of the mass of single atoms to the mass of heavy Brown's particle is of the order of 10^{-12}. From the simple conservation of linear momentum we can easily estimate that the recoil of the heavy colloidal particle resulting from the collision with single atom cannot be observed in the microscope. In the second half of the 19th century this was the main argument used by some scientist, for example by Nägeli, against the atomistic interpretation of the Brownian motion.

The great merit of Smoluchowski was his notion that the movement of the Brown's particle is a result of the fluctuation in the number of collisions with single atoms. From time to time the number of atoms pushing the Brown's particle in one direction increases significantly beyond its mean value taken over all directions. As a result the heavy particle will move in to one direction being decelerated by the internal friction force in accordance with the Stokes law. This last force arises simply from the fact that the particle moving in one direction collides on the average with more atoms moving in the opposite direction. A simple one dimensional model of Chandrasekhar [1] allows us to understand the main features of the stochastic motion and to estimate the finite value of the probability to observe fluctuations in this motion.

Let us consider the following motion of a particle along the x axis (see Fig. 2). By tossing a coin we move the particle by the unit distance a

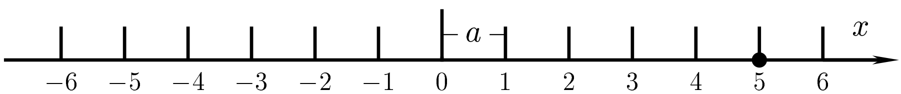

Fig. 2. The random-walk model along the axis x

towards the positive direction if the coin is falling avers up or in the negative direction if the coin is falling reverse up. This means that the movement of the coin is statistical one with the equal probability $P = 1/2$ into the positive or negative direction. We ask what is the probability $W(m,N)$ that after N steps the particle will be found at coordinate m. The probability of any distinguishable sequence of steps is $(1/2)^N$. To get the position m we need $(N+m)/2$ steps into the positive direction and $(N-m)/2$ steps into the negative one. The sequence of plus and minus steps is arbitrary. From simple combinatorics we get the total number of different sequences leading to position m as:

$$\frac{N!}{[(N+m)/2]!\,[(N-m)/2]!}\,. \tag{1}$$

For $W(m,N)$ we get:

$$W(m,N) = \frac{N!}{[(N+m)/2]!\;[(N-m)/2]!}\left(\frac{1}{2}\right)^N . \tag{2}$$

The mean value

$$\tfrac{1}{2}\langle N+m\rangle_{\rm av} = \tfrac{1}{2}N\,. \tag{3}$$

The mean square deviation is:

$$\langle[\tfrac{1}{2}(N+m) - \tfrac{1}{2}N]^2\rangle_{\rm av} = \tfrac{1}{4}N\,. \tag{4}$$

From (3) and (4) we conclude that the mean value of the shift is in accordance with simple intuition $\langle m\rangle_{\rm av} = 0$, but the mean square value $\langle m^2\rangle_{\rm av} = N$. This last relation indicates a very important property that the average value of the square of the shift along the x axis is proportional to the number of steps N. If we assume that the consecutive shifts are separated by equal time interval Δt then $N = t/\Delta t$ where t is the total time of observation. In consequence:

$$\langle m^2 \rangle_{\rm av} = \text{const.}\, \frac{1}{\Delta t}\, t = \text{const.}\, t\,. \tag{5}$$

The mean square value of the shift along x axis is proportional to the time of the observation. On the example of the simple one-dimensional random motion we can understand why Smoluchowski was happy when he found for the mean square value of the consecutive shifts of the Brown's particle a formula proportional to time. The connection between the simple proportionality to the time of the observation and the nature of the stochastic process was established. It is interesting to study the values of W for the more realistic numbers N and m assuming that N is representing the number of all atomic collisions with Brown's particle whereas m is mimicking the number of collision pushing the particle in a defined direction (fluctuation). Assuming 1 μm for the diameter of the Brown's particle and the normal condition for gas or liquid it is possible to estimate that the total number of collisions from all directions is 10^{16} s^{-1} for gas and 10^{20} s^{-1} for liquid phase. Transforming formula (2) using Stirling formula for the factorial and taking the following conditions $N \gg 1$ and $1 \ll m \ll N$ we get:

$$W(m, N) = \left(\frac{2}{\pi N}\right)^{1/2} \exp\left(-\frac{m^2}{2N}\right)\,. \tag{6}$$

From the formula (6) we see that using the realistic values for N we can expect to observe, with finite probabilities, the push in the definite direction for $m = 10^8$ and $m = 10^{10}$ molecules in the gas or liquid, respectively.

Smoluchowski solved the problem of movement of heavy Brown's particle in 3D space under the influence of the following forces:

1. Stochastic force (nondeterministic) due to the superposition of many atomic collisions (fluctuation).
2. Internal friction force in the viscous medium (Stokes Law).

The famous Smoluchowski formula (found independently by Einstein) for the mean quadratic way along x axis passed by the Brownian particle takes the form:

$$\langle x^2 \rangle_{\rm av} = \frac{kT}{3\pi\eta a}\, t = \frac{R}{N_A}\, \frac{T}{3\pi\eta a}\, t\,, \tag{7}$$

where k is the Boltzman constant, η coefficient of viscosity of the medium, a the radius of the Brown's particle, t time of the observation, R is the gas constant, $N_{\rm A}$ indicates Avogadro number. It is important to note that from the fluctuation in the stochastic process it is possible to determine the underlying number of atoms per 1 mol of substance. Using the formula (7), Perrin and Zermelo have determined the Avogadro number proving that its value is independent of the kind and mass of Brown's particle, coefficient of viscosity of the medium as well as of the temperature of the medium.

3. Fluctuation laboratory

After explaining successfully the movement of the heavy colloidal particles as generated by the thermal motion of atoms (molecules) Smoluchowski published several papers in which he applied the theory of random processes to the assemblage of Brownian particles suspended in liquid free or subject to external forces. Most of these "gedanken" experiments are possible to perform in the laboratory. Many of his theoretical investigations can be applied to the world of nuclear and subnuclear processes due to their universality.

All the formulas concerning fluctuations Smoluchowski is derived under the assumption that the considered processes are of the Markowian type *i.e.* the movements of the Brownian particles are mutually independent and all positions of the particles inside the considered volume have equal probability. These assumptions can be subsequently modified by the introduction of external fields of forces acting on the particles (*e.g.* electric fields, gravitational field). We can also consider the problems of coagulation or fragmentation of particles or other sources of mutual correlation in the movement of particles.

What I would call the Smoluchowski's laboratory of stochastic motion is simply a spacious vessel holding gaseous or liquid solution of colloidal particles performing Brownian motion. Using this "laboratory" Smoluchowski carried out many cogitated experiments, predicting their results by ingenious theoretical considerations. Many of these experiments are directly transferable to world of the subatomic physics. However, we have to keep in mind that the formulas derived by Smoluchowski are valid for phenomena concerning a large number of particles. Special care must be taken in order to apply these formulas to the systems of low number of particles.

The basic question put forward by Smoluchowski can be stated as follows: if we have on the average ν Brown's particles in a volume element ΔV of the equilibrated colloidal solution, what is the probability $W(n)$ to find n particles in this volume element ΔV at an arbitrary moment? It turned out that the solution takes the form of the Poisson distribution [1]

$$W(n) = \frac{\nu^n \exp(-\nu)}{n!}\,. \tag{8}$$

Next notion introduced by Smoluchowski is the transition probability $W(n,m)$. It answers the following question: suppose that at certain moment we have in the volume element ΔV n Brownian particles. What is the probability that after a certain time internal τ we find m particles in this element? The transition probability is from the mathematical point of view called the conditional probability. Smoluchowski called $W(n,m)$ in German as "Wahrscheinlichkeitsnachwirkung" what can be translated to English as "probability after effect". Next question considered by Smoluchowski is: what is the probability that a particle located in volume element ΔV after time interval τ leaves ΔV? Let us repeat that all this questions are formulated assuming stationary state of the system of Brownian particles, the movement of particles are independent (uncorrelated) and all positions in the volume are equally probable.

Finally for $W(n,m)$ Smoluchowski obtained the following formula:

$$W(n,m) = \sum_{x+y=m} w_1^{(n)}(x) w_2(y)\,, \tag{9}$$

where

$$w_1^{(n)} = C_x^n (1-P)^x P^{n-x}\,, \qquad (0 \le x \le n) \tag{10}$$

and

$$w_2(y) = \frac{(\nu P)^y \exp(-\nu P)}{y!}\,, \qquad (0 \le y < \infty)\,. \tag{11}$$

where $w_1^{(n)}(x)$ is the probability that x particles remain in ΔV after time τ, if at t_0 (first moment of observation) it was there n particles, $w_2(y)$ indicates the probability that after time τ, y particles enter the element ΔV. P denotes the probability that a particle being at the beginning in element ΔV after time τ leaves this element. The coefficient C_x^n denotes the number of distinct ways of selecting x particles from the initial group of n. The other symbols are the same as in Section 2. Smoluchowski was interested in the density fluctuation phenomena mostly for the purpose of studying the reversibility of processes generated by statistical thermodynamics. He derived two important formulas for the mean time of life T_n of the fluctuation state of n particles in the volume element ΔV and the mean recurrence time of n particles in ΔV Θ_n:

$$T_n = \frac{\tau}{1 - W(n,n)}\,, \tag{12}$$

and

$$\Theta_n = \frac{\tau}{1 - W(n,n)} \, \frac{1 - W(n)}{W(n)}\,. \tag{13}$$

For $n \gg \nu$

$$\Theta_n \sim \tau \, \frac{\exp(\nu)\; n!}{\nu^n}\,. \tag{14}$$

We notice from (14) that at very large value of n the recurrence time will become infinitely long. That means that observing the volume element ΔV at finite time intervals τ we have to wait infinitely long to get the same configuration n. This was for Smoluchowski an important proof of the irreversibility of statistical thermodynamics.

For our purposes more interesting is the probability density $W(\vec{r}_1, \vec{r}_2)$ that the Brownian particle in its chaotic movement after time τ will change the position from $\vec{r}_1$ to $\vec{r}_2$:

$$W(\vec{r}_1, \vec{r}_2) = \frac{1}{(4\pi D\tau)^{3/2}} \exp\left(-\frac{|\vec{r}_1 - \vec{r}_2|^2}{4D\tau}\right), \tag{15}$$

where

$$D = \frac{R}{N_A} \frac{T}{6\pi a\eta}, \tag{16}$$

D is called the diffusion coefficient. The other symbols in (15) and (16) were defined in Section 2. Integrating over the volume element ΔV we get:

$$P = \frac{1}{(4\pi D\tau)^{3/2}} \Delta V \int\int \exp\left(-\frac{|\vec{r}_1 - \vec{r}_2|^2}{4D\tau}\right) d\vec{r}_1\, d\vec{r}_2, \tag{17}$$

where the integration over $\vec{r}_1$ is stretched over the volume ΔV and over $\vec{r}_2$ is confined to space outside volume ΔV. Coefficient of diffusion D contains Avogadro number N_{A}. Through formulas (10) and (11) N_{A} is connected to the density fluctuation. We can then conclude that by observation of density fluctuation *i.e.* counting the number of particles in volume ΔV at constant time intervals τ we might also determine the number of invisible atoms in one mol of substance.

The above presented considerations can be directly transferred to the subatomic world by investigating the fluctuations of number of particles emitted in nuclear collisions from one collision to the other (event by event fluctuations). The size of those fluctuations depends on the type of the excited gas which is the source of emitted particles *e.g.* quark–gluon plasma or hadron gas. Instead of volume element ΔV in 3D space we are often considering elements in phase space, elements of solid angle, elements in momentum space or ratios of various types of emitted particles.

Smoluchowski studied also the Brownian motion in the presence of perfectly absorbing wall, perfectly reflecting walls as well as coagulation problems. This last process occurs when two Brownian particles are sticking together if their mutual distance $|\vec{R}_1 - \vec{R}_2|$ falls down below the certain radius R_c called coalescence radius. This concept is also being used in nuclear reactions in which two nucleons or two groups of nucleons with momenta $\vec{p}_1$

and $\vec{p}_2$ find themselves within the coalescence sphere in momentum space $|\vec{q}_c|$ such that $|\vec{p}_1 - \vec{p}_2| < |\vec{q}_c|$. They are then emitted as one particle. This is a full analogy of the coagulation radius of Smoluchowski.

More interesting for the application in physics of subatomic processes are the non stationary phenomena in which there is a flow of Brownian particles caused by the gradient of the particle density (diffusion) or by external electrical or gravitational fields. Those processes are governed by the so called Smoluchowski's equation:

$$\frac{\partial W}{\partial t} = \mathrm{div}\left(D\,\mathrm{grad}W - \frac{Fm}{6\pi a\eta}\,W\right), \tag{18}$$

where F is the external force acting on the unity of mass, W is the density of Brownian particles, D denotes the diffusion coefficient, a indicates the diameter of Brownian particle and η the coefficient of viscosity. The first term in parenthesis describes the stochastic diffusion process, the second one the motion caused by the external force. The Smoluchowski's diffusion equation is a limiting form of the more general Fokker–Planck equation. The works of Smoluchowski are not terribly often cited in papers concerning subatomic processes due to the simple fact that before the neutron discovery (1932) physics of nuclear structure practically did not exist. Nevertheless many of the fluctuation equations from the physics of Brownian motion and the Smoluchowski's equation have found an application to physics of nuclear collisions.

4. Application of Smoluchowski's work to subatomic physics

Most of the theoretical experiments of Smoluchowski concerning the colloidal solution of particles have found applications in the chemistry of colloids. Smoluchowski explained also the blue color of the sky as well as the critical opalescence. After the discovery of new forms of matter like nucleonic matter and more fundamental substructures of quarks and gluons it is to be expected that many Smoluchowski's considerations will be applicable to highly excited nuclear matter up to its critical state as well as in search for the early state of the universe — the quark–gluon plasma. Those states are usually created in laboratory by heavy ion collisions.

In Fig. 3 a schematic picture of heavy ion collision in the energy range from tens to few hundreds MeV per nucleon is shown. We can distinguish three regions of the reaction: projectile like, target like and the interaction region. In the interaction region a strongly excited nucleon gas is formed. To the excited nucleons the random walk model presented in Section 2 can be applied. As a result of collisions the nucleons can be transferred to projectile like fragment PLF or target like fragment TLF or escape to the continuum. If two nucleons are moving with momenta $\vec{p}_1$ and $\vec{p}_2$ which are subject to the condition $|\vec{p}_1 - \vec{p}_2| < |\vec{q}_{\rm c}|$ where $q_{\rm c}$ is the coalescence radius in momentum space then these two nucleons can be emitted as one particle with momentum $\vec{p} = \vec{p}_1 + \vec{p}_2$. This phenomenon is a full analog of coagulation of colloidal particles described by Smoluchowski [1]. The random walk model has been applied to the two stage model of heavy ion reaction by Sosin [2]. In the first stage of reaction the nucleons become free as the result of the interaction with mean field or with other nucleons. In the second stage the free nucleons can be transferred to the projectile, to the target or form a cluster if the difference of their momenta is contained in the coalescence sphere. Several

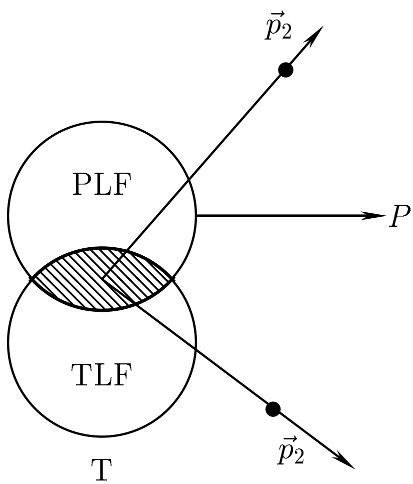

Fig. 3. Schematic picture of the collisions of two heavy ions. P — projectile, T — target, PLF — projectile like fragment, TLF — target like fragment hatched part indicates the interaction region, $\vec{p}_1$ and $\vec{p}_2$ indicate momenta of the outgoing nucleons or clusters.

attempts to determine the primary number of particles created in central heavy ions collisions from event by event fluctuations have been made. These are experiments which can be directly related to the determination of the Avogadro number by observations of the fluctuation of number of Brownian particles in a volume element ΔV. It is usually rather difficult to determine the volume element ΔV in nuclear collisions so the fluctuation considerations are shifted to studies of volume independent quantities like ratios of intensity of various kinds of particles, ratios of particles of different charges *etc.*

As an example let us take ratios of charged particles. It should be kept in mind that the charge unit in QGP phase is 1/3e whereas in the hadron phase 1e. Defining the next emitted charge by $Q = N_+ - N_-$, where N_+ indicates the number or emitted particles positively charged, and N_- the number of emitted negatively charged particles we have for the total number of charged particles $N_{\rm ch} = N_+ + N_-$. Let us define $R = N_+/N_-$ and $F = Q/N_{\rm ch}$. The mean square fluctuation of R^2 is then:

$$\langle \delta R^2 \rangle_{\rm av} = \langle R^2 \rangle_{\rm av} - \langle R \rangle_{\rm av}^2 = 4 \langle \delta F^2 \rangle_{\rm av} \,. \tag{19}$$

Jeon and Koch [3] have shown, that the parameter D defined as $\langle N_{\rm ch} \rangle \langle \delta R^2 \rangle$ for hadron gas has the value $D_h = 4$ and for quark–gluon plasma $D_{\rm QGP} = 1$. On the basis of the experimental results for π^+ and π^- emission in ^{208}Pb + ^{208}Pb collision at energy 17.6 GeV per nucleon pair in CM at the SPS accelerator in CERN the experimental value was found to be $D_{\rm exp} = 3,5$. We can then conclude that in very short time $\tau < 10^{-23}$ s the quarks are coalescing into the hadron resonances which are decaying into meson pairs. So far the value of parameter D was not calculated for strongly coagulating gas. The existence of hadrons and meson resonances in the decay of QGP was recently indicated by Broniowski and Florkowski [4]. At this point we may try to speculate a little bit about the fate of a piece of QGP. It was expected that due to the large statistical factor of the QGP (see Section 1) once formed the QGP will expand. This is due to the principle of increase of the entropy. The transformation back to the hadron phase within the same volume would contradict the law of constant increase of entropy. In view of the possibility of formation of the hadron resonances in the process of fast coagulation of quarks it is possible to explain the transformation of QGP into hadron phase within the primary volume of colliding nuclei thanks to the exponential increase of the number of hadronic states. This large number of excited resonance states provides the phase space density necessary to keep the entropy increasing within the volume of colliding nuclei what is suggested by recent results of the HBT type of experiments [5].

Quite recently with the help of the Smoluchowski's equation Świątecki and his colleagues [6] have shown how to calculate the fusion cross section for very heavy systems created in the interaction of projectiles ^{48}Ca–^{96}Kr with ^{208}Pb nuclei. This reaction is schematically show in Fig. 4. The fusion cross section was so far calculated by multiplying the cross section for reaching the sticking configuration of the colliding nuclei by the probability of survival of the nucleus left after one neutron evaporation from the compound system. It turned out that the product mentioned above has to be multiplied by the third factor which takes into account the diffusion process of nucleons in the neck region. Due to this effect part of the nucleons will move down the potential and the second part will be shifted by the diffusion processes up

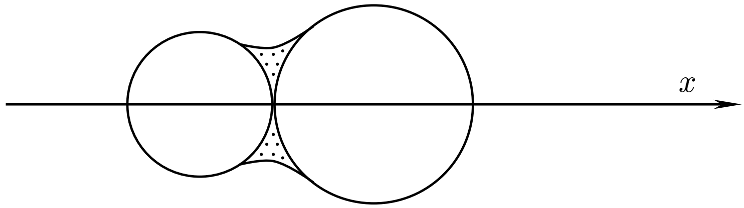

Fig. 4. Schematic picture of the fusion of two nuclei. Dotted part indicates the neck region.

the potential barrier. This last group of nucleons will contribute to fusion. For the potential of a parabolic shape $V(x) = -1/2bx^2$, the process can be described by the one dimensional Smoluchowski's equation:

$$G\frac{\partial W}{\partial t} = (bxW)' + TW''\,, \tag{20}$$

where primes denote the differentiation by the x variable, G is the friction coefficient, W is the probability that particles are at position x. The neck plays the role of the Brownian particle. The calculated cross section shows excellent agreement with experiment.

Finally we should mention the possibility of using Smoluchowski's works on the fluctuation at the critical point of nuclear matter or critical phenomena at transition point to quark–gluon plasma. Unfortunately the incomplete experimental data existing up to now on these subjects preclude at the moment full interpretation of critical phenomena in subatomic phase transitions.

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