CONNECTING NONLEPTONIC AND WEAK RADIATIVE HYPERON DECAYS

P. Żenczykowski

Department of Theoretical Physics
H. Niewodniczański Institute of Nuclear Physics
Radzikowskiego 152, 31-342 Kraków, Poland
E-mail: zenczyko@iblis.ifj.edu.pl

(Received January 17, 2003)

Using the recent measurement of the $\Xi^0 \to \Lambda\gamma$ asymmetry as an input, we reanalyse nonleptonic and weak radiative hyperon decays in a single symmetry-based framework. In this framework the old S:P problem of nonleptonic decays is automatically resolved when the most important features of weak radiative decays are taken into account as an input. Experimental data require that symmetry between the two types of hyperon decays be imposed at the level of currents, not fields. Previously established connections between hyperon decays and nuclear parity violation imply that the conflict, originally suggested by weak radiative decays, has to surface somewhere.

PACS numbers: 11.40.–q, 11.30.Hv, 13.30.–a, 14.20.Jn

For a long time weak hyperon decays have been presenting us with a couple of puzzles (see [1, 2]). These have been in particular: the question of the S:P ratio in the nonleptonic hyperon decays (NLHD) and the issue of a large negative asymmetry in the $\Sigma^+ \to p\gamma$ weak radiative hyperon decay (WRHD), the latter being indicative of either SU(3) breaking effects much larger than expected, or of Hara's theorem being violated. Violation of this theorem, although forbidden on the basis of hadron-level arguments, was also suggested by a couple of (technically correct) quark-level calculations (constituent quark model, CQM) [3–5]. In the CQM calculations the constituent quarks in the intermediate states between the action of weak interaction and the emission of a photon are essentially free. Violation of Hara's theorem followed also when NLHD and WRHD were connected via the vector-meson dominance (VMD) approach [6].

Some time ago it was pointed out [2] that the status of Hara's theorem can be clarified through the measurement of the $\Xi^0 \to \Lambda\gamma$ decay asymmetry. By yielding a large and negative value of -0.65 ± 0.19 for this asymmetry, the recent NA48 experiment [7] has decided very clearly in favour of the theorem. The experimental number disagrees very strongly with the results of the CQM calculations [4, 8], and with the VMD approach [4, 6, 9], which both yield large positive value for this asymmetry. Consequently, we are forced to conclude that

(a) constituent quark calculations do not provide us with a proper description of weak hyperon decays, and

(b) the existing description of NLHD and/or the connection between NLHD and WRHD used in the VMD-based approach of [6] do not correspond to physical reality.

The aim of this paper is to present a symmetry-based explanation of both the measured S:P ratio in NLHD and the gross structure of the observed pattern of asymmetries and branching ratios in WRHD, an explanation which maintains an intimate connection between NLHD and WRHD, and yet does not lead to the CQM/VMD results of [3, 6, 8, 9]. Fragments of this explanation have been known for over twenty years now [10, 11], but they have never been presented in a way clearly highlighting the underlying simple symmetry connection.

One generally expects that NLHD and WRHD should be related (see *e.g.* [6, 12]), and that this should hold both for the parity-conserving (p.c.) and the parity-violating (p.v.) amplitudes. Theoretical and phenomenological analyses of p.c. hyperon decay amplitudes have shown many times that there are no basic problems here, only some numerological differences. The p.c. NLHD amplitudes are sufficiently well described by the pole model, while the p.c. WRHD amplitudes may be estimated from NLHD through $\mathrm{SU}(2)_W$ spin symmetry with the help of VMD (or in another effectively equivalent way), always leading to qualitatively similar results. Probably the most reliable phenomenological evaluation of p.c. WRHD amplitudes along the symmetry lines was carried out in Ref. [9]. The real problem, as troubles with Hara's theorem indicated, is with the p.v. amplitudes.

In order to connect the asymmetries of NLHD with those of WRHD, we need to be consistent when fixing the relative signs between p.c. and p.v. amplitudes of both NLHD and WRHD. Our conventions are the same as those in Ref. [6] (Table IV, Eq. (5.2b)). For the purposes of this paper, it is sufficient to recall the following expressions for the p.c. amplitudes:

$$B(\Xi^0 \to \Lambda\pi^0) \;=\; \frac{1}{2\sqrt{3}}\left[\left(3\frac{f}{d}-1\right)\frac{F_A}{D_A}+3-\frac{f}{d}\right]C\,, \tag{1}$$

$$B(\Xi^0 \to \Lambda\gamma) \;=\; \frac{e}{g_\rho}\left[-\frac{4}{3\sqrt{3}}C\right], \tag{2}$$

$$B(\Xi^0 \to \Sigma^0\gamma) \;=\; \frac{e}{g_\rho}\left[-\frac{4}{3}C\right], \tag{3}$$

where $g_\rho = 5.0$, $F_A/D_A \approx 0.56$, and $f/d \approx -1.8$ or -1.9. The factor of e/g_ρ, reminiscent of VMD, follows from the replacement of the strong coupling with the electromagnetic one, *i.e.* its appearance does not require VMD (but is consistent with it) [4]. The $\Xi^0 \to \Lambda\gamma$ and $\Xi^0 \to \Sigma^0\gamma$ p.c. amplitudes are of the same sign. Furthermore, the ratio $B(\Xi^0 \to \Lambda\gamma)/B(\Xi^0 \to \Lambda\pi^0)$ is around $-3e/g_\rho$, *i.e.* negative.

For the p.v. amplitudes, with the experiment forcing us to abandon the constituent quark description [4, 7], we also turn to hadron-level approaches. In the approach of Ref. [6], the WRHD p.v. amplitudes were calculated using symmetry from the NLHD p.v. amplitudes through the chain of connections: $\pi \overset{\mathrm{SU}(6)_W}{\longrightarrow} \rho\ (\omega,\phi) \overset{\mathrm{VMD}}{\longrightarrow} \gamma$. The basic assumptions were VMD, $\mathrm{SU}(6)_W$, and the assumption that the p.v. NLHD amplitudes are well described by the current-algebra (CA) commutator term. In Ref. [13] it was proved that the soft meson CA approach and the SU(6)-symmetric quark-line diagram approach are totally equivalent in a group-theoretical sense. The perturbative QCD effects can yield only nonleading corrections to the simple quark-diagram scheme. The confining effects require going from quark to hadron level of description, in which quarks are treated as spin-flavour indices of effectively local hadron fields. This is how the whole $\mathrm{SU}(6)_W$ quark diagram scheme is understood. Although in principle the nonperturbative effects could affect the simple quark diagram scheme, they are not expected to do so: the quark diagram scheme works well in many places, somehow including all such effects (see also the comment on p. 338 of Ref. [1]). In conclusion, the quark-diagram approach to NLHD (Ref. [6]) was completely consistent with current algebra. When generalised to WRHD, this approach predicted large positive asymmetry for the $\Xi^0 \to \Lambda\gamma$ decay [9]. As this prediction strongly disagrees with the data, the following two questions emerge:

(i) Is the input in the chain of connections (*i.e.* the p.v. NLHD amplitudes) understood sufficiently well? (The S:P puzzle indicates there may be a problem here.)

(ii) Is the chain of connections itself correct (*i.e.* are $SU(6)_W$ and VMD applied in a proper way), and — if not — how to modify it?

The assumption of $SU(6)_W$ relates the relative sizes of contributions to the p.v. amplitudes corresponding to the diagrams shown in Fig. 1, but only for contributions from a single class: either ($b1$) or ($b2$), *etc.* It does not relate ($b1$) to ($b2$) (or to ($c1$) or ($c2$)), and it does not connect NLHD with WRHD. Fixing the relative sizes and signs of contributions from these four types of diagrams, both for NLHD and WRHD separately, as well as between NLHD and WRHD, requires additional assumptions that go beyond mere $SU(6)_W$. In other words the $SU(6)_W$ quark diagram approach has to be properly augmented, so that it indicates not only the contractions of quark indices, but also the way in which various diagrammatic amplitudes are to be combined (*i.e.* what are their relative strengths and sizes). For example, the soft meson term corresponds to a particular combination of diagrammatic amplitudes, as adopted in [13]. However, CA admits a correction to this term (the correction being proportional to meson momentum), which corresponds to a different combination of diagrammatic amplitudes [14]. Similarly, the constituent quark model fixes (in disagreement with experiment [4]) the relative signs and sizes of all (b)-type NLHD and WRHD amplitudes.

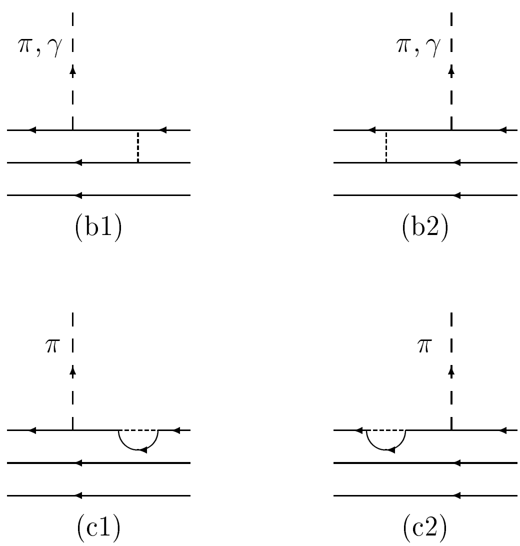

Fig. 1. $SU(6)_W$ diagrams for weak hyperon decays.

The $SU(6)_W$ coefficients with which different amplitude types contribute to different decays were calculated in the past (see [15]), and are gathered in Table I for NLHD, and in Table II for WRHD. Table II shows only the coefficients appropriate for (b)-type diagrams as the smallness of the measured $\Xi^- \to \Sigma^-\gamma$ branching ratio implies that the total single-quark contribution (which involves (c)-type diagrams) is negligible.

TABLE I

$\mathrm{SU}(6)_W$ coefficients for NLHD

	Decay k	$b_1(k)$	$b_2(k)$	$c_1(k)$	$c_2(k)$
Σ_0^+	$\Sigma^+ \to p\pi^0$	0	$\frac{1}{2\sqrt{2}}$	$-\frac{1}{6\sqrt{2}}$	0
Σ_-^-	$\Sigma^- \to n\pi^-$	0	$-\frac{1}{2}$	$\frac{1}{6}$	0
Λ_0^0	$\Lambda \to n\pi^0$	0	$\frac{1}{4\sqrt{3}}$	$-\frac{1}{4\sqrt{3}}$	0
Ξ_0^0	$\Xi^0 \to \Lambda^0\pi^0$	0	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{4\sqrt{3}}$	0

TABLE II

$\mathrm{SU}(6)_W$ coefficients for WRHD

Decay k	$b_1(k)$	$b_2(k)$
$\Sigma^+ \to p\gamma$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$
$\Lambda \to n\gamma$	$\frac{1}{6\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$
$\Xi^0 \to \Lambda\gamma$	0	$-\frac{1}{3\sqrt{3}}$
$\Xi^0 \to \Sigma^0\gamma$	$\frac{1}{3}$	0

Experimental p.v. NLHD amplitudes $A_{\mathrm{NL}}(k)$ are phenomenologically very well described by

$$A_{\mathrm{NL}}(k) = b_2(k)\ b_{\mathrm{NL}} + c_1(k)\ c_{\mathrm{NL}} \tag{4}$$

with

$$b_{\mathrm{NL}} = -5\,, \tag{5}$$
$$c_{\mathrm{NL}} = 12 \tag{6}$$

(in units of 10^{-7}), which corresponds to the S-wave SU(3) parameters

$$d_S = b_{\mathrm{NL}}\,, \tag{7}$$
$$f_S = -b_{\mathrm{NL}} + \frac{2}{3}c_{\mathrm{NL}}\,, \tag{8}$$

satisfying

$$\frac{f_S}{d_S} = -1 + \frac{2}{3}\frac{c_{\rm NL}}{b_{\rm NL}} = -2.6\,. \tag{9}$$

Experimental values of the $\Xi^0 \to \Lambda\gamma$ and $\Xi^0 \to \Sigma^0\gamma$ branching ratios are well described when, after factorising the b_i coefficients and replacing the electromagnetic couplings in the WRHD amplitudes with their strong counterparts, the moduli of the (rescaled) SU(6)$_W$ p.v. WRHD amplitudes for diagrams (b1) and (b2) (Fig. 1) are both *numerically* (approximately) equal to $|b_{\rm NL}|$:

$$|b_{\rm WR}(b1)| = |b_{\rm WR}(b2)| \approx |b_{\rm NL}| \tag{10}$$

(the two branching ratios in question do *not* depend on the signs of $b_{\rm WR}(b1)$ and $b_{\rm WR}(b2)$, see [2]). Incidentally, this agreement shows that the effects of SU(3) breaking in these two decays are not large, as discussed in Refs. [2,15] (the dominant terms in the p.v. amplitudes are SU(3) symmetric). Furthermore, with p.c. amplitudes of these decays being of equal sign (Eqs. (2), (3)), it follows from Table II that equal signs of their experimental asymmetries require that the total p.v. WRHD amplitudes $A_{\rm WR}$ be proportional to the difference $b_1 - b_2$:

$$A_{\rm WR}(k) = \frac{e}{g_\rho}(-b_1(k) + b_2(k))\ b_{\rm WR} \tag{11}$$

with factor $\frac{e}{g_\rho}$ accounting for the replacement of the strong coupling with the electromagnetic one, just as in p.c. amplitudes (Eqs. (2), (3)). Finally, with p.c. amplitudes of $\Xi^0 \to \Lambda\pi^0$ and $\Xi^0 \to \Lambda\gamma$ decays being of opposite signs (Eqs. (1), (2)), from the equal signs of their experimental asymmetries it follows (using Tables I and II and Eqs. (5), (6)) that

$$b_{\rm WR} \approx -b_{\rm NL}\,. \tag{12}$$

The form of Eq. (11) ensures that Hara's theorem holds for exact SU(3) since the SU(3)-symmetric b_i terms cancel there (Table II). As proposed in Ref. [11], the large value of the experimental $\Sigma^+ \to p\gamma$ asymmetry is presumably due to a substantial SU(3) breaking effect, expected to be of the order of $\delta s/\delta\omega$, with the SU(3) breaking mass difference $\delta s \approx m_s - m_d \approx 190$ MeV, and $\delta\omega \approx 570$ MeV being the energy difference between the first excited $1/2^-$ state and the ground state.

We now proceed to the question of the theoretical connection between NLHD and WRHD, *i.e.* between Eq. (4) and Eq. (11). Current algebra expresses the p.v. NLHD amplitudes $A_{\rm NL}(k)$ in terms of the contribution

$C(k)$ from the CA commutator and the correction $q^\mu R_\mu(k)$ proportional to the momentum of the emitted pion:

$$A_{\rm NL}(k) = C(k) + q^\mu R_\mu(k)\,. \tag{13}$$

Ref. [4] proves that the b-diagram-dependent part of $C(k)$ is proportional to $b_1(k) + b_2(k)$. It may be shown using the derivative form of the strong coupling of π to baryons (see [4, 14]), that the b-diagram-dependent part of the $q^\mu R_\mu(k)$ term is proportional to $-b_1(k) + b_2(k)$. Namely, by CP invariance and hermiticity, in the parity-violating CP-conserving couplings of CP $= -1$ neutral pseudoscalar mesons to baryons

$$g^{(0)}_{fi}\bar{u}_f u_i P^0 + g^{(1)}_{fi} q^\mu \bar{u}_f \gamma_\mu u_i P^0\,, \tag{14}$$

in the convention of [14], the coefficients $g^{(n)}$ are imaginary, with $g^{(0)}_{fi}$ ($g^{(1)}_{fi}$) antisymmetric (symmetric) under $i \leftrightarrow f$ interchange. When translated into the language of b_i coefficients, this leads to b_1+b_2 and $-b_1+b_2$ structures for the q-independent and q-dependent terms, respectively [4,14]. $\mathrm{SU}(2)_W$ spin symmetry relates contributions to NLHD and WRHD from terms proportional to $-b_1(k) + b_2(k)$. Since experimentally the total contribution of all single-quark diagrams (including the (c)-type ones) is negligible in WRHD, for NLHD we expect from symmetry with WRHD that a substantial contribution from (c)-type diagrams arises from the commutator term only.

Thus, the soft-meson CA approach of [13] is augmented with a correction term, due to a non-zero value of pion momentum and estimated from WRHD by symmetry:

$$A_{\rm NL}(k) = (b_1(k)+b_2(k))\ b_{\rm com} + (c_1(k)+c_2(k))\ c_{\rm com} + (-b_1(k)+b_2(k))\ b_{\rm WR}\,, \tag{15}$$

where the first two terms describe the commutator $C(k)$, and the symmetry between NLHD and WRHD is used for the $-b_1 + b_2$ term.

Consequently,

$$b_{\rm NL} = b_{\rm com} + b_{\rm WR}\,, \tag{16}$$

$$c_{\rm NL} = c_{\rm com}\,. \tag{17}$$

Note that for NLHD all $b_1(k)$ are zero and, consequently, without knowing $b_{\rm WR}$ we cannot extract $b_{\rm com}$ directly from the data. Clearly, we cannot have $b_{\rm com} = 0$ as suggested by the constituent quark model calculations combined with Hara's theorem [4], or else we would have $b_{\rm NL} = b_{\rm WR}$, and the $\Xi^0 \to (\Lambda, \Sigma^0)\gamma$ asymmetries would be predicted as positive, in disagreement with experiment.

Using Eq. (12) we obtain

$$b_{\rm com} \approx 2b_{\rm NL} \tag{18}$$

and for the commutator we have

$$d_{\rm com} = 2b_{\rm NL}\,, \tag{19}$$

$$f_{\rm com} = -2b_{\rm NL} + \frac{2}{3}c_{\rm NL}\,. \tag{20}$$

Consequently

$$\frac{d_{\rm com}}{d_S} = 2\,, \tag{21}$$

$$\frac{f_{\rm com}}{f_S} \approx 1.4\,, \tag{22}$$

$$\frac{f_{\rm com}}{d_{\rm com}} = -1 + \frac{1}{3}\frac{c_{\rm NL}}{b_{\rm NL}} = -1.8\,. \tag{23}$$

Thus, the values of $d_{\rm com}$ and $f_{\rm com}$ (extracted from the S-wave amplitudes) agree with the values of SU(3) parameters d_P and f_P needed to describe the P-wave amplitudes. The resolution of the S:P problem and the description of WRHD are interconnected. The mechanism by which the S-wave amplitudes are reduced from their commutator values is closely related to the explanation proposed in Ref. [10]. In Ref. [10] the downward correction is due to the $(70, 1^-)$ intermediate states. In our approach explicit intermediate states are not used. However, the symmetry properties of the correction term in Ref. [10] and in this paper are identical in the symmetry limit. The difference is that in this paper, instead of estimating the overall size of the correction in a model as the authors of Ref. [10] do, we extract both its size and sign from WRHD.

There still remains a question how to understand the absence in WRHD of a term proportional to $b_1 + b_2$ (*i.e.* the analogue of the CA commutator term as obtained in the constituent quark model calculations). We observe that if in the presence of weak (p.v.) perturbation $\mathcal{L}^{\rm p.v.}$ the symmetry is imposed between axial and vector currents J^μ_A and J^μ_V, the resulting couplings to photons and pions are obtained from

$$A_\mu T(J^\mu_V(x)\mathcal{L}^{\rm p.v.}(0)) \tag{24}$$

$$\partial_\mu T(J^\mu_A(x)\mathcal{L}^{\rm p.v.}(0)) = T(\partial_\mu J^\mu_A(x)\mathcal{L}^{\rm p.v.}(0)) + \text{commutator} \tag{25}$$

with the pion field appearing via PCAC in the first term on the r.h.s. of Eq. (25). As shown in Eqs. (24), (25), the symmetry is not between the pion field $\pi \propto \partial_\mu J_A^\mu$ and the photon field A_μ (or the vector-meson field through VMD current-field identity $J_V^\mu \propto V^\mu$) but rather between the currents J_V, J_A appearing on the l.h.s. This bring us back to the original Gell-Mann's paper [16].

This identification of symmetry necessary for a successful joint description of nonleptonic and radiative weak hyperon decays leads to problems elsewhere, however. Namely, our present understanding of nuclear parity violation (*cf.* Ref. [17]) is based on symmetry of weak couplings between the *fields* of pseudoscalar and vector mesons (and not on symmetry between the axial and vector *currents*). According to Refs. [17, 18] the explanation of data on nuclear parity violation requires the dominance of the weak rho-nucleon coupling of the form $\bar{u}_N \gamma_\mu \gamma_5 u_N \rho^\mu$. Via the current-field identity (VMD) this leads to photon-nucleon coupling $\bar{u}_N \gamma_\mu \gamma_5 u_N A^\mu$ and the violation of Hara's theorem in weak radiative hyperon decays [2, 6]. Since the negative asymmetry of the $\Xi^0 \to \Lambda\gamma$ decay means that Hara's theorem is satisfied, it follows that either the current-field identity is not universal or our present understanding of nuclear parity violation (*i.e.* [17]) is not fully correct.

In conclusion:

1. The simple constituent quark model may produce unphysical results in higher order calculations if free constituent quarks are used in intermediate states. Consequently, it is an idealisation that goes too far, and should be used and interpreted with care. The constituent quark model should better be regarded as a method of evaluating symmetry properties of simplest hadronic couplings and transitions.

2. The connection between nonleptonic and weak radiative hyperon decays should be formulated at the level of hadronic currents J_A, J_V (and not at the level of fields π, ρ, γ) in agreement with Gell-Mann's paper [16].

3. The sizes and signs of the p.v. WRHD amplitudes are correlated with those of the *correction* to the commutator term in NLHD. When WRHD data are used to estimate this correction, the old S:P problem in NLHD is resolved. The explanation of the large asymmetry in $\Sigma^+ \to p\gamma$ presumably requires more detailed SU(3)-breaking considerations (*e.g.* [11]).

4. The current-field identity suggests that vector mesons do not couple to baryons through the $\bar{u}\gamma_\mu\gamma_5 u V^\mu$ term. This is in conflict with our understanding of nuclear parity violation [18]. Thus, either this understanding is not fully correct, or current-field identity is not universal.

This work was supported in part by the Polish State Committee for Scientific Research (KBN) grant 5 P03B 050 21.

REFERENCES

[1] J.F. Donoghue, E. Golowich, B. Holstein, *Phys. Rep.* **131**, 319 (1986).

[2] J. Lach, P. Żenczykowski, *J. Mod. Phys.* **A10**, 3817 (1995).

[3] A.N. Kamal, Riazuddin, *Phys. Rev.* **D28**, 2317 (1983).

[4] P. Żenczykowski, *Acta Phys. Pol. B* **33**, 1253 (2002).

[5] In a gauge-invariant framework, the violation of Hara's theorem requires that essential nonlocality be present in the parity-violating photon coupling to baryons (see [4]).

[6] P. Żenczykowski, *Phys. Rev.* **D40**, 2290 (1989).

[7] S.A. Schmidt, Die Zerfallsasymmetrie des radiativen Hyperonzerfalls $\Xi^0 \to \Lambda\gamma$, Ph.D. Thesis, Johannes-Gutenberg Universität Mainz, D-55099 Mainz, November 2001.

[8] R.C. Verma, A. Sharma, *Phys. Rev.* **D38**, 1443 (1988).

[9] P. Żenczykowski, *Phys. Rev.* **D44**, 1485 (1991).

[10] A. LeYaouanc *et al., Nucl. Phys.* **B149**, 321 (1979).

[11] M.B. Gavela *et al., Phys. Lett.* **B101**, 417 (1981).

[12] R.W. Brown, E.A. Paschos, *Nucl. Phys.* **B319**, 623 (1989).

[13] J.G. Körner, T. Gudehus, *Nuovo Cimento* **A11**, 597 (1972); J.G. Körner *et al., Z. Phys.* **C2**, 117 (1979).

[14] P. Żenczykowski, *Acta Phys. Pol. B* **30**, 271 (1999).

[15] P. Żenczykowski, *Phys. Rev.* **D50**, 3285 (1994).

[16] M. Gell-Mann, *Physics* **1**, 63 (1964).

[17] B. Desplanques, J.F. Donoghue, B. Holstein, *Ann. Phys.* (N.Y.) **124**, 449 (1980).

[18] B. Desplanques, *Nucl. Phys.* **A586**, 607 (1995).