FUSION BARRIERS DERIVED FROM THE HARTREE–FOCK FUNCTIONAL WITH SKYRME INTERACTIONS*

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Using effective nucleon-nucleon interactions of the Skyrme type and the semi-classical extended Thomas-Fermi approach we describe heavy-ion fusion barriers in the sudden approximation. We review in particular a large number of fusion reactions leading to the superheavy element $Z = 114$.

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The aim of our present investigation is to propose a simple approach permitting the evaluation of fusion barriers encountered in heavy-ion reaction, approach in which the nuclear force between the two colliding ions is obtained using the Hartree–Fock energy functional for effective nucleon–nucleon interactions of the Skyrme type [1]. Since only the radii and the tails of the density distributions of target and projectile decide about the height of the fusion barriers, we have chosen the semi-classical approximation known as the extended Thomas–Fermi (ETF) method [2] to determine in a self-consistent way the structure of projectile and target nuclei. This approach allows to describe the energy as a functional of the local density: $E^{(i)} = E^{(i)}[\rho]\,,\ i = 1, 2$, where i labels the colliding nuclei. This simple result, founded on the famous Hohenberg–Kohn theorem [3], is obtained in the ETF approach through the fact that for Skyrme forces the energy of a nucleus can be described as a functional of the local densities ρ_q, the kinetic

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energy densities τ_q and the spin–orbit densities $\vec{J}_q$, $q=\{n,p\}$:

$$\begin{aligned} E &= \int d^3r\ \mathcal{E}\left[\rho_q, \tau_q, \vec{J}_q\right] \\ &= \int d^3r \left\{ \frac{\hbar^2}{2m}(\tau_n+\tau_p) + B_1\rho^2 + B_2(\rho_n^2+\rho_p^2) + B_3\rho\tau \right. \\ & +B_4(\rho_n\tau_n+\rho_p\tau_p) - B_5(\vec{\nabla}\rho)^2 - B_6\left[(\vec{\nabla}\rho_n)^2+(\vec{\nabla}\rho_p)^2\right] + \rho^\alpha\left[B_7\rho^2\right. \\ & \left.\left. +B_8(\rho_n^2+\rho_p^2)\right] - B_9\left[\vec{J}\cdot\vec{\nabla}\rho + \vec{J}_n\cdot\vec{\nabla}\rho_n + \vec{J}_p\cdot\vec{\nabla}\rho_p\right] + \mathcal{E}_{\rm Coul}(\vec{r}) \right\} \quad (1) \end{aligned}$$

and the fact that the ETF approach allows to express τ and $\vec{J}$ as functions of the local density ρ and its derivatives. At lowest order in the ETF expansion one has *e.g.* the well-known Thomas–Fermi kinetic energy density

$$\tau_q^{(\rm TF)}[\rho_q] = \frac{3}{5}(3\pi^2)^{2/3}\ \rho_q^{5/3} \quad (2)$$

but in the self-consistent semi-classical calculations, as in Eq. (1), the full ETF functionals up to order $\hbar^4$ have, of course, been used. The constants B_i in Eq. (1) are combinations of the usual Skyrme-force parameters t_j and x_j and non indexed quantities correspond to the sum of neutron and proton densities, as: $\rho = \rho_n + \rho_p$.

As a consequence of the ETF approximation, the total energy is a functional of the local densities alone (we write $E[\rho]$, meaning $E[\rho_n, \rho_p]$), and the density-variational calculation gives estimates of the semi-classical densities. It has been shown [2, 4] that modified Fermi function of the form

$$\rho_q(r) = \rho_0^{(q)}\ \left(1 + {\rm e}^{(r-R_q)/a_q}\right)^{-\gamma_q} \quad (3)$$

is a very good approximation to the full variational solution.

After having explained how the structure of the nuclei is determined, we turn now to the description of the fusion potential. Using the "*sudden approximation*", *i.e.* neglecting rearrangement effects of the two colliding nuclei, we describe this potential as [5]

$$V_{12} = E\left[\rho^{(1)} + \rho^{(2)}\right] - E\left[\rho^{(1)}\right] - E\left[\rho^{(2)}\right], \quad (4)$$

where $E[\rho]$ consists (see Eq. (1)) of a nuclear and a Coulomb contribution.

For distances s between the equivalent sharp surfaces of the two colliding ions which are significantly larger then the range of the nuclear forces, the nuclear part of the interaction V_{12} vanishes. On the other hand, our sudden approximation is, of course, only valid for not too small distances ($s \gtrsim 0$).

The fusion barrier appears as the result of the competition between the long-range repulsive Coulomb interaction $\mathcal{V}_{\rm Coul}$ and the short-range attractive nuclear forces $\mathcal{V}_{\rm nucl}$ as is illustrated on Fig. 1, where the total fusion barrier is plotted as a function of the distance s of the half-density surfaces. One notices that for distance $s \gtrsim 3$ fm the attractive nuclear part of the fusion potential vanishes and the barrier is determined by the repulsive Coulomb potential alone. The diffuseness of the charge-density distribution reduces the Coulomb energy at the fusion barrier by ≈1 MeV only. When,

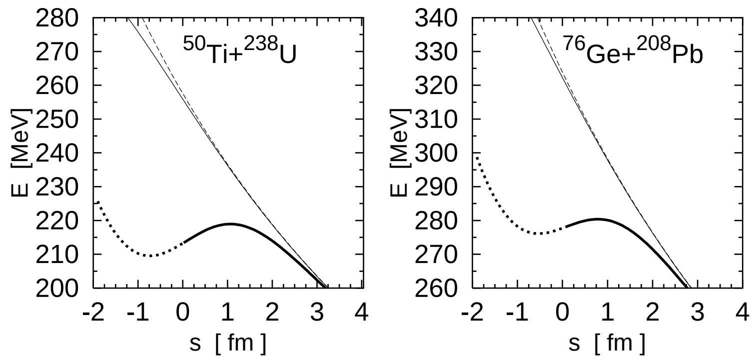

Fig. 1. The shapes of fusion barriers V_{12} as a function of the inter-nuclear distance s for two reactions leading to two different isotopes of the nucleus $Z = 114$. Comparison between the full (nuclear + Coulomb) potential (solid line), the Coulomb barrier (thin line) and the Coulomb barrier of two point charges (dashed line)

on the other hand, the overlap of the densities of the colliding nuclei becomes large ($s \lesssim -1$ fm) the sudden approximation used here becomes certainly more and more questionable. Our aim, however, is not to give a precise description of the fusion potential for large negative s values but rather to obtain some reasonable estimates of the height E_B of the fusion barrier.

Fig. 2 shows E_B obtained in the sudden approximation and neglecting the effect of deformation of the colliding ions for 54 different reactions leading all to different isotopes of the element $Z = 114$. The thick lines always correspond to the barriers obtained with the Skyrme force SLy4 [6], while the medium thick lines are the ones using the SkM* force [7]. The thin lines illustrate the heights of the fusion barriers obtained in the Myers–Swiatecki proximity model [8]. Barrier heights evaluated using the prescription of Ref. [9] are very close to the former for the Sr + ^{192}Os reactions and differ by not more than 1.5 MeV for Ge + ^{208}Pb.

As one can see, the heights of the barriers calculated with two widely used Skyrme forces differ from each other about 1.3 to 3 MeV and are higher by some 3 to 5 MeV as compared to the barriers obtained with the proximity force of Ref. [8].

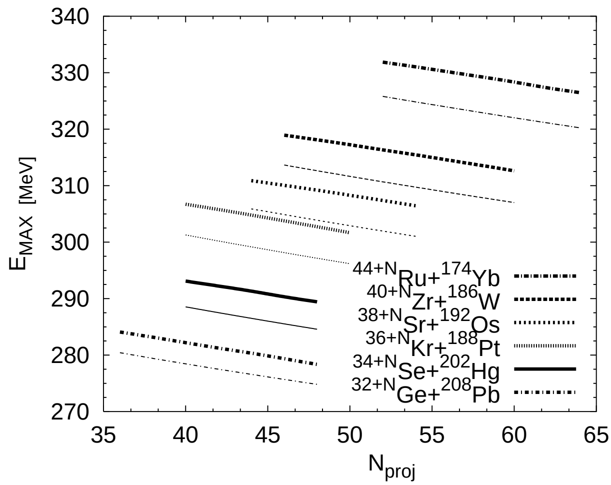

Fig. 2. Hight of fusion barriers as function of the projectile neutron number.

We have presented a model allowing a systematic investigation of different reactions leading to the same heavy element and hope that such a study can be used as a guide-line for the synthesis of superheavy elements.

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