# RARE $\boldsymbol{B}_{\boldsymbol{s}} \boldsymbol{\rightarrow} \boldsymbol{\nu} \overline{\boldsymbol{\nu}} \boldsymbol{\gamma}$ DECAY BEYOND THE STANDARD MODEL 

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Using the most general model independent form of the effective Hamiltonian the rare decay $B_{s} \rightarrow \nu \bar{\nu} \gamma$ is studied. The sensitivity of the photon energy distribution and branching ratio to new Wilson coefficients are investigated.

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## 1. Introduction

The experimental observation of the $b \rightarrow s \gamma[1]$ and $B \rightarrow X_{s} \gamma$ [2] processes opened a window to study possible rare $B$ meson decays, which will give important information about Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. In the Standard Model rare $B_{s} \rightarrow \nu \bar{\nu}$ decay is forbidden due to the helicity conservation. When photon is emitted from structure dependent part (see below) the helicity conservation allows the decay $B_{s} \rightarrow \nu \bar{\nu} \gamma$. Therefore, the investigation of the $B_{s} \rightarrow \nu \bar{\nu} \gamma$ rare decay becomes interesting.

The main interest in studying $B$ meson decays is to test the Standard Model (SM) predictions at loop level and extract new physics effects beyond the SM. The loop induced rare decays $B_{s}\left(B_{d}\right) \rightarrow \nu \bar{\nu} \gamma$ within the SM have been studied using constituent quark model and pole model and the branching ratios are found to be $10^{-8}$ for $B_{s} \rightarrow \nu \bar{\nu} \gamma$ and $10^{-9}$ for $B_{d} \rightarrow \nu \bar{\nu} \gamma$ [3].

The decay rate of the $B_{s} \rightarrow \nu \bar{\nu} \gamma$ might have an enhancement comparing with the pure leptonic modes of $B$ meson. The new physics effects beyond SM can be also probed by studying this $B_{s} \rightarrow \nu \bar{\nu} \gamma$ rare decay properties.

In this paper, we study the sensitivity of the physically measurable quantities, branching ratio, photon energy distribution, to the new physics effects using the most general form of the effective Hamiltonian. The effects of the new Wilson coefficients $C_{X}$ on branching ratio and photon energy distribution are also investigated.

## 2. Effective Hamiltonian

The most general model independent form of the effective Hamiltonian for the process $b \rightarrow q \nu \bar{\nu}$ can be written in the following form [4,5];

$$
\begin{align*}
H_{\mathrm{eff}}= & \frac{G_{\mathrm{F}} \alpha V_{t b} V_{t s}^{*}}{4 \sqrt{2} \sin ^{2} \theta_{w}}\left\{C_{L L}^{\mathrm{tot}} \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{\nu} \gamma^{\mu}\left(1-\gamma_{5}\right) \nu\right. \\
& +C_{L R}^{\mathrm{tot}} \bar{s} \gamma_{\mu}\left(1-\gamma_{5}\right) b \bar{\nu} \gamma^{\mu}\left(1+\gamma_{5}\right) \nu \\
& +C_{R L} \bar{s} \gamma_{\mu}\left(1+\gamma_{5}\right) b \bar{\nu} \gamma^{\mu}\left(1-\gamma_{5}\right) \nu+C_{R R} \bar{s} \gamma_{\mu}\left(1+\gamma_{5}\right) b \bar{\nu} \gamma^{\mu}\left(1+\gamma_{5}\right) \nu \\
& +C_{L R L R} \bar{s}\left(1+\gamma_{5}\right) b \bar{\nu}\left(1+\gamma_{5}\right) \nu+C_{R L L R} \bar{s}\left(1-\gamma_{5}\right) b \bar{\nu}\left(1+\gamma_{5}\right) \nu \\
& +C_{L R R L} \bar{s}\left(1+\gamma_{5}\right) b \bar{\nu}\left(1-\gamma_{5}\right) \nu+C_{R L R L} \bar{s}\left(1-\gamma_{5}\right) b \bar{\nu}\left(1-\gamma_{5}\right) \nu \\
& \left.+C_{T} \bar{s} \sigma_{\mu \nu} b \bar{\nu} \sigma^{\mu \nu} \nu+i C_{T E} \in^{\mu \nu \alpha \beta} \bar{s} \sigma_{\mu \nu} b \bar{\nu} \sigma_{\alpha \beta} \nu\right\} \tag{1}
\end{align*}
$$

where the projection operators $L$ and $R$ in Eq. (1) are defined as $L=\left(1-\gamma_{5}\right) / 2, R=\left(1+\gamma_{5}\right) / 2$, and $C_{X}$ are the coefficients of the fourFermi interactions. In this work, we restrict ourselves by considering only Dirac neutrino in order to avoid additional lepton-number-violating operators. Furthermore, the states $\nu_{L}$ and $\nu_{R}$ are well separated in the massless Dirac neutrino case. However, for the massive neutrino case we can change chirality with help of the mass, and therefore in this case it is necessary the right-handed neutrino to be heavy. In general, the Wilson coefficients for $b \rightarrow s l^{+} l^{-}$and $b \rightarrow s \nu \bar{\nu}$ processes are different. However, operator structures in both case must be the same, since for massive neutrino case charged lepton and neutrino except their electric charge are the same with respect to the $\mathrm{SU}(2)$. First four terms in the effective Hamiltonian are the vector interactions. The interaction terms with coefficients $C_{L L}^{\mathrm{tot}}$ and $C_{L R}^{\mathrm{tot}}$ which are present in the SM are given in the form,

$$
\begin{aligned}
& C_{L L}^{\mathrm{SM}}=C_{9}^{\mathrm{eff}}-C_{10} \\
& C_{L R}^{\mathrm{SM}}=C_{9}^{\mathrm{eff}}+C_{10}
\end{aligned}
$$

The contributions from the new physics can be described by redefining the Wilson coefficients as $C_{L L(L R)}^{\mathrm{tot}}=C_{L L(L R)}^{\mathrm{SM}}-C_{X}$. The coefficients $C_{L R L R}$, $C_{R L L R}, C_{L R R L}$ and $C_{R L R L}$ describe the scalar type interactions which disappear in our calculations for $B_{s} \rightarrow \nu \bar{\nu} \gamma$ process. The remaining last two coefficients in Eq. (1), correspond to tensor type interactions. In general,
these operators are possible and they can appear from the exchange of spin-2 particles. It is obvious that the effects of the $C_{T}$ and $C_{T E}$ operators will be larger, because there are more Lorentz indices, which means that summing over them will lead to larger effects. In order to calculate the matrix element for $B_{s} \rightarrow \nu \bar{\nu} \gamma$ decay we use the general form of the effective Hamiltonian and the standard definitions for the matrix elements [6-8];

$$
\begin{align*}
\langle\gamma(q)| \bar{s} \gamma_{\mu}\left(1 \mp \gamma_{5}\right) b\left|B\left(p_{B}\right)\right\rangle= & \frac{e}{m_{B}^{2}}\left\{\epsilon_{\mu \nu \lambda \sigma} \varepsilon^{* \nu} p^{\lambda} q^{\sigma} g\left(p^{2}\right) \pm i\left[\varepsilon^{* \mu}(p q)\right.\right. \\
& \left.\left.-\left(\varepsilon^{*} p\right) q^{\mu}\right] f\left(p^{2}\right)\right\}  \tag{2}\\
\langle\gamma(q)| \bar{s} \sigma_{\mu \nu} b\left|B\left(p_{B}\right)\right\rangle= & \frac{e}{m_{B}^{2}} \epsilon_{\mu \nu \lambda \sigma}\left[G \varepsilon^{* \lambda} q^{\sigma}+H \varepsilon^{* \lambda} p^{\sigma}+N\left(\varepsilon^{*} p\right) p^{\lambda} q^{\sigma}\right],  \tag{3}\\
\langle\gamma(q)| \bar{s}\left(1 \mp \gamma_{5}\right) b\left|B\left(p_{B}\right)\right\rangle= & 0 \tag{4}
\end{align*}
$$

where $\varepsilon_{\mu}^{*}$ is the polarization vector of the photon. The $p, q$ and $p_{B}$ are the transfer momentum, photon momentum and the momentum of $B$ meson, respectively. Using Eqs. (2), (3) and (4) the matrix element for the process can be calculated as follows:

$$
\begin{align*}
M= & \frac{\alpha G_{\mathrm{F}}}{4 \sqrt{2}} V_{t b} V_{t s}^{*} \frac{e}{m_{B}^{2}}\left\{\bar{\nu} \gamma^{\mu}\left(1-\gamma_{5}\right) \nu\right. \\
& \times\left[A_{1} \in_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p^{\alpha} q^{\beta}+i A_{2}\left(\varepsilon_{\mu}^{*}(p q)-\left(\varepsilon^{*} p\right) q_{\mu}\right)\right] \\
& +\bar{\nu} \gamma^{\mu}\left(1+\gamma_{5}\right) \nu\left[B_{1} \in_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p^{\alpha} q^{\beta}+i B_{2}\left(\varepsilon_{\mu}^{*}(p q)-\left(\varepsilon^{*} p\right) q_{\mu}\right)\right] \\
& +i \in_{\mu \nu \alpha \beta} \bar{\nu} \sigma^{\mu \nu} \nu\left[G \varepsilon^{* \alpha} q^{\beta}+H \varepsilon^{* \alpha} p^{\beta}+N\left(\varepsilon^{*} p\right) p^{\alpha} q^{\beta}\right] \\
& +i \bar{\nu} \sigma_{\mu \nu} \nu\left[G_{1}\left(\varepsilon^{* \mu} q^{\nu}-\varepsilon^{* \nu} q^{\mu}\right)+H_{1}\left(\varepsilon^{* \mu} p^{\nu}-\varepsilon^{* \nu} p^{\mu}\right)\right. \\
& \left.\left.+N_{1}\left(\varepsilon^{*} p\right)\left(p^{\mu} q^{\nu}-p^{\nu} q^{\mu}\right)\right]\right\} \tag{5}
\end{align*}
$$

where we have used the definitions [6];

$$
\begin{aligned}
A_{1} & =\left(C_{L L}^{\mathrm{tot}}+C_{R L}\right) g, & A_{2} & =\left(C_{L L}^{\mathrm{tot}}-C_{R L}\right) f, \\
B_{1} & =\left(C_{L R}^{\mathrm{tot}}+C_{R R}\right) g, & B_{2} & =\left(C_{L R}^{\mathrm{tot}}-C_{R R}\right) f \\
G & =4 C_{T} g_{1}, & N & \\
& & =-4 C_{T} \frac{\left(f_{1}+g_{1}\right)}{p^{2}}, & H=N(p q) \\
G_{1} & =-8 C_{T E} g_{1}, & N_{1}=8 C_{T E} \frac{\left(f_{1}+g_{1}\right)}{p^{2}}, & H_{1}=N_{1}(p q) .(6)
\end{aligned}
$$

with $p^{2}=m_{B}^{2}(1-x), p q=m_{B}^{2} x / 2, x=2 E_{\gamma} / m_{B}$ where $E_{\gamma}$ is the photon energy. The double differential decay width of the $B_{s} \rightarrow \nu \bar{\nu} \gamma$ process in the rest frame of the $B$ meson can be written in the form

$$
\begin{equation*}
\frac{d \Gamma}{d x d E_{1}}=\frac{1}{128 \pi^{3}}|M|^{2} \tag{7}
\end{equation*}
$$

The bounds of the final neutrino energy $E_{1}$ and the dimensionless parameter $x$ for photon energy are determined from the following inequalities

$$
\begin{equation*}
\frac{m_{B}}{2}-E_{\gamma} \leq E_{1} \leq \frac{m_{B}}{2}, \quad 0 \leq x \leq 1 \tag{8}
\end{equation*}
$$

In our calculations, we consider hard photon in the process $B_{s} \rightarrow \nu \bar{\nu} \gamma$. For the experimental observability of photon we impose a cut on the minimum energy to be greater than 25 MeV which corresponds to $x \geq 0.01$ [6,8,9]. We integrate the differential decay width over the neutrino energy $E_{1}$ to get the photon energy distribution

$$
\begin{align*}
& \frac{d \Gamma}{d x}=-\left|\frac{\alpha G_{\mathrm{F}}}{4 \sqrt{2}} V_{t b} V_{t s}^{*}\right|^{2} \frac{\alpha}{(2 \pi)^{3}} \frac{\pi}{4} m_{B} x^{3}\left\{-4\left[\left|H_{1}\right|^{2}(1-x)+\operatorname{Re}\left(G_{1} H_{1}^{*}\right) x\right] \frac{1-x}{x^{2}}\right. \\
& -4\left[|H|^{2}(1-x)+\operatorname{Re}\left(G H^{*}\right) x\right] \frac{1-x}{x^{2}} \\
& +\frac{1}{3} m_{B}^{2}\left[2 \operatorname{Re}\left(G N^{*}\right)+m_{B}^{2}|N|^{2}(1-x)\right](1-x) \\
& +\frac{1}{3} m_{B}^{2}\left[2 \operatorname{Re}\left(G_{1} N_{1}^{*}\right)+m_{B}^{2}\left|N_{1}\right|^{2}(1-x)\right](1-x) \\
& \left.-\frac{2}{3} m_{B}^{2}\left[\left(\left|A_{1}\right|^{2}+\left|A_{2}\right|^{2}+\left|B_{1}\right|^{2}+\left|B_{2}\right|^{2}\right)(1-x)\right]-\frac{4}{3}\left(|G|^{2}+\left|G_{1}\right|^{2}\right)\right\} .(9) \tag{9}
\end{align*}
$$

The form factors $f, g, f_{1}$ and $g_{1}$, which we have used in our numerical calculations, appearing in Eq. (6) can be obtained in the framework of light cone QCD sum rules given in $[7,8]$ and their $x$ dependence with a good accuracy:

$$
\begin{align*}
f(x)=\frac{0.8 \mathrm{GeV}}{\left(1-\frac{(4.8)^{2}(1-x)}{(6.5)^{2}}\right)^{2}}, & g(x)=\frac{1 \mathrm{GeV}}{\left(1-\frac{(4.8)^{2}(1-x)}{(5.6)^{2}}\right)^{2}} \\
f_{1}(x)=\frac{0.68 \mathrm{GeV}^{2}}{\left(1-\frac{(4.8)^{2}(1-x)}{30}\right)^{2}}, & g_{1}(x)=\frac{3.74 \mathrm{GeV}^{2}}{\left(1-\frac{(4.8)^{2}(1-x)}{40.5}\right)^{2}} \tag{10}
\end{align*}
$$

## 3. Results and discussions

We use the main input parameters $m_{B}=5.28 \mathrm{GeV},\left|V_{t b} V_{t s}^{*}\right|=0.045$, $\alpha^{-1}=137, G_{\mathrm{F}}=1.17 \times 10^{-5} \mathrm{GeV}^{-2}$ in the numerical results. For the Wilson coefficients we take the values $C_{9}^{\text {eff }}=4.344$ and $C_{10}=4.6242$ given in $[6,10]$.

In this work, we assume that all new Wilson coefficients $C_{X}$ are real and vary in the region $-4 \leq C_{X} \leq+4$. The experimental bounds on the branching ratios of the $B \rightarrow K^{\star} \mu^{+} \mu^{-}$and $B_{s} \rightarrow \mu^{+} \mu^{-}$[11] suggest that this is the right order of magnitude range for the vector and scalar interactions
coefficients. Therefore, we assume that all new Wilson coefficients change in this range. The integrated branching ratio for the rare $B_{s} \rightarrow \nu \bar{\nu} \gamma$ decay depending on the new Wilson coefficients $C_{T}, C_{T E}, C_{R R}, C_{R L}, C_{L L}, C_{L R}$ is plotted in Fig. 1. It is clear from this figure that branching ratio increases when all new Wilson coefficients $C_{X}>0$ increase, for example, in the region $-4 \leq C_{T}, C_{T E} \leq 0$ branching ratio decreases and it increases in the region $0 \leq C_{T}, C_{T E} \leq+4$. Furthermore, the branching ratio remain approximately unchanged when the coefficients $C_{R R}, C_{L R}$ varies and there is no sensitivity to these coefficients. The branching ratio weakly depends on the $C_{R R}, C_{R L}, C_{L R}$ coefficients.


Fig. 1. The branching ratio of the rare $B_{s} \rightarrow \nu \bar{\nu} \gamma$ decay depending on the new Wilson coefficients $C_{X}$ for the parameter cut $x_{\min }=0.01$.

However, one can conclude that the branching ratio is more sensitive to the tensor type $C_{T}$ and $C_{T E}$ coefficients. We find the branching ratio $B\left(B_{s} \rightarrow \nu \bar{\nu} \gamma\right)=1.2 \times 10^{-8}$ for new Wilson coefficients are set to zero. The branching ratio for the contact interactions give symmetrical distribution with respect to zero. Measuring this branching ratio can give an information about the magnitude of this type tensor interactions, see Fig. 2. In addition, the $C_{R L}$ and $C_{L L}$ distribution can give an opportunity to detect the sign of these coefficients. We see that in dependence of the sign of the tensor interaction $C_{T E}$ differential branching ratio can be larger or smaller than SM prediction.


Fig. 2. The differential branching ratio for rare $B_{s} \rightarrow \nu \bar{\nu} \gamma$ depending on the dimensionless variable $x=2 E_{\gamma} / m_{B}$ for the different values of tensor interaction coefficient $C_{T E}$.

The photon energy distribution can also give information about new physics effects. In Fig. 3, we present the differential branching ratio for the rare $B_{s} \rightarrow \nu \bar{\nu} \gamma$ decay as a function of dimensionless variable $x$ for different values of coefficient $C_{L L}$. In this view the differential branching ratio measurement could give important information about the sign of new Wilson coefficient. The higher sensitivity can be obtained when the photon energy reaches $\sim 0.6 G e V$.

In conclusion, using a general model independent effective Hamiltonian for the process $B_{s} \rightarrow \nu \bar{\nu} \gamma$, the branching ratio and the photon energy distribution are found to be sensitive to the existence of new physics beyond the SM . Within a reasonable range of branching ratios, it would be possible to detect the rare processes in the future B-factories. At planning LHCB and B TeV hadronic machines $10^{11}-10^{12} b b$ pair per years [12] will be produced. Therefore, the number of expected events are $N \approx 10^{3}-10^{4}$, which quite detectable this decay in the above mentioned colliders. Note that signature of this decay will be single photon and missing energy.


Fig. 3. The differential branching ratio for rare $B_{s} \rightarrow \nu \bar{\nu} \gamma$ depending on the dimensionless variable $x=2 E_{\gamma} / m_{B}$ for the different values of vector interaction coefficient $C_{L L}$.

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## REFERENCES

[1] R. Ammar et al., [CLEO Colloboration], Phys. Rev. Lett. 71, 674 (1993).
[2] M.S. Alam et al., [CLEO Colloboration], Phys. Rev. Lett. 74, 2885 (1995).
[3] C.D. Lü, D.X. Zhang, Phys. Lett. B381, 348 (1996).
[4] T.M. Aliev, C.S. Kim, Y.G. Kim, Phys. Rev. D62, 014026 (2000).
[5] T.M. Aliev, D.A. Demir, M. Savci, Phys. Rev. D62, 074016 (2000).
[6] T.M. Aliev, A. Özpineci, M. Savci, Phys. Lett. B520, 69 (2001).
[7] G. Eilam, I. Halperin, R.R. Mendel, Phys. Lett. B361, 137 (1995).
[8] T.M. Aliev, A. Özpineci, M. Savci, Phys. Rev. D55, 7059 (1997).
[9] T.M. Aliev, A. Özpineci, M. Savci, Phys. Lett. B393, 141 (1997).
[10] F. Krüger, L.M. Sehgal, Phys. Lett. B380, 199 (1996).
[11] T. Affolder et al. ., CDF Collaboration, Phys. Rev. Lett. 83, 3278 (1999); Yongsheng Gao et al. ., [CLEO Collaboration], hep-ph/0108005.
[12] N. Harnew, Proc. 8th Heavy Flavour Conference, Southampton (1999).

