# THE ALPHA-DEFORMED SUPERHEAVY NUCLEUS INTERACTION POTENTIAL* 

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The $\alpha$ particle emission process through the multidimensional barrier potential has been considered. The deformed nucleus- $\alpha$ particle interaction energy including proximity approximation in the nuclear potential and Coulomb terms have been proposed. The alpha decay of very heavy arbitrarily deformed nuclear systems have been studied within the above potential. The curvature effects have been also investigated.

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## 1. Introduction

Theoretical predictions of existence of deformed superheavy nuclei (SHE) [1-3] promote the studies of alpha decay process, which is predominant in this area. The form of interaction potential required for description of light fragments emission from the deformed nuclear shapes is proposed in our paper. The method of spherical potential expansion [4] provides in the description additional degrees of freedom associated with higher orders deformation of parent nucleus as well as with different positions of alpha particles. As commonly known even a small difference in the barrier height can change life time of a few orders of magnitude. Exact calculations of barrier heights and widths are essential in estimation of life times considering this decay. However, the existence of deformed SHE has not been experimentally confirmed yet and precise theoretical studies in this area are necessary.

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## 2. Shape parametrization

The paper presents parametrization of the shape of the parent nucleus $(M)$ and the $\alpha$ particle which proceeds in the following way:

$$
\vec{r}_{M(\alpha)}=\left\{\begin{array}{l}
R_{M(\alpha)}\left(\theta_{M(\alpha)}\right) \sin \left(\theta_{M(\alpha)}\right) \cos \left(\phi_{M(\alpha)}\right),  \tag{1}\\
R_{M(\alpha)}\left(\theta_{M(\alpha)}\right) \sin \left(\theta_{M(\alpha)}\right) \sin \left(\phi_{M(\alpha)}\right), \\
R_{M(\alpha)}\left(\theta_{M(\alpha)}\right) \cos \left(\theta_{M(\alpha)}\right)
\end{array}\right.
$$

The standard form of nucleus surface $R_{M}$ expansion into a series of spherical harmonics in the $\beta_{l}$ deformation parameters is used. The vector connecting the mass centers of alpha particle and parent nucleus is recorded in the spherical reference system associated with this nucleus $(R, \Theta, \Phi)$.

$$
\vec{R}=\left\{\begin{array}{l}
R \sin (\Theta) \cos (\Phi)  \tag{2}\\
R \sin (\Theta) \sin (\Phi) \\
R \cos (\Theta)
\end{array}\right.
$$

As only the emission of alpha particle from the axially symmetric deformation of nuclei is considered $\Phi=0$ and $\phi_{M}=0$. It is additionally assumed that the $\alpha$ particle does not affect deformation of the nucleus from which it was emitted.

## 3. Coulomb energy

Expansion of spherical formula of the Coulomb potential for the deformed parent nucleus with the atomic and mass numbers $Z_{M}\left(A_{M}\right)$ and the $\alpha$ particle is of the form:

$$
\begin{equation*}
V_{c}=\frac{2 Z_{M} e^{2}}{R}+\frac{e}{\sqrt{\pi}} \sum_{l_{M}=2}^{\infty} \frac{1}{R^{l_{M}+1}} C_{l_{M}, 0}^{0,0} Y_{l_{M}}^{0}(\Theta) Q_{l_{M}}^{0} \tag{3}
\end{equation*}
$$

Here $e$ is the charge unit and $C_{l_{M}, 0}^{0,0}$ are the Wigner coefficients. For the application presented here, it is sufficient to consider only the outer region where the barriers are located and the Coulomb expansion is absolutely convergent. We adopt here the following definition of the multipole moments in the spherical harmonics $Y_{l_{M}}^{0}\left(\theta_{M}\right)$ :

$$
\begin{equation*}
Q_{l_{M}}^{0}=\int_{\nu_{M}} \rho_{M}\left(\vec{r}_{M}\right) r_{M}^{l_{M}} Y_{l_{M}}^{0}\left(\theta_{M}\right) d \tau_{M} \tag{4}
\end{equation*}
$$

where $\rho_{M}\left(\vec{r}_{M}\right)$ is a charge distribution with uniform density, with the "equivalent sharp radius" [5]. Integration over parent nucleus volume ( $\nu_{M}$ ) in the coordinates defined by Eq. (1) is carried out. Let us note that the first term in the presented multipole expansion corresponds to the case of interaction when both fragments are spherical.

## 4. Nuclear energy

To calculate the nuclear part of $\alpha$ particle-deformed parent nucleus interaction we used the suitable form of "proximity" approximation [6],

$$
\begin{equation*}
V_{n}=\gamma K \psi(s) \tag{5}
\end{equation*}
$$

with the surface tension coefficient $\gamma=0.9517\left(1-1.7826 I^{2}\right) \mathrm{MeV} / \mathrm{fm}^{2}$, where $I=(N-Z) / A$. This hypothesis is based on the assumption, that the nuclear interaction takes place between two infinitesimal volume elements located closest for a given configuration. It allows to separate nucleus interaction formula into two independent factors, geometrical one ( $K$ ) connected with the surface curvature and the universal function $\psi(s)$ independent of the system geometry. In this paper the version of this function given in [7] is used. Extension of "proximity" interaction for the case of deformed nuclei consists in redefining of the geometrical factor $K$ only because, as already pointed out, the universal function does not depend on the system geometry.

Geometrical prefactor $K$ is given by "reduced main radii of the curvature" $k_{1 M(\alpha)}$ and $k_{2 M(\alpha)}$ :

$$
\begin{equation*}
K=2 \pi \sqrt{k_{1 M(\alpha)} k_{2 M(\alpha)}} \tag{6}
\end{equation*}
$$

which are calculated in the point of the nucleus surface for which the distance between the parent nucleus and the emitted particle surfaces (for a given $R$ and $\Theta)$ is the smallest $s=\min \left|\vec{R}+\vec{r}_{\alpha}-\vec{r}_{M}\right|$.

$$
\begin{equation*}
k_{1 M(\alpha)}=\frac{R_{1 M} R_{1 \alpha}}{R_{1 M}+R_{1 \alpha}} ; \quad k_{2 M(\alpha)}=\frac{R_{2 M} R_{2 \alpha}}{R_{2 M}+R_{2 \alpha}} . \tag{7}
\end{equation*}
$$

From the minimization procedure we find the parameters (angles) $\theta_{M}^{\min }, \theta_{\alpha}^{\min }$ for which we are able to calculate the first $R_{1 M(\alpha)}$ and second $R_{2 M(\alpha)}$ main radius curvatures, with earlier calculation of the fundamental forms of the first and the second order of the nucleus surface for parametrization presented in Section 2.

## 5. Results

The model presented above is used for calculation of interaction energy between the alpha particle and the parent nucleus as a function of the particle position $(\Theta)$ and the distance between mass centers $(R)$ of both fragments. The calculations were made for exemplary deformed $\left(\beta_{2}=0.2\right.$, $\beta_{4}=-0.08$ ) superheavy nucleus ${ }^{278} 114$. The equilibrium deformations were found by minimizing energies in the multi-dimensional deformation space. Fig. 1(a) presents exemplary interactions of energy for the distance between
the mass centers $R=11 \mathrm{fm}$. The maximal difference of energy is about 10 MeV in the case $\beta_{4}=-0.08$ and two times more ( $\sim 20 \mathrm{MeV}$ ) when $\beta_{4}$ has the opposite sign.


Fig. 1. (a) Interaction energy between the $\alpha$ particle and the parent nucleus for the distance between the mass centers $R=11 \mathrm{fm}$. (b) Full barriers for the emission of particle $\alpha$ as the distance function for $\Theta=90^{\circ}$ (dotted lines), and $\Theta=0^{\circ}$ (dashed line).

Full shapes of barriers $\left(V_{n}+V_{c}\right)$ are presented in Fig. 1(b) only for two extremal orientations. The difference between the barrier heights in these extremal cases where $\Theta=0^{\circ}$ ("tips" position) or $\Theta=90^{\circ}$ ("equatorial" position) is $\Delta V_{\max }=1.54 \mathrm{MeV}$. The corresponding difference in the barrier radius for the superheavy element under discussion is $R_{\max }=1.34 \mathrm{fm}$. There can be observed the influence of emitting nucleus curvature on the obtained shapes of the barriers. Distinct differences are visible particularly at small distances of the emitted particle alpha where dominant contribution comes from the nuclear part. Large energy difference for various directions $(\Theta)$ of the emitted particle has a significant effect on the obtained life times for this process.

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