QUADRUPOLE COLLECTIVE EXCITATIONS IN MEDIUM HEAVY TRANSITIONAL NUCLEI WITHIN A SELFCONSISTENT APPROACH WITH SKYRME FORCES*

L. Próchniak[a], P. Quentin[b,c], D. Samsoen[b], and J. Libert[d]

[a]Institute of Physics, Maria Curie-Skłodowska University
pl. M. Curie-Skłodowskiej 1, 20–031 Lublin, Poland
[b]CEN Bordeaux Gradignan, BP 120, 33175 Gradignan, France
[c]T Division, LANL, PB1663, Los Alamos, NM 87544 (USA)
[d]IPN, 91406 Orsay, France

(Received January 31, 2003)

We present an attempt to describe collective quadrupole excitations in medium heavy transitional nuclei starting from HF-BCS approach with Skyrme SIII forces. The collective dynamics is treated through the Bohr Hamiltonian with mass parameters and moments of inertia calculated microscopically in the cranking approximation. Theoretical energy levels and $B(E2)$ transition probabilities for ^{102}Zr, ^{104}Mo, ^{110}Ru, ^{110}Pd, ^{124}Xe and ^{126}Ba nuclei are compared with experiment.

PACS numbers: 21.10.Re, 21.60.Jz, 23.20.Lv, 27.60.+j

1. Introduction

It is known that medium heavy nuclei in the region $A = 100$–130, both neutron rich and neutron deficient ones, are interesting examples of transitional, γ soft nuclei where the concept of a triaxial deformation is important. Moreover recent experimental techniques such as in beam spectroscopy allows us to analyze in much detail the structure of low lying levels, *cf. e.g.* [1, 2]. Several theoretical approaches have been applied in studies of collective excitations in the discussed nuclear region. Some of them employ the Bohr Hamiltonian however their prescriptions for obtaining a potential energy and inertial functions are different from ours. The energy and the inertial functions are parametrized in a certain way (General Collective

* Presented at the XXXVII Zakopane School of Physics “Trends in Nuclear Physics”, Zakopane, Poland, September 3–10, 2002.

(Frankfurt) Model [3]) or are derived from a schematic interaction (as in Kumar–Baranger dynamic deformation model [4]) or from phenomenological one particle Nilsson potential [5,6]. Another alternative is the Interacting Boson Model [2] with all parameters fitted to experimental data.

In this paper we aim at providing a full dynamical description of the quadrupole modes in the discussed nuclei. We also use the Bohr Hamiltonian but we obtain the functions which enter the Hamiltonian from the effective nucleon–nucleon Skyrme interaction. On the other hand, the possibility of a coupling of the vibrational and rotational degrees of freedom differs our approach significantly from calculations applying the generator coordinator method (GCM) to the similar microscopic forces in Ref. [7–9].

The Bohr Hamiltonian is determined by seven functions: the potential energy, three moments of inertia and three mass parameters. The method used to obtain these functions from self consistent calculations is based on the Adiabatic Time Dependent Hartree Fock (Bogolyubov) approach [10–14]. However going this route we make use of one further approximation: we neglect so called Thouless–Valatin terms coming from the time-odd part of the density matrix. As a consequence we get the Inglis–Belayev formula for the moments of inertia and so called $M(Q)$ expression for the mass parameters [14,15]:

$$B_{kj} = \frac{\hbar^2}{2}\left(M_{(1)}^{-1} M_{(3)} M_{(1)}^{-1}\right)_{kj}, \tag{1}$$

$$M_{(n),kj} = \sum_{\mu,\nu} \frac{\langle\mu|Q_k|\nu\rangle\langle\nu|Q_j|\mu\rangle}{(e_\mu + e_\nu)^n}(u_\mu v_\nu + u_\nu v_\mu)^2, \tag{2}$$

where Q_j are the components of the mass quadrupole operator.

2. Details of the calculations

The results presented below have been obtained using the Skyrme SIII interaction and a seniority force in the p–p channel. Pairing strengths $G_{n,p} = g_{n,p}/(11 + \{N,Z\})$ have been fitted separately for neutron rich and neutron deficient nuclei and are given below.

TABLE I

Pairing strength parameters.

	g_n [MeV]	g_p [MeV]
^{102}Zr, ^{104}Mo, ^{110}Ru, ^{110}Pd	17.1	16.5
^{124}Xe, ^{126}Ba	18.0	17.5

Numerical calculations have been performed using the code described in [16], ie employing the harmonic oscillator eigenfunctions in the cylindrical coordinates. However the only symmetries imposed on the self consistent solutions are parity and signature thus allowing for triaxial shapes.

The collective β, γ variables are defined as $\beta\cos\gamma = q_0\sqrt{\pi/5}/A\langle r^2\rangle$, $\beta\sin\gamma = q_2\sqrt{3\pi/5}/A\langle r^2\rangle$, where q_0 and q_2 are expectation values of the Q_{20} and Q_{22} components of the quadrupole mass tensor.

The potential energy, the mass parameters and the moments of inertia have been calculated on a regular mesh with a step equal to 0.05 (6°) along the β (γ) direction. The mesh contains 144 points and covers the $(0, 0.65) \times (0°, 60°)$ sextant of the β, γ plane. To get a HF+BCS solution with a given deformation we use linear constraints on the components of the quadrupole mass tensor. Given the potential energy and mass parameters as the functions of the β, γ variables the eigenproblem of the Bohr Hamiltonian is solved using the method presented in [5].

3. Results

3.1. Energy levels

The figures below (Figs. 1, 2, 3) show a comparison between experimental [17] and calculated energy levels for the considered nuclei. The levels are grouped into g.s., quasi γ and quasi β bands (most of the nuclei are really γ soft, thus justifying the presence of the "quasi" prefix). The general conclusion is that the theoretical spectra are stretched significantly as compared with experiment, however we should remember that our calculations do not contain any free parameter. Such stretching can be explained by too small values of the inertial functions entering the Bohr Hamiltonian, this point will be discussed briefly in the last section.

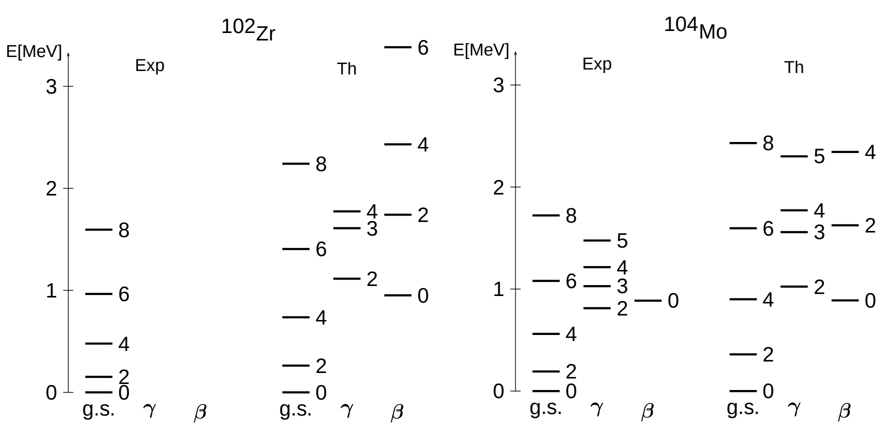

Fig. 1. Experimental [17] and theoretical energy levels for ^{102}Zr and ^{104}Mo nuclei.

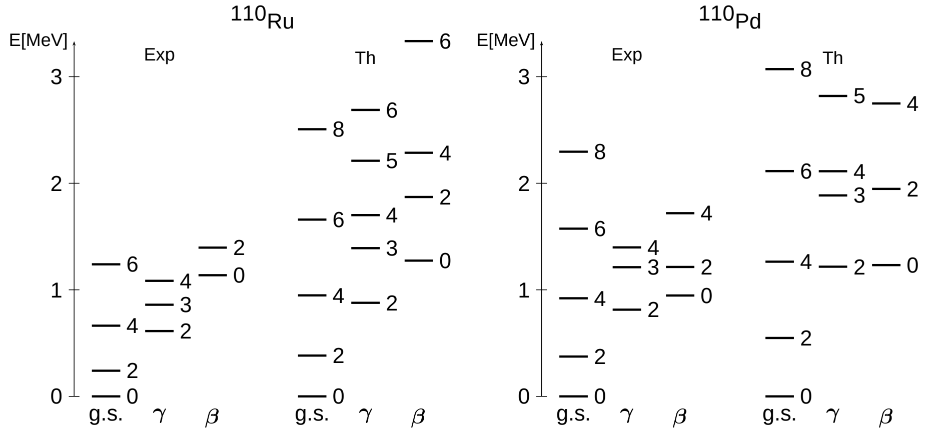

Fig. 2. Experimental [17] and theoretical energy levels for ^{110}Ru and ^{110}Pd nuclei.

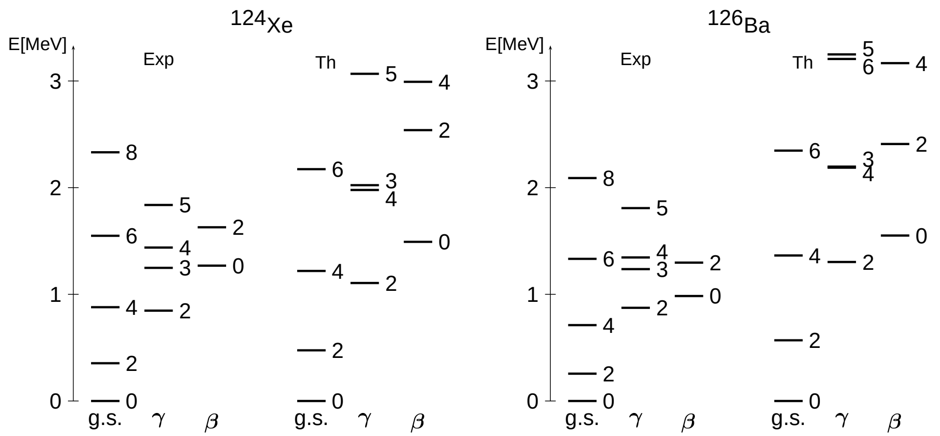

Fig. 3. Experimental [17] and theoretical energy levels for ^{124}Xe and ^{126}Ba nuclei.

3.2. E2 transition probabilities

The electromagnetic E2 transitions give us important information on the nuclear wavefunctions. In this subsection we present a sample of the results, namely $B(\mathrm{E2})$ probabilities for $2_1^+ \to$ g.s. transitions in the considered nuclei and transitions between low lying states in the ^{110}Pd isotope. Here we also do not fit any parameters. Figure 4 shows quite a good agreement with experiment, better than what was obtained for the energies.

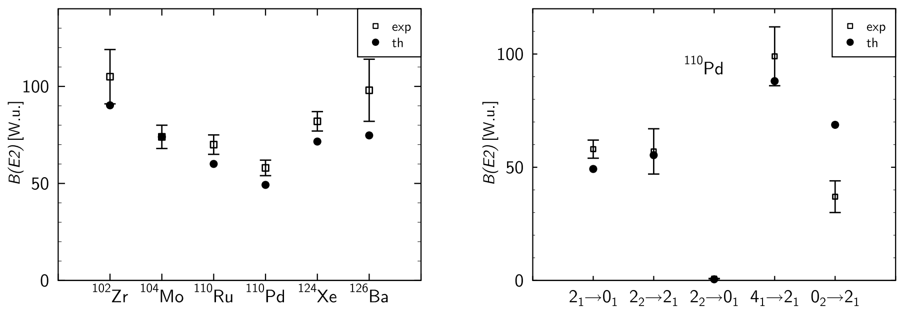

Fig. 4. Left panel: $B(E2)$ probabilities for $2_1^+ \rightarrow$ g.s.; right panel: $B(E2)$ transition probabilities in ^{110}Pd nucleus. Experimental data taken from [17].

4. Conclusions

Results of Section 3 show that in the presented approach general features of quadrupole collective excitations of the considered nuclei can be understood assuming only the well established microscopic nucleon–nucleon interaction. However the problem of too small mass parameters (too large energies) remains. Two different sources of corrections yielding an increase of the mass parameters have been discussed recently. The first source, characteristic for self consistent calculations, are the Thouless–Valatin terms mentioned in the Introduction. Preliminary, rough estimates (*cf.* [13,15,18]) give the 10%-15% increase of the mass parameters, which leads to decrease of the collective energies of a significant amount. The second source, proposed in [5,19], is the renormalization of the pairing gaps due to the coupling with the pairing degrees of freedom. The work on more extensive studies of these problems is in progress.

Present work is supported in part by the Polish State Committee for Scientific Research (KBN) grant No 2 P03B04119 and by the Convention IN2P3 — Lab. Polonaises No 99–95.

REFERENCES

[1] J.C. Wang *et al.*, *Phys. Rev.* **C61**, 044308 (2000).

[2] V. Werner, H. Meise, I. Wiedenhover, A. Gade, P. von Brentano, *Nucl. Phys.* **A692**, 451 (2001).

[3] P. Petkov, A. Dewald, W. Andrejtscheff, *Phys. Rev.* **C51**, 2511 (1995).

[4] K. Kumar, J.B. Gupta, *Nucl. Phys.* **A694**, 199 (2001).

[5] L. Próchniak, K. Zajac, K. Pomorski, S.G. Rohoziński, J. Srebrny, *Nucl. Phys.* **A648**, 181 (1999).

[6] K. Zajac, L. Próchniak, K. Pomorski, S.G. Rohoziński, J. Srebrny, *Nucl. Phys.* **A653**, 71 (1999).

[7] P. Bonche, J. Dobaczewski, H. Flocard, P.-H. Heenen, *Nucl. Phys.* **A530**, 149 (1991).

[8] J. Skalski, P.-H. Heenen, P. Bonche, *Nucl. Phys.* **A559**, 221 (1993).

[9] P.-H. Heenen, P. Bonche, J. Dobaczewski, H. Flocard, *Nucl. Phys.* **A561**, 367 (1993).

[10] M. Baranger, M. Vénéroni, *Ann. Phys. (NY)* **114**, 123 (1978).

[11] M.J. Giannoni, F. Moreau, P. Quentin, D. Vautherin, M. Vénéroni, D.M. Brink, *Phys. Lett.* **65B**, 305 (1976).

[12] M.J. Giannoni, P. Quentin, *Phys. Rev.* **C21**, 2060 (1980).

[13] M.J. Giannoni, P. Quentin, *Phys. Rev.* **C21**, 2076 (1980).

[14] E.Kh. Yuldashbaeva, J. Libert, P. Quentin, M. Girod, *Phys. Lett.* **461B**, 1 (1999).

[15] J. Libert, M. Girod, J.-P. Delaroche, *Phys. Rev.* **C60**, 054301 (1999).

[16] D. Samsoen, P. Quentin, J. Bartel, *Nucl. Phys.* **A652**, 34 (1999).

[17] Data extracted using the NNDC On-Line Data Service from the ENSDF database, file revised as of April 2002.

[18] P. Quentin, H. Lafchiev, D. Samsoen, I.N. Mikhailov, paper in preparation.

[19] S. Piłat, K. Pomorski, *Nucl. Phys.* **A554**, 413 (1993).