THE LMA SOLAR SOLUTION AND FERMION UNIVERSALITY*

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We suggest that the Large Mixing Angle MSW solar solution, whose unique physical status is confidently supported by the recent results from KamLAND experiments, gets its justification in the fermion universality, interpreted for neutrinos and charged leptons in a straightforward way, most readily in the framework of seesaw mechanism. To this end, we consider an explicit seesaw model, where Dirac and (righthanded) Majorana neutrino masses are simultaneously measurable, and *both* are conjectured to be proportional to charged-lepton masses. However, the LMA solar solution is *also* not inconsistent with the simple option, where neutrinos are Dirac particles carrying masses proportional to those of charged leptons.

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As is known, the first results from the KamLAND long-baseline experiments for reactor $\bar{\nu}_e$'s [1] shows that the Large Mixing Angle MSW solution can be confidently considered [2–6] as the unique oscillation solution to the problem of solar ν_e's. The best-fit estimate is $\Delta m^2_{21} \equiv m^2_2 - m^2_1 \sim 7 \times 10^{-5}$ eV^2 and $\tan^2\theta_{12} \sim 0.42$ ($\theta_{12} \sim 33°$). The bilarge mixing matrix for active neutrinos $\nu_{e\mathrm{L}}$, $\nu_{\mu\mathrm{L}}$, $\nu_{\tau\mathrm{L}}$,

$$U \simeq \left(\begin{array}{ccc} c_{12} & s_{12} & 0 \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{array} \right) , \tag{1}$$

where $c_{23} = 1/\sqrt{2} = s_{23}$ ($\theta_{23} = 45°$) and $s_{13} = 0$, describes then correctly the deficits of both solar ν_e's and atmospheric ν_μ's [7] as well as the absence

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of oscillations of reactor $\bar{\nu}_e$'s in the Chooz experiment [8]. It gives, however, no LSND effect for accelerator $\bar{\nu}_\mu$'s and ν_μ's [9], unless a third independent neutrino mass-squared scale of the order $O(1\mathrm{eV}^2)$ is introduced beside Δm^2_{21} and Δm^2_{32} into the theory. This effect will be reinvestigated soon in the Mini-BOONE experiment. The SuperKamiokande experiments for atmospheric ν_μ's [7] lead to the best-fit estimate $\Delta m^2_{32} \equiv m^2_3 - m^2_2 \sim 3 \times 10^{-3}$ eV^2 and $\sin^2 2\theta_{23} \sim 1$ $(\theta_{23} \sim 45^\circ)$.

In the flavor representation (used hereafter), where the charged-lepton mass matrix is diagonal, the mixing matrix U is diagonalizing at the same time the active-neutrino effective mass matrix [as given in Eq. (4)].

The unique experimental status of Large Mixing Angle MSW solar solution requires now its theoretical explanation or, at least, its phenomenological correlation with other elements of neutrino physics. In this note we suggest that such a justification follows from the fermion universality interpreted for neutrinos and charged leptons in a straightforward way, where *both* Dirac and Majorana masses are involved.

To this end, let us consider an explicit model for the seesaw mechanism [10], where the lefthanded, Dirac and righthanded 3×3 components of the neutrino generic 6×6 mass matrix

$$\left(\begin{array}{cc} M^{(\mathrm{L})} & M^{(\mathrm{D})} \\ M^{(\mathrm{D})\,\mathrm{T}} & M^{(\mathrm{R})} \end{array} \right) \tag{2}$$

are

$$M^{(\mathrm{L})} = 0\,,\; M^{(\mathrm{D})} = U \,\mathrm{diag}(\lambda_1, \lambda_2, \lambda_3) U^\dagger\,,\; M^{(\mathrm{R})} = -U \,\mathrm{diag}(\Lambda_1, \Lambda_2, \Lambda_3) U^\dagger\,, \tag{3}$$

respectively, with U as given in Eq. (1) [note that $M^{(\mathrm{D})}$ and $M^{(\mathrm{R})}$ commute *i.e.*, are simultaneously measurable, what characterizes the neutrino texture (3)]. Here, all $\lambda_1, \lambda_2, \lambda_3 \ll$ all $\Lambda_1, \Lambda_2, \Lambda_3$ and all ≥ 0. Then, the effective 3×3 mass matrices for active and (conventional) sterile neutrinos, $\nu_{\alpha\mathrm{L}}$ and $\nu_{\alpha\mathrm{R}}$ $(\alpha = e, \mu, \tau)$, are

$$M^{(\mathrm{L})\,\mathrm{eff}} = -M^{(\mathrm{D})} \frac{1}{M^{(\mathrm{R})}} M^{(\mathrm{D})\mathrm{T}} = U \;\mathrm{diag}\left(\frac{\lambda_1^2}{\Lambda_1}, \frac{\lambda_2^2}{\Lambda_2}, \frac{\lambda_3^2}{\Lambda_3} \right) U^\dagger \tag{4}$$

and

$$M^{(\mathrm{R})\,\mathrm{eff}} = M^{(\mathrm{R})} = -U \,\mathrm{diag}(\Lambda_1, \Lambda_2, \Lambda_3) U^\dagger\,, \tag{5}$$

respectively. Hence,

$$m_i = \frac{\lambda_i^2}{\Lambda_i}\,,\; M_i = -\Lambda_i \tag{6}$$

are masses (being Majorana masses) of mass neutrinos $\nu_{i\mathrm{L}}$ and $\nu_{i\mathrm{R}}$ $(i = 1, 2, 3)$, respectively, connected with the flavor neutrinos $\nu_{\alpha\mathrm{L}}$ and $\nu_{\alpha\mathrm{R}}$

$(\alpha = e, \mu, \tau)$ through the unitary transformations

$$\nu_{\alpha\mathrm{L}} = \sum_i U_{\alpha i} \nu_{i\mathrm{L}} \ , \ \nu_{\alpha\mathrm{R}} = \sum_i U_{\alpha i} \nu_{i\mathrm{R}} \ , \tag{7}$$

where $U = (U_{\alpha i})$ gets the form (1).

The generic neutrino mass term in the Lagrangian is

$$-\mathcal{L}_{\mathrm{mass}} = \frac{1}{2} \sum_{\alpha\beta} \left(\overline{\nu_{\alpha\mathrm{L}}} \, , \, \overline{(\nu_{\alpha\mathrm{R}})^{\mathrm{c}}} \right) \left(\begin{array}{cc} M^{(\mathrm{L})}_{\alpha\beta} & M^{(\mathrm{D})}_{\alpha\beta} \\ M^{(\mathrm{D})}_{\beta\alpha} & M^{(\mathrm{R})}_{\alpha\beta} \end{array} \right) \left(\begin{array}{c} (\nu_{\beta\mathrm{L}})^{\mathrm{c}} \\ \nu_{\beta\mathrm{R}} \end{array} \right) + \mathrm{h.\,c.} \tag{8}$$

and leads, in the case of active neutrinos, to the effective mass term

$$-\mathcal{L}^{(\mathrm{L})\,\mathrm{eff}}_{\mathrm{mass}} = \frac{1}{2} \sum_{\alpha\beta} \overline{\nu_{\alpha\mathrm{L}}} M^{(\mathrm{L})\,\mathrm{eff}}_{\alpha\beta} (\nu_{\beta\mathrm{L}})^c + \mathrm{h.\,c.} \tag{9}$$

with $M^{(\mathrm{L})\,\mathrm{eff}}$ as given in Eq. (4).

Notice that λ_i and Λ_i appearing in Eq. (3) are (simultaneously measurable) Dirac and righthanded Majorana masses, respectively, for the set of six mass neutrinos arising from the set of six flavor neutrinos which includes three active $\nu_{\alpha\mathrm{L}}$ and three (conventional) sterile $\nu_{\alpha\mathrm{R}}$. The fermion universality applied to neutrinos and charged leptons may mean that the proportionality (at least approximate) occurs between their Dirac masses,

$$\lambda_1 : \lambda_2 : \lambda_3 \simeq m_e : m_\mu : m_\tau \, , \tag{10}$$

implying due to the first Eq. (6) the relation

$$m_1 : m_2 : m_3 \simeq \frac{m_e^2}{\Lambda_1} : \frac{m_\mu^2}{\Lambda_2} : \frac{m_\tau^2}{\Lambda_3} \, . \tag{11}$$

Here, $m_e = 0.510999$ MeV, $m_\mu = 105.658$ MeV and $m_\tau = 1776.99^{+0.29}_{-0.26}$ MeV [11]. Hence,

$$\begin{aligned} \Delta m^2_{21} \; &\simeq \; m_2^2 \left(1 - \frac{m_e^4 \Lambda_2^2}{m_\mu^4 \Lambda_1^2} \right) = m_2^2 \left(1 - 5.471 \times 10^{-10} \, \frac{\Lambda_2^2}{\Lambda_1^2} \right) \, , \\ \Delta m^2_{32} \; &\simeq \; m_3^2 \left(1 - \frac{m_\mu^4 \Lambda_3^2}{m_\tau^4 \Lambda_2^2} \right) = m_3^2 \left(1 - 1.250 \times 10^{-5} \, \frac{\Lambda_3^2}{\Lambda_2^2} \right) \end{aligned} \tag{12}$$

and

$$\begin{aligned} \frac{\Delta m^2_{21}}{\Delta m^2_{32}} \; &\simeq \; \frac{m_\mu^4 \Lambda_3^2}{m_\tau^4 \Lambda_2^2} \, \frac{1 - m_e^4 \Lambda_2^2 / m_\mu^4 \Lambda_1^2}{1 - m_\mu^4 \Lambda_3^2 / m_\tau^4 \Lambda_2^2} \\ &= \; 1.250 \times 10^{-5} \, \frac{\Lambda_3^2}{\Lambda_2^2} \, \frac{1 - 5.471 \times 10^{-10} \, \Lambda_2^2/\Lambda_1^2}{1 - 1.250 \times 10^{-5} \, \Lambda_3^2/\Lambda_2^2} \, , \end{aligned} \tag{13}$$

what gives

$$\Delta m_{21}^2 \sim 3.7 \times 10^{-8} \frac{\Lambda_3^2}{\Lambda_2^2} \, \frac{1 - 5.471 \times 10^{-10}\, \Lambda_2^2/\Lambda_1^2}{1 - 1.250 \times 10^{-5}\, \Lambda_3^2/\Lambda_2^2} \;\mathrm{eV}^2 \tag{14}$$

with the use of the SuperKamiokande estimate $\Delta m_{32}^2 \sim 3 \times 10^{-3}\;\mathrm{eV}^2$.

Since Λ_i are much larger than λ_i, it may seem that Λ_i are nearly degenerate: $\Lambda_1 \simeq \Lambda_2 \simeq \Lambda_3$. In this case, $\Delta m_{21}^2 \simeq m_2^2$ and $\Delta m_{32}^2 \simeq m_3^2$ from Eqs. (12), while Eq. (14) predicts the value

$$\Delta m_{21}^2 \sim 3.7 \times 10^{-8}\;\mathrm{eV}^2 \tag{15}$$

lying in the range of the LOW MSW solar solution and so, being much smaller than the correct Large Mixing Angle MSW value $\Delta m_{21}^2 \sim 7 \times 10^{-5} \mathrm{eV}^2$. Here,

$$m_1^2 \sim 2.0 \times 10^{-17} \mathrm{eV}^2\,, \quad m_2^2 \sim 3.7 \times 10^{-8} \mathrm{eV}^2\,, \quad m_3^2 \sim 3 \times 10^{-3} \mathrm{eV}^2 \tag{16}$$

and, when normalizing $\lambda_1 = m_e$ (*i.e.*, $\Lambda_1 = \lambda_1^2/m_1 = m_e^2/m_1$), one gets

$$\Lambda_1 \simeq \Lambda_2 \simeq \Lambda_3 \sim 5.8 \times 10^{10} \mathrm{GeV}\,. \tag{17}$$

In such a situation, let us conjecture tentatively that the fermion universality in the context of neutrinos and charged leptons has to be interpreted as the proportionality (at least approximate) between *both* their Dirac and Majorana [12] masses,

$$\lambda_1 : \lambda_2 : \lambda_3 \simeq \Lambda_1 : \Lambda_2 : \Lambda_3 \simeq m_e : m_\mu : m_\tau\,, \tag{18}$$

implying through the first Eq. (6) that

$$m_1 : m_2 : m_3 \simeq \frac{m_e^2}{\Lambda_1} : \frac{m_\mu^2}{\Lambda_2} : \frac{m_\tau^2}{\Lambda_3} \simeq m_e : m_\mu : m_\tau. \tag{19}$$

In this case,

$$\begin{aligned} \Delta m_{21}^2 &\simeq m_2^2 \left(1 - \frac{m_e^2}{m_\mu^2}\right) = m_2^2 \left(1 - 2.339 \times 10^{-5}\right)\,, \\ \Delta m_{32}^2 &\simeq m_3^2 \left(1 - \frac{m_\mu^2}{m_\tau^2}\right) = m_3^2 \left(1 - 3.535 \times 10^{-3}\right) \end{aligned} \tag{20}$$

and

$$\frac{\Delta m_{21}^2}{\Delta m_{32}^2} \simeq \frac{m_\mu^2 - m_e^2}{m_\tau^2 - m_\mu^2} = 3.548 \times 10^{-3}\,, \tag{21}$$

predicting the value

$$\Delta m_{21}^2 \sim 1.1 \times 10^{-5} \text{ eV}^2 \,, \tag{22}$$

when the SuperKamiokande estimate $\Delta m_{32}^2 \sim 3 \times 10^{-3}$ eV^2 is used. Here,

$$m_1^2 \sim 2.5 \times 10^{-10} \,\text{eV}^2 \,, \quad m_2^2 \sim 1.1 \times 10^{-5} \,\text{eV}^2 \,, \quad m_3^2 \sim 3 \times 10^{-3} \,\text{eV}^2 \tag{23}$$

and, when normalizing $\lambda_1 = m_e$ (*i.e.*, $\Lambda_1 = \lambda_1^2/m_1 = m_e^2/m_1$), one obtains

$$\Lambda_1 \sim 1.7 \times 10^7 \,\text{GeV} \,, \quad \Lambda_2 \sim 3.4 \times 10^9 \,\text{GeV} \,, \quad \Lambda_3 \sim 5.8 \times 10^{10} \,\text{GeV} \,. \tag{24}$$

Concluding, we can see that the prediction (22) is not very different from the correct Large Mixing Angle MSW value $\Delta m_{21}^2 \sim 7 \times 10^{-5}$ eV^2. In order to get this value more precisely, one ought to put in Eq. (14)

$$\frac{\Lambda_3^2}{\Lambda_2^2} \sim \frac{7 \times 10^{-5} \text{ eV}^2}{3.7 \times 10^{-8} \text{ eV}^2} = 1.9 \times 10^3 = 6.6 \frac{m_\tau^2}{m_\mu^2} \tag{25}$$

in place of the simple proportion $\Lambda_3^2/\Lambda_2^2 \simeq m_\tau^2/m_\mu^2$, where $m_\tau^2/m_\mu^2 = 282.9$. Then, $\Lambda_3/\Lambda_2 \sim 43 \simeq 2.6 m_\tau/m_\mu$ in place of $\Lambda_3/\Lambda_2 \simeq m_\tau/m_\mu$, where $m_\tau/m_\mu = 16.82$.

Eventually, it is interesting to remark that, if neutrinos were Dirac particles rather than Majorana particles *i.e.*, $M^{\text{eff}} = M^{(\text{D})} = U \,\text{diag}(\lambda_1, \lambda_2, \lambda_3) U^\dagger$ with $m_i = \lambda_i$, the fermion universality for neutrinos and charged leptons might be interpreted as the proportionality (at least approximate) between their (Dirac) masses,

$$\lambda_1 : \lambda_2 : \lambda_3 \simeq m_e : m_\mu : m_\tau \,, \tag{26}$$

where $m_i = \lambda_i$. Then, also in this case Eq. (21) would hold, predicting the value (22) for Δm_{21}^2 [13] which is not so different from the correct Large Mixing Angle MSW value $\Delta m_{21}^2 \sim 7 \times 10^{-5}$eV^2. One should stress that the value (22) is a parameter-free prediction following from the proportionality (18) or (26) for Majorana or Dirac neutrinos, respectively, as they are investigated in the present neutrino-oscillation experiments.

Finally, let us note that writing

$$m_1 = \overset{0}{m} - \delta \,, \quad m_2 = \overset{0}{m} + \delta \,, \quad m_3 = \overset{0}{m} + \Delta \tag{27}$$

and using Eq. (1) for U we obtain the formula [14]

$$M^{(L)\,\text{eff}} = U \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U^\dagger = \overset{0}{m} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \Delta \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$
$$+ \delta \begin{pmatrix} -\cos 2\theta_{12} & \frac{1}{\sqrt{2}} \sin 2\theta_{12} & -\frac{1}{\sqrt{2}} \sin 2\theta_{12} \\ \frac{1}{\sqrt{2}} \sin 2\theta_{12} & \frac{1}{2} \cos 2\theta_{12} & -\frac{1}{2} \cos 2\theta_{12} \\ -\frac{1}{\sqrt{2}} \sin 2\theta_{12} & -\frac{1}{2} \cos 2\theta_{12} & \frac{1}{2} \cos 2\theta_{12} \end{pmatrix} . \tag{28}$$

In the case of prediction (22) we have from Eq. (23)

$$m_1 \sim 1.6 \times 10^{-5}\,\mathrm{eV}\,, \quad m_2 \sim 3.3 \times 10^{-3}\,\mathrm{eV}\,, \quad m_3 \sim 5.5 \times 10^{-2}\,\mathrm{eV}\,, \quad (29)$$

while we get

$$m_2 \sim 8.4 \times 10^{-3}\,\mathrm{eV}\,, \quad m_3 \sim 5.5 \times 10^{-2}\,\mathrm{eV} \quad (30)$$

if $m_1^2 \ll m_2^2$ and we use the experimental estimate $\Delta m_{21}^2 \sim 7{\times}10^{-5}$ eV^2 (and also $\Delta m_{32}^2 \sim 3 \times 10^{-3}$ eV^2, as before). Due to the experimental estimate $\theta_{12} \sim 33^\circ$ we can put in the formula (28) $\cos 2\theta_{12} \sim 0.41$ and $\sin 2\theta_{12} \sim 0.91$.

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