# COSMOLOGICAL IMPLICATIONS OF LOW SCALE QUARK–LEPTON UNIFICATION

T.L. Yoon and R. Foot

School of Physics, Research Centre for High Energy Physics
The University of Melbourne, Victoria 3010, Australia

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There is a unique $\mathrm{SU}(4) \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{SU}(2)_\mathrm{R}$ gauge model which allows quarks and leptons to be unified at the TeV scale — thereby making the model testable and avoiding the gauge hierarchy problem. In its minimal form, this model could quite naturally accommodate simultaneous solutions to the solar and LSND neutrino oscillation data. The atmospheric neutrino anomaly can be easily accommodated by mirror-symmetrising the minimal model. The model also contains three right-handed neutrinos, with masses in the range 1 keV to $\sim$ 1 GeV. We investigate the implications of these right-handed neutrinos for early Universe cosmology. It is shown that the minimal model is inconsistent with some of the standard assumptions of the Big Bang model. This motivates an examination of non-standard Big Bang cosmology, such as a low reheating temperature scenario with $T_\mathrm{RH} \sim$ MeV. In such a Universe, peaceful co-existence between low-scale quark–lepton gauge unification and early Universe cosmology is possible.

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## 1. Introduction

The similarities between the quarks and leptons does hint at the possibility of a symmetry between them. The first example of such a theory is the Pati–Salam model [1], which is based on the $\mathrm{SU}(4) \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{SU}(2)_\mathrm{R}$ gauge group. While a very good idea, this particular model has a number of serious drawbacks. One problem is that experiments constrain the high symmetry breaking scale to be greater than 20 TeV [2], making direct tests of the model impossible, at least within the next quarter century. Moreover, the presence of a high symmetry breaking scale, significantly greater than a TeV, becomes theoretically problematic since it leads to the gauge hierarchy problem. This is quite unlike the situation in the standard model where there is no "hierarchy problem"' as such because there is only one scale in the Higgs potential[1]. However, in extensions of the standard model involving two (or more) symmetry breaking scales, there can be a significant fine-tuning problem if the scales are widely separated and appear in the Higgs potential. In that case, the fine-tuning problem can be alleviated if one also assumes that low energy supersymmetry exists. Unfortunately, this does not completely solve the problem, though, since one still needs to arrange the hierarchy at tree level. Also, low energy supersymmetry generates a host of new problems such as sparticle mediated FCNC, rapid proton decay, $\mu$-problem, *etc.*, so that it ends up creating more problems than it solves — clearly an unsatisfactory situation.

Perhaps an interesting question is the following one: Is it possible to build a simple gauge model which unifies quarks and leptons at low scales ($\lesssim$ few TeV) so that the gauge hierarchy problem is avoided? The first such model was written down some time ago [5], whereby a leptonic $\mathrm{SU}(3)_\ell$ gauge group was introduced allowing for a discrete quark–lepton (spontaneously broken) symmetry to exist. The main problem with the discrete symmetry approach comes from neutrino masses. The lightness of the neutrino masses in that model (as with the usual Pati–Salam model) suggests a high symmetry breaking scale ($\gg$ TeV) if the usual see-saw mechanism is employed, and while there are alternatives [6], they are somewhat complicated. Searches for new ideas led to the alternative $\mathrm{SU}(4)\otimes\mathrm{SU}(2)_\mathrm{L}\otimes\mathrm{SU}(2)_\mathrm{R}$ gauge model [7] which allows for TeV scale quark–lepton unification without any problems for existing experiments with *necessarily* tiny neutrino masses. This "alternative 422 model" predicts a multitude of new phenomenology including: rare $B, K$ decays, baryon number violation as well as non-zero neutrino masses, all of which are within current bounds, despite the low symmetry breaking scale of a few TeV (see Refs. [8, 9] for more details of the phenomenological implications of the model).

In Ref. [9] the extent to which the minimal alternative 422 model could accommodate solutions to the neutrino physics anomalies was investigated. While it did not seem possible to simultaneously accommodate solutions to all three classes of neutrino physics anomalies (*i.e.* LSND [10], solar [11–13] and atmospheric [14, 15] neutrino anomalies), it was shown that the minimal alternative 422 model can quite naturally accommodate the LSND and atmospheric neutrino anomalies. The solar neutrino problem could be

[1] It is sometimes asserted in the literature that a hierarchy problem also exists between the weak scale and the "Planck" scale; however, in this case things are much less clear. Firstly, the physics of the Planck scale is really not understood at the moment so it is not yet clear whether this (as yet unknown) physics will lead to a fine-tuning problem. Another line of argument asserts that there exists a fine-tuning problem between the weak scale and a momentum "cut-off". However, that argument depends on the regulator scheme used [3], (see also Ref. [4]) and should not be taken too seriously.

explained if the model was extended with a mirror sector [9,16]. Since that time, there has been an important new development. SNO has now measured both neutral- and charged-current solar neutrino fluxes, providing strong evidence that large angle active–active oscillations are occurring [13]. In view of this new development, the available information suggests an essentially unique picture [17]

$$\begin{aligned} &\nu_e \;\rightarrow\; \nu_\tau \text{ large angle oscillations explain the solar neutrino problem,}\\ &\nu_\mu \;\rightarrow\; \nu_{\rm s} \text{ large angle oscillations explain the atmospheric neutrino anomaly,}\\ &\bar\nu_e \;\rightarrow\; \bar\nu_\mu \text{ small angle oscillations explain LSND data,} \end{aligned} \tag{1}$$

where $\nu_{\rm s}$ is a hypothetical effectively sterile neutrino. While the atmospheric neutrino data prefer the $\nu_\mu \rightarrow \nu_\tau$ channel, it is also true that the $\nu_\mu \rightarrow \nu_s$ possibility is only mildly disfavoured, at the $\sim 1.5\sigma$–$3\sigma$ level, depending on how the data are analysed [18,19]. The overall goodness of fit (g.o.f) of the above scheme has recently been explicitly calculated in Ref. [20], where it was found to be 0.26. That is, there is a 26% probability of obtaining a worse global fit to the neutrino data. This shows that the above scheme still provides a reasonable fit to the totality of the neutrino oscillation data[2]. Although this scheme is not particularly popular, it at least has the virtue that it will be tested in the near future: MiniBooNE will test the oscillation explanation of the LSND anomaly, while the forthcoming long baseline experiments (MINOS and CNGS) will discriminate between the $\nu_\mu \rightarrow \nu_{\rm sterile}$ and $\nu_\mu \rightarrow \nu_\tau$ channels used to resolve the atmospheric neutrino anomaly.

The oscillation scheme (Eq. (1)) is essentially unique in the sense that it is the simplest scheme involving only two-flavour oscillations explaining the totality of the data and also specific features such as SNO's neutral current/charge current solar flux measurement [13]. Of course, other, but more complicated schemes involving multi-flavour oscillations, are possible because they can also provide an acceptable fit to the data for a range of parameters. For example, one can have an additional parameter, $\sin^2\omega$, where $\sin^2\omega = 0$ corresponds to the scheme, Eq. (1), $\sin^2\omega = 1$ is similar to Eq. (1) with $\nu_\tau$ interchanged with $\nu_{\rm s}$ and intermediate values of $\sin^2\omega$ corresponds to mixed active + sterile oscillations [22]. Such schemes are called 2 + 2 models because they feature two pairs of almost degenerate states separated by the LSND mass gap. While the scheme of Eq. (1) could be viewed as a particular 2 + 2 scheme with $\sin^2\omega = 0$, it could alternatively

[2] Recently, Ref. [21] has argued that all 4-neutrino models of the (2 + 2) variety are "ruled out" by the totality of neutrino oscillation data (solar, atmospheric and LSND). However, the g.o.f. obtained by Ref. [21] (g.o.f. $= 10^{-6}$) is not really the g.o.f but some other quantity. Indeed, as shown in Ref. [20], it disagrees with the actual g.o.f by more than 5 orders of magnitude. For more discussion on this issue, see Ref. [20].

be viewed as motivating the following hypothesis [17]: *The fundamental theory of neutrino mixing, whatever it is, features (i) large (or even maximal)* $\nu_\mu \to \nu_{\rm s}$ *mixing, (ii) small-angle active–active mixing except for the* $\nu_e \to \nu_\tau$ *channel which is large.*

This hypothesis is the one which we adopt in this paper. We shall show that the minimal alternative 422 model is a candidate for the new physics required to explain the active–active oscillations suggested by the LSND and solar neutrino data within the above hypothesis. The atmospheric neutrino anomaly, as explained above, will be assumed to be due to $\nu_\mu \to \nu_{\rm s}$ oscillations. The 422 model does not have any suitable candidates for the needed light sterile neutrino (it does have effectively sterile right-handed neutrinos, but it turns out that they are too heavy [9]).

It is known [16] that three light effectively sterile neutrinos $(\nu'_e, \nu'_\mu, \nu'_\tau)$ maximally mixed with their active partners is predicted to exist if mirror symmetry is an exact fundamental symmetry. Thus, if we mirror symmetrise the model, we can easily accommodate the atmospheric neutrino anomaly, via $\nu_\mu \to \nu'_\mu$ oscillations. If the oscillation lengths of the $\nu_{e,\tau} \to \nu'_{e,\tau}$ oscillations are longer than the Earth–Sun distance for solar neutrinos, then this 3 active + 3 mirror neutrino model effectively reduces to the required four-neutrino scheme, Eq. (1). It is also possible to have the oscillation lengths of the $\nu_{e,\tau} \to \nu'_{e,\tau}$ oscillations shorter than the Earth–Sun distance. This would mean a large sterile component ($\sim 50\%$) in the solar neutrino flux, which is still allowed by the data [23]. (It would also require some modifications to the solar model, such as larger boron flux *etc.*).

It turns out that the consistency between the low symmetry breaking scale and the solutions to these neutrino anomalies imposes some constraints on the possible forms that could be assumed by the Majorana mass matrix of the right-handed neutrinos (which form part of the particle content of the model). As a result the masses of the right-handed neutrinos in the alternative 422 model are constrained to be in the ranges of 1 keV–10 keV and 4 MeV to $\sim$ 1 GeV (see the forthcoming Eq. (32) discussed in Sec. 2).

These particles can potentially contribute significantly to the matter density of the Universe. Within the framework of the standard Big Bang model of cosmology, an important constraint is that a given particle species $X$ must satisfy the cosmological energy density bound

$$\Omega_X \equiv \frac{\rho_X}{\rho_{\rm c}} \lesssim 1\,, \tag{2}$$

where $\Omega_X$ is the contribution of their present density $\rho_X$ normalised to the critical density, $\rho_{\rm c} = 10^4 h^2$ eVcm$^{-3}$ ($h = H_0/100\,{\rm kms}^{-1}\,{\rm Mpc}^{-1}$ is the normalised Hubble constant). It has been a routine practice to check if new particle species contained in a given extension of the standard model are compatible with standard cosmology and other astrophysical bounds. This "consistency check" will form the content of the first half of the present paper. It is found that even the lightest right-handed neutrinos contained in this model are *not* consistent with standard Big Bang cosmology.

Despite the claims that cosmology has ushered into an era of unprecedented precision, there are still many unsettled issues at the interface of particle physics and cosmology (see, for example, Ref. [24] for a general account of pending issues facing particle cosmology). The inconsistency of a particle physics model with standard cosmology does not necessarily mean that a given particle physics model is not realistic. The standard Big Bang cosmology is generally not robust against plausible modifications (either by new observations or theoretical input) to a set of standard assumptions. Hence it requires some cautious attitude when one is to use standard cosmology to "rule out" or support a given extension of the standard model of particle physics. For example, the standard cosmological model contains an implicit but not observationally justified assumption that the reheating temperature (corresponding to the highest temperature during the radiation-dominated epoch) of the early Universe is much higher than the characteristic temperatures of cosmological processes under investigation (*e.g.* the freeze-out temperature pertinent to a given particle species). However, Refs. [25, 26] show that the reheating temperature that is consistent with the observational light element abundances could be as low as 0.7 MeV. The possibility of a low reheating temperature scenario has prompted many works since then (see for example Refs. [27–30]). For instance, Refs. [27, 28] have shown that in a low reheating cosmology the relationship between the relic density of an exotic particle species normalised to the present energy density of the Universe deviates from that of the standard case. As a result, well-known constraints (such as the Cowsik–McClelland bound [31]) previously imposed on the masses of the ordinary neutrinos can be greatly relaxed in such a scenario.

In the low reheating temperature scenario, we shall re-analyse the cosmological constraints imposed on the right-handed neutrinos contained in the alternative 422 model. We find that a low reheating temperature cosmology is consistent with the alternative 422 model, as there is some parameter space in which the right-handed neutrinos can accommodate the cosmological and other astrophysical bounds. We also point out that the lightest right-handed neutrinos of the model provide an interesting dark matter candidate.

The plan of this paper is as follows. In Sec. 2 we first briefly explain how the minimal version of the alternative 422 model accommodates the solutions to the LSND and solar neutrino data, which consistency requires the mass spectrum of the right-handed neutrinos to fall in a specific range. Then, in Sec. 3 we examine the cosmological implications of these right-handed neutrinos in the framework of standard cosmology. Having shown that the right-handed neutrinos are inconsistent with standard cosmology, we proceed (in Sec. 4) to examine the right-handed neutrinos in the non-standard low reheating temperature scenario with $T_{\rm RH} \sim$ a few MeV. Taking the reheating temperature $T_{\rm RH}$ as a free parameter with a rough lower bound of $T_{\rm RH} \gtrsim 0.7$ MeV, we show that the right-handed neutrinos could circumvent the cosmological energy density bound for some parameter range. In Sec. 5 we conclude.

## 2. The masses of the right-handed neutrinos in the alternative 422 model

We first revise the details of the model and explain how it can accommodate the large angle $\nu_e \to \nu_\tau$ and small angle $\nu_\mu \to \nu_e$ oscillations suggested by the solar and LSND anomalies. We refer the reader to Ref. [9] for further details.

The gauge symmetry of the alternative 422 model is $\mathrm{SU}(4) \otimes \mathrm{SU}(2)_{\rm L} \otimes \mathrm{SU}(2)_{\rm R}$. Under this gauge symmetry the fermions of each generation transform in the anomaly-free representations:

$$Q_{\rm L} \sim (4,2,1), \quad Q_{\rm R} \sim (4,1,2), \quad f_{\rm L} \sim (1,2,2)\,. \tag{3}$$

The minimal choice of scalar multiplets which can both break the gauge symmetry correctly and give all of the charged fermions mass is

$$\chi_{\rm L} \sim (4,2,1), \quad \chi_{\rm R} \sim (4,1,2), \quad \phi \sim (1,2,2)\,. \tag{4}$$

These scalars couple to the fermions as follows:

$$\begin{aligned}\mathcal{L} \;=\; & \lambda_1 \mathrm{Tr}\left[\,\overline{Q_{\rm L}}(f_{\rm L})^c \tau_2 \chi_{\rm R}\right] + \lambda_2 \mathrm{Tr}\left[\,\overline{Q_{\rm R}} f_{\rm L}^T \tau_2 \chi_{\rm L}\right] + \lambda_3 \mathrm{Tr}\left[\,\overline{Q_{\rm L}}\phi \tau_2 Q_{\rm R}\right] \\ & + \lambda_4 \mathrm{Tr}\left[\,\overline{Q_{\rm L}}\phi^c \tau_2 Q_{\rm R}\right] + \mathrm{h.c.}\,,\end{aligned} \tag{5}$$

where the generation index has been suppressed and $\phi^c = \tau_2 \phi^* \tau_2$. The model reduces to the standard model following the spontaneous symmetry breaking pattern:

$$\begin{array}{c} \mathrm{SU}(4) \otimes \mathrm{SU}(2)_{\rm L} \otimes \mathrm{SU}(2)_{\rm R} \\ \downarrow \langle \chi_{\rm R} \rangle \\ \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_{\rm L} \otimes \mathrm{U}(1)_Y \\ \downarrow \langle \phi \rangle, \langle \chi_{\rm L} \rangle \\ \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_Q \end{array} \quad . \tag{6}$$

Note that the SU(4) group has a maximal $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_T$ subgroup with the 4 representation having the branching rule $4 = 3(\frac{1}{3}) + 1(-1)$. The vacuum expectation values (VEVs) can be conveniently expressed in terms of the $T, I_{3\mathrm{R}}, I_{3\mathrm{L}}$ charges as

$$\begin{aligned} \langle\chi_{\mathrm{R}}(T=-1, I_{3\mathrm{R}}=1/2)\rangle &= w_{\mathrm{R}}\,, \quad \langle\chi_{\mathrm{L}}(T=-1, I_{3\mathrm{L}}=1/2)\rangle = w_{\mathrm{L}}\,, \\ \langle\phi(I_{3\mathrm{L}}=-I_{3\mathrm{R}}=-1/2)\rangle &= u_1\,, \qquad \langle\phi(I_{3\mathrm{L}}=-I_{3\mathrm{R}}=1/2)\rangle = u_2\,. \end{aligned} \tag{7}$$

Note that $Y = T + 2I_{3\mathrm{R}}$ is the linear combination of $T$ and $I_{3\mathrm{R}}$ which annihilates $\langle\chi_{\mathrm{R}}\rangle$ (*i.e.* $Y\langle\chi_{\mathrm{R}}\rangle = 0$) and $Q = I_{3\mathrm{L}} + Y/2$ is the generator of the unbroken electromagnetic gauge symmetry. Observe that in the limit where $w_{\mathrm{R}} \gg w_{\mathrm{L}}, u_1, u_2$, the model reduces to the standard model at low energies.

The identity of the particles in the fermion multiplets, Eq. (3), can now be made explicit. We have the known quarks and leptons, along with some exotic heavy leptons $\{E^0, E^-\}$ and a right-handed neutrino, $\tilde{\nu}_{\mathrm{R}}$:

$$Q_{\mathrm{L}}^{\alpha,\gamma} = \begin{pmatrix} \tilde{U} & \tilde{E}^0 \\ D & \tilde{E}^- \end{pmatrix}_{\mathrm{L}}, Q_{\mathrm{R}}^{\beta,\gamma} = \begin{pmatrix} \tilde{U} & \tilde{\nu} \\ \tilde{D} & l \end{pmatrix}_{\mathrm{R}}, \; f_{\mathrm{L}}^{\alpha,\beta} = \begin{pmatrix} (\tilde{E}_{\mathrm{R}}^-)^c & \tilde{\nu}_{\mathrm{L}} \\ (\tilde{E}_{\mathrm{R}}^0)^c & l_{\mathrm{L}} \end{pmatrix}. \tag{8}$$

In the above representation, $\alpha = \pm\frac{1}{2}$, $\beta = \pm\frac{1}{2}$ index the $\mathrm{SU}(2)_{\mathrm{L}}$ and $\mathrm{SU}(2)_{\mathrm{R}}$ components, respectively. The SU(4) decomposition into the $\mathrm{SU}(3)_c \otimes \mathrm{U}(1)_T$ maximal subgroup is indexed by $\gamma = \gamma'(\frac{1}{3}) \oplus 4(-1)$, where $\gamma' = y, g, b$ is the usual colour index for $\mathrm{SU}(3)_c$ and $\gamma' = 4$ refers to the fourth colour. The number in the bracket refers to the $T$ charge of the subgroup $\mathrm{U}(1)_T$. In the above matrices the first row of $Q_{\mathrm{L}}$ and $f_{\mathrm{L}}$ ($Q_{\mathrm{R}}$) is the $I_{3\mathrm{L}}$ ($I_{3\mathrm{R}}$) $= \alpha$ ($\beta$) $= \frac{1}{2}$ component while the second row is the $I_{3\mathrm{L}}$ ($I_{3\mathrm{R}}$) $= \alpha$ ($\beta$) $= -\frac{1}{2}$ component. The columns of $Q_{\mathrm{L}}, Q_{\mathrm{R}}$ are the $\gamma'(\frac{1}{3})$ and $4(-1)$ components of SU(4), and the columns of $f_{\mathrm{L}}$ are the $I_{3\mathrm{R}} = \beta = \pm\frac{1}{2}$ components. Each field in the multiplets Eq. (8) represents $3 \times 1$ column vector of three generations. The tilde in the fermion fields signify that they are the weak eigenstates, which are generally not aligned with the corresponding mass eigenstates.

A set of theoretically arbitrary CKM-type unitary matrices are introduced into the theory to relate the weak eigenstates with their corresponding mass eigenstates. The basis is chosen such that

$$\begin{aligned} E_{\mathrm{R}} &= V^\dagger \tilde{E}_{\mathrm{R}}\,, \qquad E_{\mathrm{L}} = U^\dagger \tilde{E}_{\mathrm{L}}\,, \\ U_{\mathrm{R}} &= Y_{\mathrm{R}}^\dagger \tilde{U}_{\mathrm{R}}\,, \qquad U_{\mathrm{L}} = Y_{\mathrm{L}}^\dagger \tilde{U}_{\mathrm{L}}\,, \\ D_{\mathrm{R}} &= K' \tilde{D}_{\mathrm{R}}\,, \end{aligned} \tag{9}$$

and $\tilde{D}_{\mathrm{L}} = D_{\mathrm{L}}, \tilde{l}_{\mathrm{L}} = l_{\mathrm{L}}, \tilde{l}_{\mathrm{R}} = l_{\mathrm{R}}$. The matrix $Y_{\mathrm{L}}^\dagger \equiv K_{\mathrm{L}}$ is the usual CKM matrix (as in the standard model), whereas $Y_{\mathrm{R}}^\dagger K'^\dagger \equiv K_{\mathrm{R}}$ is the analogue of the CKM-type matrix for the right-handed charged quarks in the $\mathrm{SU}(2)_{\mathrm{R}}$ sector. The matrix $K'$ is the analogue of the CKM-type matrix in the SU(4) sector pertaining to lepto-quark interactions.

The model contains new gauge bosons: $W'$, $W_{\rm R}^{\pm}$ and $Z'$. The masses of these new bosons and the exotic leptons $\{E^0, E^-\}$ are constrained to be within the range:

$$\begin{aligned} 0.5(1.0)\ \text{TeV} &\lesssim M_{W_{\rm R}}\,, \quad M_{Z'}(M_{W'}) \lesssim 10\ \text{TeV}, \\ 45\ \text{GeV} &\lesssim M_{E_i} \lesssim 10\ \text{TeV}\,. \end{aligned} \tag{10}$$

Note that the lower limit on the mass of the $E$ leptons arises from LEP measurements of the $Z^0$ width, whereas the lower bound on the masses of $Z'$, $W_{\rm R}$ is obtained from the consistency of the model with LEP data [7]. The upper bound is a rough theoretical limit — the scale of symmetry breaking cannot be much greater than a few TeV, otherwise we would have a gauge hierarchy problem. By this we mean a real fine-tuning problem in the Higgs potential, not a hypothetical problem with the Plank scale or artificial cut-off parameter.

At tree level, mixing between $\tilde{\nu}_{\rm R}$ with $E^0_{\rm L,R}$ in the Lagrangian density of Eq. (5) generates the $3\times 3$ Majorana mass matrix $M_{\rm R}$ for the right-handed neutrinos after spontaneous symmetry breaking (SSB)

$$M_{\rm R} \simeq \left(M_l V M_E^{-1} U^\dagger Y_{\rm L} M_u Y_{\rm R}^\dagger\right) + \left(M_l V M_E^{-1} U^\dagger Y_{\rm L} M_u Y_{\rm R}^\dagger\right)^\dagger, \tag{11}$$

where $M_l, M_u, M_E$ are the $3\times 3$ diagonal mass matrices for the standard charged leptons, up-type quarks and the exotic $E$ leptons. We know that the CKM matrix is approximately diagonal, $Y_{\rm L} \approx \boldsymbol{I}_3$, whereas the rest of the CKM-type matrices (which are not present in the standard model) are poorly constrained by experiments (except for the matrix $K'$ which is constrained to only a few specific forms by limits on rare $K^0$ and $B^0$ decays mediated by the $W'$ bosons, see Ref. [8]). In the special case of decoupled generations (*e.g.* $Y_{\rm L} = Y_{\rm R} = U = V = \boldsymbol{I}$), $M_{\rm R}$ reduces to a diagonal matrix

$$M_{\rm R} = \text{diag}\left\{\frac{2m_u m_e}{M_{E_1}}, \frac{2m_c m_\mu}{M_{E_2}}, \frac{2m_t m_\tau}{M_{E_3}}\right\}.$$

At the one-loop level the gauge interactions from the charged ${\rm SU(2)_L}$ gauge bosons $W_{\rm L}^{\pm}$ and ${\rm SU(2)_R}$ gauge bosons $W_{\rm R}^{\pm}$ give rise to $\overline{\tilde{\nu}_{\rm L}}(\tilde{\nu}_{\rm L})^c$ Majorana mass $m_{\rm M}$, $\overline{\tilde{\nu}_{\rm L}}\tilde{\nu}_{\rm R}$ Dirac mass $m_{\rm D}$ and $\overline{\tilde{\nu}_{\rm L}}(E^0_{\rm L})^c$ mass mixing term $m_{\nu E}$ [3]. The Majorana mass $m_{\rm M}$ is generically very small in comparison to $m_{\rm D}$ and $m_{\nu E}$, and can be ignored in the subsequent discussion. The Dirac mass

[3] There is also radiative neutrino masses arising from the scalar sector of the model. But because of the larger arbitrariness in the scalar interactions we do not consider them in depth.

matrix $m_{\mathrm{D}}$ is generated (Fig. 1) via the diagonal (*i.e.* no cross generation mixing) gauge interactions

$$\frac{g_{\mathrm{L}}}{\sqrt{2}}\,\overline{\tilde{\nu}_{\mathrm{L}}}\,\not{W}_{\mathrm{L}}^{+} l_{\mathrm{L}} + \frac{g_{\mathrm{R}}}{\sqrt{2}}\,\overline{\tilde{\nu}}_{\mathrm{R}}\,\not{W}_{\mathrm{R}}^{+} l_{\mathrm{R}} + \text{ h.c.}, \tag{12}$$

where $g_{\mathrm{L}}$ is the usual $\mathrm{SU}(2)_{\mathrm{L}}$ coupling constant, and $g_{\mathrm{R}}$ is the $\mathrm{SU}(2)_{\mathrm{R}}$ coupling constant. As discussed in Ref. [7], $g_{\mathrm{R}}(M_{W'}) \approx g_{\mathrm{L}}(M_{W'})/\sqrt{3}$.

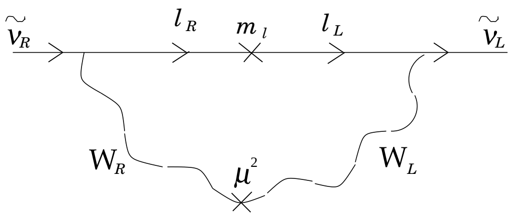


Fig. 1. Dirac mass generated by gauge interactions leading to the mass term $\overline{\tilde{\nu}_{\mathrm{L}}}\tilde{\nu}_{\mathrm{R}}$. $\mu^2 \equiv g_{\mathrm{R}} g_{\mathrm{L}} u_1 u_2$ is the $W_{\mathrm{L}} - W_{\mathrm{R}}$ mixing mass squared.

The mass matrix $m_{\mathrm{D}}$ is diagonal, and is parametrised by $0 < \eta < 1$ such that

$$m_{\mathrm{D}} = M_l \eta S, \tag{13}$$

where

$$S = S(M_{W_{\mathrm{R}}}) = \frac{g_{\mathrm{R}} g_{\mathrm{L}}}{8\pi^2}\,\frac{1}{2}\sqrt{3}\,\frac{M_{W_{\mathrm{L}}}^2}{M_{W_{\mathrm{R}}}^2}\,\frac{m_b}{m_t}\,\ln\left(\frac{M_{W_{\mathrm{R}}}^2}{M_{W_{\mathrm{L}}}^2}\right) \sim 10^{-7}\left(\frac{\mathrm{TeV}}{M_{W_{\mathrm{R}}}}\right)^2 . \tag{14}$$

The mass mixing term $m_{\nu E}$ (Fig. 2) arises at one-loop level via the gauge interactions

$$\frac{g_{\mathrm{L}}}{\sqrt{2}}\,\overline{E_{\mathrm{L}}^0}\,\not{W}_{\mathrm{L}}^{+} E_{\mathrm{L}}^{-} + \frac{g_{\mathrm{R}}}{\sqrt{2}}\,\overline{(E_{\mathrm{R}}^{-})^c}V^{\dagger}\,\not{W}_{\mathrm{R}}^{+}\tilde{\nu}_{\mathrm{L}} + \text{h.c.} \tag{15}$$

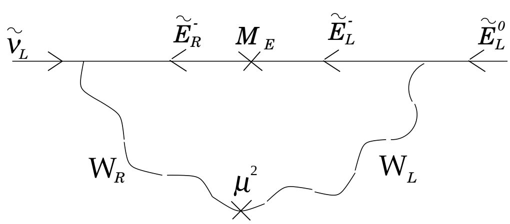


Fig. 2. $\overline{\tilde{\nu}_{\mathrm{L}}}(\tilde{E}_{\mathrm{L}}^0)^c$ neutrino mixing term generated by gauge interactions leading to the mass term $m_{\nu E}$.

In contrast to the mass matrix $m_{\rm D}$, the involvement of the matrix $V$ in the interactions Eq. (15) may mediate cross generational mixing so that the matrix $m_{\nu E}$ is non-diagonal in general. However, in the special case of decoupled generations (*i.e.* $V \to \boldsymbol{I}$), the mass matrix $m_{\nu E}$ reduces to the diagonal form

$$\lim_{V\to \boldsymbol{I}} m_{\nu E} = \eta M_E S\,. \tag{16}$$

Hence, we obtain an effective Lagrangian density for the mass matrix of the neutrinos (which are approximately decoupled from the heavy $E$ leptons),

$$\mathcal{L}_{\rm eff} \;=\; \tfrac{1}{2}\left(\ \overline{\tilde{\nu}_{\rm L}} \quad \overline{(\tilde{\nu}_{\rm R})^c}\ \right) M_\nu \left(\begin{array}{c} (\tilde{\nu}_{\rm L})^c \\ \tilde{\nu}_{\rm R} \end{array}\right) + \text{ h.c.}, \tag{17}$$

where the matrix $M_\nu$ is given by

$$M_\nu \simeq \left(\begin{array}{cc} 0 & m'_{\rm D} \\ (m'_{\rm D})^\dagger & M_{\rm R} \end{array}\right), \tag{18}$$

with

$$m'_{\rm D} = m_{\rm D} + m_{\nu E} M_E^{-1} V^\dagger M_l\,, \tag{19}$$

and $M_{\rm R}$ is given in Eq. (11).

In the see-saw limit where the eigenvalues of $M_{\rm R}$ are much larger than the eigenvalues of $m'_{\rm D}$ (which is generally valid), the right and left neutrino states are effectively decoupled:

$$\mathcal{L}^{\rm see-saw} \simeq \tfrac{1}{2}\ \overline{\tilde{\nu}_{\rm L}}\ m_{\rm L}(\tilde{\nu}_{\rm L})^c + \tfrac{1}{2}\ \overline{(\tilde{\nu}_{\rm R})^c}\ M_{\rm R}\tilde{\nu}_{\rm R} + \text{ h.c.}\,, \tag{20}$$

where

$$m_{\rm L} \;\simeq\; -m'_{\rm D} M_{\rm R}^{-1} (m'_{\rm D})^\dagger\,. \tag{21}$$

Knowledge of the mass matrix for the light neutrinos, $m_{\rm L}$, allows us to work out the oscillation parameters (*i.e.* the mixing angles and $\delta m^2$) among the light neutrinos $\nu_{\rm L}$, and thereby make contact with the neutrino data. For the sake of simplicity, we will drop the tilde in the neutrino fields in the subsequent discussion whenever no confusion could arise. Unless otherwise stated, the symbols $\nu_{\rm L}, \nu_{\rm R}$ are all (approximately) flavour eigenstates.

One way to obtain large $\nu_{e\rm L} \to \nu_{\tau\rm L}$ oscillations as suggested by the SNO data and other solar neutrino experiments and small mixing angle $\nu_{\mu\rm L} \to \nu_{e\rm L}$ oscillations for the LSND data is for the mass matrix $m_{\rm L}$ in Eq. (21) to have the approximate form

$$m_{\rm L} \approx \left(\begin{array}{ccc} 0 & 0 & m_1 \\ 0 & m_2 & 0 \\ m_1 & 0 & 0 \end{array}\right), \tag{22}$$

where the "0" elements are not strictly zero, but much smaller than the $m_i$. Clearly the mass matrix $m_{\rm L}$ has 3 eigenvalues, $\lambda'_1 \approx \lambda'_3 \approx m_1$, $\lambda'_2 \approx m_2$. If this mass matrix is to be consistent with the solar and LSND neutrino data, then it is required that

**A** $\lambda'_1$ and $\lambda'_3$ have to be split in such a way that $|\lambda'^2_1 - \lambda'^2_3|$ is identified with $\delta m^2_{\rm solar}$,

$$|\lambda'^2_1 - \lambda'^2_3| \equiv \delta m^2_{\rm solar}. \tag{23}$$

**B** For the sake of naturalness, the absolute mass scale of $m_1^2$ has to be much larger than $\delta m^2_{\rm solar}$,

$$m_1^2 \gg \delta m^2_{\rm solar}. \tag{24}$$

**C** We require that

$$\delta m^2_{\rm LSND} = |\lambda'^2_1 - \lambda'^2_2| \simeq |m_1^2 - m_2^2|, \tag{25}$$

where $0.2\ {\rm eV}^2 \lesssim \delta m^2_{\rm LSND} \lesssim 10\ {\rm eV}^2$.

Our strategy now is to find the forms of the CKM matrices (and the corresponding Yukawa matrices $\lambda_1 - \lambda_4$) which lead to a neutrino mass matrix $m_{\rm L}$ of the form Eq. (22), and also satisfy the requirements **A, B** and **C**. It turns out that the above conditions do not lead to a unique solution. However, if we impose an additional condition on the CKM-type unitary matrices that

**D** There is a left-right similarity between the CKM matrix $K_{\rm L}$ and the corresponding CKM-type matrix $K_{\rm R}$ for the SU(2)$_{\rm R}$ interactions of the right-handed quarks, *i.e.*

$$K_{\rm L} \approx K_{\rm R} (\equiv Y_{\rm R}^\dagger K'^\dagger) \approx \boldsymbol{I}, \tag{26}$$

then apparently a unique picture emerges. That is, it is found that there is a simple set of CKM-type matrices where the theory is consistent with a SU(4) $\otimes$ SU(2)$_{\rm L}$ $\otimes$ SU(2)$_{\rm R}$ symmetry breaking scale of less than a few TeV and reproduces the form of $m_{\rm L}$ as in Eq. (22) with the conditions Eqs. (23)–(26) satisfied[4]:

[4] The form of these matrices could be derived following a similar procedure as was done in Ref. [9]. Recall that in Ref. [9] it was assumed that the atmospheric neutrino anomaly is solved by $\nu_{\mu\rm L} \to \nu_{\tau\rm L}$ oscillations, the LSND data is solved by $\nu_{\mu\rm L} \to \nu_{e\rm L}$ oscillations with the solar neutrino anomaly solved by $\nu_{e\rm L} \to \nu_{\rm sterile}$ oscillations. While in the present (post SNO) paper the solar neutrino anomaly is solved by $\nu_{e\rm L} \to \nu_{\tau\rm L}$ oscillations, the LSND data is solved by $\nu_{\mu\rm L} \to \nu_{e\rm L}$ oscillations and the atmospheric neutrino anomaly is solved by $\nu_{\mu\rm L} \to \nu_{\rm sterile}$ oscillations. The difference is essentially a transformation of $\nu_{e\rm L} \leftrightarrow \nu_{\mu\rm L}$.

$$V \approx \boldsymbol{I_3}\,, \qquad U \approx \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad K' \approx Y_{\rm R}'^{\dagger} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{27}$$

Eq. (26) amounts to suggesting that the non-diagonal $K'$ (required for low symmetry breaking scale, see [8]) and non-diagonal $Y_{\rm R}^{\dagger}$ (required to obtain large mixing angle $\nu_{e{\rm L}} \to \nu_{\tau{\rm L}}$ oscillations) may have a common origin.

The set of CKM-type matrices (Eq. (27)) would arise in the theory if we make the following ansatz for the Yukawa matrices $\lambda_1, \lambda_3, \lambda_4$:

$$\lambda_1 \approx \begin{pmatrix} 0 & 0 & \times \\ \times & 0 & 0 \\ 0 & \times & 0 \end{pmatrix},\ \lambda_3 \approx \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix},\ \lambda_4 \approx \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}, \tag{28}$$

with the matrix $\lambda_2$ approximately diagonal. In other words, if $\lambda_1, \ldots, \lambda_4$ have the above form then a low symmetry breaking scale ($\lesssim$ few TeV) is phenomenologically viable, and additionally, the model can accommodate the large mixing angle $\nu_{e{\rm L}} \to \nu_{\tau{\rm L}}$ solution to the solar neutrino problem as well as small angle $\nu_{\mu{\rm L}} \to \nu_{e{\rm L}}$ oscillations as suggested by the LSND experiment. In this solution scheme, the absolute scales of the mass squared of the left-handed neutrinos turns out to be[5]

$$\begin{aligned} m_1^2 &\approx 16\eta^4 S^4 M_{E_3}^2 \left(\frac{m_e}{m_u}\right)^2 \sim 10^{-1}\eta^4 \left(\frac{M_{E_3}}{\rm TeV}\right)^2 {\rm eV}^2, \\ m_2^2 &\approx 4\eta^4 S^4 M_{E_2}^2 \left(\frac{m_\mu}{m_t}\right)^2 \sim 10^{-6}\eta^4 \left(\frac{M_{E_2}}{\rm TeV}\right)^2 {\rm eV}^2. \end{aligned} \tag{29}$$

The matrices in Eqs. (26), (27) are translated (via the matrix $M_{\rm R}$ in Eq. (11)) into the following mass spectrum for the right-handed neutrinos:

$$\begin{aligned} m_{\nu R1} \approx m_{\nu R3} &= \frac{m_\tau m_u}{M_{E_3}} + \frac{m_e m_{\rm c}}{M_{E_1}} \sim \frac{m_\tau m_u}{M_{E_3}}, \\ m_{\nu R2} &= \frac{2 m_\mu m_t}{M_{E_2}}. \end{aligned} \tag{30}$$

Among $M_{E_i}$, $M_{E3}$ is constrained by the requirement to accommodate the solution to the LSND result (Eq. (25) and Eq. (29)) which implies the lower bound

$$M_{E_3} \gtrsim {\rm TeV}. \tag{31}$$

[5] For numerical definiteness, we have assumed $S$ to take on a value near its allowed upper bound, $S \sim 10^{-6}$, see Eq. (14).

This bound on the mass of $M_{E_3}$ in turn constrains the mass spectrum of $m_{\nu_{\rm R}}$ to the ranges:

$$\begin{aligned} 1\ \text{keV} &\lesssim m_{\nu R1} \approx m_{\nu R3} \lesssim 10\ \text{keV}, \\ 4\ \text{MeV} &\lesssim m_{\nu R2} \lesssim 1\ \text{GeV}\,. \end{aligned} \tag{32}$$

In other words, in the present solution scheme there are two approximately degenerate light right-handed neutrinos $\nu_{\rm R1}, \nu_{\rm R3}$ (with masses in the range keV–10 keV) and a heavy $\nu_{\rm R2}$ with mass in the range of $\sim$ MeV to $\sim$ 1 GeV.

The mixing between the left and right neutrino sectors, though suppressed, could be detected in a tell-tale kink (slope discontinuity) in the otherwise continuous spectrum of a low $Q$-value nuclear beta decay (such as the tritium beta decay) [32]. For ordinary-sterile neutrino two-state mixing, the weak eigenstates $\{\tilde{\nu}_{\rm L}, (\tilde{\nu}_{\rm R})^c\}$ will be linear combinations of the two mass eigenstates $\{\nu_{\rm L}, (\nu_{\rm R})^c\}$:

$$\tilde{\nu}_{\rm L} = \cos\psi \nu_{\rm L} + \sin\psi (\nu_{\rm R})^c, \quad (\tilde{\nu}_{\rm R})^c = -\sin\psi \nu_{\rm L} + \cos\psi (\nu_{\rm R})^c, \tag{33}$$

where the mixing angle between the $i$ left-handed state and $j$ right-handed state is given by

$$\sin^2\psi_{ij} \sim \left|\frac{m_i}{m_{\nu_{{\rm R}j}}}\right|, \tag{34}$$

which is of significant interest only for $ij = \{13\}, \{31\}$, *i.e.*,

$$\sin^2\psi_{13} \simeq \sin^2\psi_{31} \sim 10^{-3}\eta^2 \left(\frac{\text{keV}}{m_{\nu_{\rm R1}}}\right)^2 . \tag{35}$$

The mixing between $\tilde{\nu}_{\mu{\rm L}} - (\tilde{\nu}_{\rm R2})^c$ is much smaller and we can ignore it (as with the other $ij \neq \{13\}, \{31\}$ channels). We will take the convention

$$\delta m_{ij}^2 \equiv m_{\nu_{{\rm R}j}}^2 - m_i^2 \tag{36}$$

so that $\delta m_{ij}^2$ is positive. Due to the mixing, the spectrum of a weak decay that includes $\nu_{e{\rm L}}$ in the final state will consist of two components corresponding to $m_1$ and $m_{\nu_{\rm R3}}$. In the limit $m_{\nu_{\rm R3}} \gg m_1$, the observed beta spectrum can be expressed as the product of the massless neutrino spectrum and a massive neutrino shape factor $S(E)$,

$$\frac{dN}{dE} \propto \frac{dN(E, m_1 = 0)}{dE}\, S(E)\,, \tag{37}$$

with

$$S(E) = \begin{cases} 1 + \tan^2\psi_{13}\left[1 - \frac{m_{\nu_{\rm R3}}^2}{(Q-E)^2}\right]^{1/2} & \text{for } E \leq Q - m_{\nu_{\rm R3}}\,, \\ 1 & \text{for } E > Q - m_{\nu_{\rm R3}}\,, \end{cases} \tag{38}$$

where $E$ is the beta decay's energy and $Q$ is the total decay energy. The mixed spectrum will display a kink at $E = Q - m_{\nu_{\rm R3}}$ corresponding to the mass $m_{\nu_{\rm R3}}$ and mixing angle of Eq. (35), which is a signature predicted by the 422 model. At present, the sensitivity of the experimental searches for such kink in nuclear beta decays, which only sets an upper bound of $\sin^2\psi_{13} \lesssim$ few $10^{-3}$ for $m_{\nu_{\rm R3}}$ in the range of a few keV [33], is on the verge of detecting the presence of the keV component predicted by the 422 model. If the sensitivity of the experiments could increase by two orders of magnitude in the future, we will be able to verify (or falsify) the model by detecting (or not detecting) this signature.

Having addressed the significance of the laboratory signature from the mixing between the left- and right-handed neutrinos, we now turn to investigate the cosmological implications of these right-handed neutrinos which are potentially in conflict with the standard cosmological energy density bound[6].

## 3. The right-handed neutrinos in the framework of standard cosmology

In this section we would like to determine if standard Big Bang cosmology is consistent with the minimal 422 model. Within the context of the standard Big Bang model, the present energy density of a given particle species $X$, $\rho_X$ must not be much larger than the critical density of the Universe $\rho_{\rm c}$, see Eq. (2). The first check of the implications of the minimal alternative 422 model on the standard cosmological picture is therefore, to estimate the present energy density of the right-handed neutrinos in this model. In the following we will estimate the present day relic density of $\nu_{\rm R1}, \nu_{\rm R3}$ with the assumption that they are "hot" (*i.e.* particle species that freeze-out from the thermal plasma while still relativistic) and approximately stable in the standard early Universe scenario. The heavier $\nu_{\rm R2}$ neutrinos will be discussed separately in Sec. 4.2.

In the alternative 422 model, the dominant decay mode of $\nu_{\rm R1}, \nu_{\rm R3}$ is the $Z^0$-mediated tree-level process,

$$\nu_{R1,R3} \xrightarrow{Z^0} \nu_{\tau,e} + \bar{\nu}_\alpha \nu_\alpha \;\; (\alpha = e, \mu, \tau), \tag{39}$$

with a lifetime of the order

$$\tau_{\nu_{R1,R3}} \sim \frac{\tau_\mu}{\sin^2\psi_{13}} \left(\frac{m_\mu}{m_{\nu_{\rm R1}}}\right)^5 \sim \frac{10^{22}}{\eta^2} \left(\frac{\rm keV}{m_{\nu_{\rm R1}}}\right)^3 {\rm s}, \tag{40}$$

[6] Note that the exotic leptons $\{E^0, E^-\}$ do not lead to a cosmological energy density problem. This is because their masses are heavy enough to allow them to rapidly decay into quarks and leptons via the gauge interactions.

(where $\tau_\mu \approx 10^{-6}$ s is the muon decay lifetime) which is much larger that the age of the Universe, $t_{\rm U} \sim 10^{17}$ s. Therefore, $\nu_{\rm R1}, \nu_{\rm R3}$ could be taken as approximately stable as far as their contribution to the energy density of the Universe is concerned[7].

The relic density is a function of the masses of the right-handed neutrinos. Obviously, a heavier relativistic particle has a larger contribution to the relic density. Because various observations limit the relic density to be less than the critical density of the Universe, this will in turn suggest an upper limit on the masses of these particles which can be compared with the mass range of Eq. (32). It is fairly straightforward to estimate the constraint imposed on the masses of the right-handed neutrinos from $\Omega_{\nu_{\rm R}} h^2 \lesssim 1$ within the standard cosmological framework (see for example Sec. 5 in Ref. [35]).

In order to find out the relic abundance of the lightest right-handed neutrino we need to estimate its production rate. Production of right-handed neutrinos can occur via two distinct mechanisms. First there is a direct production by $Z'$- and $W_{\rm R}$-mediated interactions, such as[8]

$$e_{\rm R}\overline{e_{\rm R}} \overset{Z'}{\longleftrightarrow} \overline{\nu_{\rm R}}\nu_{\rm R}. \tag{41}$$

Second, right-handed neutrinos can be produced via oscillations of $\nu_{\rm L} \leftrightarrow (\nu_{\rm R})^c$. However, for the purpose of this section, we will ignore the second production mechanism because it will not modify our general conclusions.

To estimate the relic abundance of the $\nu_{\rm R1}, \nu_{\rm R3}$ from direction production, we have to sum over the averaged direct production rates, *i.e.*

$$\Gamma = \sum_\psi \langle \Gamma_{\bar\psi\psi \to \overline{\nu_{\rm R}}\nu_{\rm R}} \rangle \quad \text{for } \psi = e_{\rm L}, e_{\rm R}, \nu_{\alpha {\rm L}}\,, \tag{42}$$

(see Fig. 3).

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[7] In addition to the tree-level decay of Eq. (39), there also exists a sub-dominant radiative decay mode $\nu_{\rm R} \to \nu_{\rm L}\gamma$ at one-loop level. Since the transition moment is generated at one-loop level, its decay rate is suppressed. From Ref. [34] it can be estimated to be $\Gamma_\gamma/\Gamma_\mu \sim \frac{9\alpha}{64\pi}\sin^2\theta_{\rm mix}(m_{\nu_{\rm R}}/m_\mu)^5(m_\tau/M_{W_{\rm L}})^4\sin^2\phi \sim 10^{-43}\eta^2\sin^2\phi(m_{\nu_{\rm R}}/{\rm keV})^5$, where $|\sin\theta_{\rm mix}| = \frac{\eta}{2\sqrt{3}}\frac{M^2_{W_{\rm L}}}{M^2_{W_{\rm R}}}\frac{m_b}{m_t} \lesssim 10^{-4}$ is the $W_{\rm L} - W_{\rm R}$ mixing angle and $\sin\phi$ is the relevant leptonic $K_{\rm R}$ mixing angle. The radiative lifetime of $\nu_{\rm R} \to \nu_{\rm L}\gamma$ should be larger than $10^{24}$ s, which is required by the astrophysical constraint from the diffuse photon background for a radiatively decaying species $X$ in the mass range of $m_X \sim$ keV [35]. Not surprisingly, we find that this is in fact the case for the light right-handed neutrinos for all parameter space of interest, and thus this decay mode can be safely ignored.

[8] In addition to the process in Eq. (41), there are also other channels that contribute to the production of the right-handed neutrinos, including $W_{\rm R}$ mediated processes. The contribution from $W_{\rm R}$ mediated exchange will be at most the same order to that of the $Z'$ exchange channel. For simplicity sake, in the following calculation we shall only take the $Z'$ exchange channel into account for estimating the production of $\nu_{\rm R}$. Ignoring the $W_{\rm R}$ channel would not effect the conclusions of the present paper.

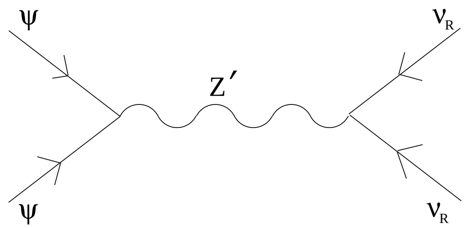


Fig. 3. Direct production of the right-handed neutrinos via scatterings of $\psi\overline{\psi}\leftrightarrow\overline{\nu_{\mathrm{R}}}\nu_{\mathrm{R}}$ mediated by $Z'$, where $\psi = e_{\mathrm{L}}, e_{\mathrm{R}}, \nu_{\alpha\mathrm{L}}$ $(\alpha = e, \mu, \tau)$.

As the Universe expands, the temperature (denoted by $T$) decreases, and so does the rate of direct right-handed neutrino production. When the temperature drops below a certain value $T_{\mathrm{F}}$ (the freeze-out temperature) the production rate would not be large enough to thermalise the right-handed neutrinos with the thermal background. The right-handed neutrino is said to freeze-out from the thermal background, leaving behind its abundance frozen at the value it had when last in thermal equilibrium. The freeze-out temperature is approximately determined from the condition that the scattering rate $\Gamma$ has decreased to that of the Hubble expansion rate $H$,

$$\Gamma(T) \approx H(T)\,, \tag{43}$$

where the Hubble expansion rate is given by

$$H = 1.66\sqrt{g_*}\,\frac{T^2}{M_{\mathrm{Pl}}}\,, \tag{44}$$

and the scattering rate is given by

$$\Gamma(T) = \sum_{\psi}\langle \Gamma_{\bar{\psi}\psi\to\overline{\nu_{\mathrm{R}}}\nu_{\mathrm{R}}}\rangle \approx \tfrac{1}{4}{G'_{\mathrm{F}}}^2 T^5\,. \tag{45}$$

In Eq. (44), $M_{\mathrm{Pl}}$ is the Planck mass and $g_*$ counts the relativistic degrees of freedom that contribute to the energy density of the Universe, as defined in the usual way, by $\rho_{\mathrm{R}} = (\pi/30)g_* T^4$. In Eq. (45), $G'_{\mathrm{F}}$ is a Fermi constant-like quantity that characterises the strength of the $Z'$ neutral current interaction,

$$G'_{\mathrm{F}} = \left(\frac{M_{W_{\mathrm{L}}}}{M_{Z'}}\right)^2 G_{\mathrm{F}}\,, \tag{46}$$

where $G_{\mathrm{F}} \simeq 10^{-5}$ GeV$^{-2}$ is the Fermi constant. The freeze-out temperature $T_{\mathrm{F}}$ can be estimated by solving Eq. (43), which gives

$$T_{\mathrm{F}} \sim 50 \text{ MeV} \ \text{ for } M_{Z'} \sim \text{TeV}\,. \tag{47}$$

Comparing Eq. (47) with the mass spectrum Eq. (32) we see that the light right-handed neutrinos are indeed relativistic at $T \approx T_{\rm F}$, which is what we have assumed.

The right-handed neutrinos, after freezing out from the thermal background, would still maintain their equilibrium distribution at temperature $T_{\nu_{\rm R}}$ which eventually becomes smaller than the background photon temperature $T$ by a factor of

$$\frac{T_{\nu_{\rm R}}}{T} = \left(\frac{g_{*S}^{\rm after}}{g_{*S}^{\rm before}}\right)^{1/3} \lesssim 1\,, \tag{48}$$

when the entropy from the $e^{\pm}$ annihilation is transferred to the photons at $T \approx m_e$. In Eq. (48), $g_{*S}^{\rm before}$ counts the total number of relativistic degrees of freedom that contribute to the entropy density $s$ of the Universe (as defined by $s = (2\pi^2/45)\, g_{*S}T^3$) just a moment before $T = m_e$, while $g_{*S}^{\rm after}$ counts the number of relativistic degrees of freedom at some later time. Standard calculation for the present energy density of the $\nu_{\rm R1}, \nu_{\rm R3}$ right-handed neutrinos (normalised to the critical density) leads to

$$\Omega_{\nu_{\rm R1}+\nu_{\rm R3}}h^2 = \frac{n_{\nu_{\rm R}} m_{\nu_{\rm R1}} h^2}{\rho_{\rm c}} = \frac{3}{4}\left(\frac{g_{*S}^{\rm after}}{g_{*S}^{\rm before}}\right)\frac{n_\gamma m_{\nu_{\rm R1}} h^2}{\rho_c}\,, \tag{49}$$

where

$$\frac{n_{\nu_{\rm R}}}{n_\gamma} = \frac{3}{4}\left(\frac{g_{*S}^{\rm after}}{g_{*S}^{\rm before}}\right)$$

is the ratio of the number density of the $\nu_{\rm R}$'s to that of the photons in the cosmic microwave background (CMBR). At present, $n_\gamma = 422~{\rm cm}^{-3}$. The constraint $\Omega_{\nu_{\rm R1}+\nu_{\rm R3}}h^2 \lesssim 1$ restricts $m_{\nu_{\rm R1}}$ to the range

$$m_{\nu_{\rm R1}} \lesssim 30\ \frac{g_{*S}^{\rm before}}{g_{*S}^{\rm after}}\ {\rm eV}, \tag{50}$$

which is clearly not consistent with the mass spectrum of the right-handed neutrinos in Eq. (32), as the factor $(g_{*S}^{\rm before}/g_{*S}^{\rm after})$ is constrained to be around $16/5.82 = 2.75$ for $M_{Z'} \sim$ few TeV. Therefore, we are led to the conclusion that, in the framework of standard cosmology, even the lightest right-handed neutrinos are not consistent with the cosmological energy density bound of Eq. (2). They will over-close the Universe.

Confronted with the inconsistency with the cosmological energy density bound, one could attempt to modify the particle physics to get around the conflict with standard cosmology. A popular way to do this is by introducing a massless Majoron $J$ that arises from the spontaneous breaking of an imposed global symmetry. Things could be arranged in such a way that the coupling of $J$ with $\nu_{\mathrm{R}}$ opens up an invisible decay channel of $\nu_{\mathrm{R}}$ into the undetected Majoron (and $\nu_{\mathrm{L}}$) that is rapid enough to alleviate the cosmological bounds. However, in our opinion, such a remedy is rather desperate, and we would not pursue it further to avoid spoiling the elegance of the 422 model. Instead of modifying the model we prefer to explore an alternative cosmology scenario to find a way out of the conflict. This will be done in the next section.

## 4. Right-handed neutrinos in the framework of non-standard cosmology with low reheating temperature

It is usually assumed that the radiation-dominated era commences after a period of inflation, and that the cold Universe at the end of inflation becomes the hot Universe of the radiation-dominated era in a process known as reheating. During reheating, a thermal bath of relativistic particles (*e.g.* electrons and photons) is slowly formed as the coherent oscillations of a condensate of zero-momentum massive scalar field $\phi$ decays [35]. The completion of the $\phi$ decay marks the commencement of radiation-dominated era at an initiation temperature $T_{\mathrm{RH}}$. From a phenomenological point of view, the reheating temperature $T_{\mathrm{RH}}$, which is given in terms of $\Gamma_\phi$, the lifetime of the massive scalar field $\phi$ [35],

$$T_{\mathrm{RH}} = \sqrt{M_{\mathrm{Pl}}\Gamma_\phi}\left[\frac{90}{8\pi^3 g_*(T_{\mathrm{RH}})}\right]^{1/4}, \tag{51}$$

could be treated as a free parameter that is model-dependent (*i.e.* via the dynamics of the $\phi$ physics in an expanding cosmic background). It is an *a priori* assumption in standard big bang cosmology that at the initiation of the radiation-dominated era, thermal and chemical equilibrium prevail as an initial condition, which is equivalent to the hypothesis that $T_{\mathrm{RH}}$ is higher than the freeze-out temperature of the cosmological process under consideration (in our case here the pertinent process is the production of the right-handed neutrinos, with the freeze-out temperature $T_{\mathrm{F}} \sim 50$ MeV). However, there is no empirical evidence of the radiation-dominated era before the epoch of BBN, *i.e.* temperature above $\sim 1$ MeV. The only real constraint on $T_{\mathrm{RH}}$ is that suggested by BBN which implies that $T_{\mathrm{RH}}$ could be as low as 0.7 MeV [25, 26].

If the reheating temperature is indeed only of order $\sim$ MeV, interesting modifications to some standard cosmological bounds on particle physics would be necessitated. For example, with such a low reheating temperature scenario a given dark matter species $X$ may never achieve chemical equilibrium with the thermal radiation background, resulting in a relic abundance that is much lower than that predicted by standard cosmology. It may, therefore, be possible that the 422 model might be reconciled with cosmology if $T_{\rm RH}$ is low enough (the calculation of the previous section only holds in the limit of high $T_{\rm RH}$). We now study this possibility.

### *4.1. Relic abundance of the light $\nu_{\rm R}$ in the low reheating Universe*

In this subsection we would like to answer the question of whether the alternative 422 model is consistent with the low reheating Universe by first estimating the relic density of the $\nu_{\rm R1}, \nu_{\rm R3}$ neutrinos produced via collisional processes in the non-standard cosmological scenario. In such a low reheating scenario

$$T_{\rm RH} < T_{\rm F} \sim 50 \text{ MeV}, \tag{52}$$

which implies that chemical equilibrium of $\nu_{\rm R}$ is not attained. This means that $n_{\nu_{\rm R}} \ll n_{\nu_{\rm R}}^{\rm eq}$ (where $n_{\nu_{\rm R}}$, $n_{\nu_{\rm R}}^{\rm eq}$ are the actual and equilibrium densities). The heavier $\nu_{\rm R2}$ right-handed neutrino will be treated separately in Sec. 4.2.

There are two distinct types of collisional processes. First, we have the direct production, as already considered in Sec. 3. Second, we have the effect of collisions on the oscillating neutrino ensemble. Our purpose is to obtain the present day abundance of the right-handed neutrinos by integrating the corresponding Boltzmann equation that governs the time evolution of these production mechanisms.

Assuming $n_{\nu_{\rm R}} = n_{\overline{\nu_{\rm R}}}$, the Boltzmann equation pertinent to the production of the right-handed neutrinos via collisional processes in an expanding cosmic background is given by [35]

$$\frac{dn_{\nu_{\rm R}}}{dt} + 3Hn_{\nu_{\rm R}} = -\langle\sigma|v|\rangle \left[n_{\nu_{\rm R}}^2 - (n_{\nu_{\rm R}}^{\rm eq})^2\right], \tag{53}$$

where $\langle\sigma|v|\rangle$ is the total thermal averaged cross section times velocity for the relevant collisional process that produces the $\nu_{\rm R}$ state. It is useful to scale out the effect of the expansion of the Universe by considering the evolution of the number of particles in a comoving volume $Y_{\nu_{\rm R}} \equiv n_{\nu_{\rm R}}/s$, where $s$ is the entropy density. Introducing an independent parameter that explicitly depends on the temperature $T$, $x \equiv m_{\nu_{\rm R}}/T$, Eq. (53) can be expressed in the form

$$\frac{x}{Y_{\nu_{\rm R}}^{\rm eq}} \frac{dY_{\nu_{\rm R}}}{dx} = -\frac{\Gamma}{H}\left[\left(\frac{Y_{\nu_{\rm R}}}{Y_{\nu_{\rm R}}^{\rm eq}}\right)^2 - 1\right], \tag{54}$$

where $\Gamma \equiv n_{\nu_{\rm R}}^{\rm eq}\langle\sigma|v|\rangle$. If the reheating temperature is lower than $T_{\rm F} \sim$ 50 MeV (or equivalently, $\Gamma \ll H$), then the interactions are too weak for chemical equilibrium to be attained. In this case, the $Y_{\nu_{\rm R}}$ term in the right-handed side of Eq. (54) could be dropped (to a good approximation) and the equation recast into the form

$$\frac{4x}{3Y_\gamma}\frac{dY_{\nu_{\rm R}}}{dx} \simeq \frac{\Gamma}{H}\,. \tag{55}$$

In arriving at Eq. (55) we have made use of the ratio of fermion ($\nu_{\rm R}$) and boson (photon) at thermal equilibrium, $Y^{\rm eq}_{\nu_{\rm R}}/Y_\gamma = n^{\rm eq}_{\nu{\rm R}}/n_\gamma = 3/4$, where the possible factor of $(T_{\nu{\rm R}}/T)^3 = g^*_{\rm after}/g^*_{\rm before}$ (which is not significantly different than 1) is ignored.

In principle, to evaluate the present density of the right-handed neutrinos we have to integrate Eq. (55) for both reheating era plus radiation-dominated era, *i.e.* $\int dY_{\nu{\rm R}} = \int_{\rm reheating} dY_{\nu{\rm R}} + \int_{\rm radiation} dY_{\nu{\rm R}}$. However, $\int_{\rm reheating} dY_{\nu{\rm R}}$ is governed by model-dependent physics of reheating and its contribution to the production of the right-handed neutrinos is not determined with definiteness. We, therefore, consider only the right-handed neutrinos produced in the radiation-dominated era (*i.e.* for $T \leq T_{\rm RH}$) and approximate $\int dY_{\nu{\rm R}} \approx \int_{T_{\rm RH}}^{T_0} dY_{\nu{\rm R}}$, in which the Hubble expansion rate $H$ scales with temperature as in Eq. (44).

Hence the present day relic abundance of the $\nu_{\rm R1}, \nu_{\rm R3}$ neutrinos is taken as $Y_{\nu_{\rm R0}} \approx Y_{\nu_{\rm R}}(T = m_{\nu_{\rm R1}})$, obtained by integrating Eq. (55) from $T = T_{\rm RH}$ to $T = m_{\nu_{\rm R1}}$ by employing the appropriate relativistic form of the "effectiveness of production" term, $\Gamma/H$[9]. The collisional production rate, $\Gamma \equiv \Gamma^{\rm col}$ is the sum of the direct production term and a decoherence term (which arises for the effects of collisions on the oscillating neutrino ensemble)

$$\Gamma^{\rm col} = \Gamma^{\rm dp} + \Gamma^{\rm col-os}\,. \tag{56}$$

As was considered in Sec. 3, the relativistic form of $\Gamma^{\rm dp}$ for the direct production mechanism is given by Eq. (45). The production rate, $\Gamma^{\rm col-osc}$, is given by [36]

$$\begin{aligned}\Gamma^{\rm col-os} &= (D_{\nu_e} + D_{\nu_\tau})\left\langle \sin^2\left(\frac{\tau_{\rm coll}}{\tau_{\rm osc}}\right)\right\rangle \sin^2 2\psi \\ &\simeq \frac{yG_{\rm F}^2T^5}{2}\frac{1}{2}\sin^2 2\psi\,,\end{aligned} \tag{57}$$

where [37]

$$D_{\nu_\alpha} = \frac{1}{2}\Gamma_{\nu_\alpha} \simeq \frac{1}{2}y_{\nu_\alpha}G_{\rm F}^2T^5 \tag{58}$$

[9] The contribution to $Y_{\nu_{\rm R}}$ from the non-relativistic regime $T = m_{\nu_{\rm R1}} \to T = T_0$ is very tiny compared to that from the relativistic regime, and hence can be ignored.

is the thermally averaged collision frequencies between the left-handed $\alpha$-neutrinos with the background plasma, and $y = y_{\nu_e} + y_{\nu_\tau} = 4.0 + 2.9 = 6.9$ [38]. The time scales characterising the period of oscillation of $\nu_{\rm L} - (\nu_{\rm R})^c$, $\tau_{\rm osc}$, and that of the collisions between the $\nu_{\rm L}$'s with the thermal background, $\tau_{\rm coll}$, are given by

$$\begin{aligned} \langle \tau_{\rm osc} \rangle &\equiv \frac{4\langle E \rangle}{\delta m_{ij}^2} \sim 10^{-14} \left( \frac{\rm keV}{m_{\nu_{\rm R1}}} \right)^2 \left( \frac{T}{\rm MeV} \right) \, {\rm s}, \\ \langle \tau_{\rm coll} \rangle &\sim \frac{1}{D_{\nu_\alpha}} \sim {\rm few} \left( \frac{\rm MeV}{T} \right)^5 \, {\rm s}\,. \end{aligned} \tag{59}$$

Clearly, the oscillation period is very short compared to the average interval between collisions, *i.e.* $\langle \tau_{\rm osc} \rangle \ll \langle \tau_{\rm coll} \rangle$ for all of the temperature range of interest, so that many oscillations occur between successive collisions. As a result, the collisions will not significantly destroy the coherence of the ensemble and we have $\langle \sin^2(\tau_{\rm coll}/\tau_{\rm osc}) \rangle = \frac{1}{2}$ in Eq. (57).

Because the right-handed neutrinos are much heavier than the left-handed states (so that the sign of $\delta m_{ij}^2 \equiv m_{\nu_{{\rm R}j}}^2 - m_i^2 \approx + m_{\nu_{{\rm R}j}}^2$ as defined via Eq. (36) is positive), no oscillation neutrino asymmetry amplification can occur via the oscillations [39, 40]. Furthermore, since $\delta m_{ij}^2 \gtrsim 10^6\ {\rm eV}^2$ (given the mass range of $m_{\nu_{\rm R}}$, Eq. (32)), it turns out that we can neglect matter effects for the temperature range of interest[10]. This justifies the use of Eq. (57).

We can now plug in the collision rates for both collisional mechanisms, Eq. (45) and Eq. (57) into Eq. (55) for $\Gamma$ and integrate it to obtain the present number densities of the $\nu_{\rm R1}, \nu_{\rm R3}$ right-handed neutrinos,

$$\frac{n_{\nu_{\rm R0}}^{\rm dp}}{n_\gamma} \sim 10^{-6} \left( \frac{\rm TeV}{M_{Z'}} \right)^4 \left( \frac{T_{\rm RH}}{\rm MeV} \right)^3,$$

[10] In general, due to interactions that discriminate between the active and sterile neutrinos by the thermal background (*i.e.* the matter effect) [41, 42], an effective potential [43] will be induced and felt by the $\alpha$-flavour active neutrinos, thus changing the dynamics of the coherent oscillations. The effective potential can be put into the convenient form [40]

$$V_\alpha = (-a + b) \frac{\delta m_{ij}^2}{2(3.15T)}\,,$$

where the dimensionless variables $a$ and $b$ are given by

$$a = -2.5 \times 10^{-5} L^{(\alpha)} \left( \frac{{\rm keV}^2}{\delta m_{ij}^2} \right) \left( \frac{T}{\rm MeV} \right)^4, \; b = -10^{-6} \left( \frac{T}{13\ {\rm MeV}} \right)^6 \left( \frac{{\rm keV}^2}{\delta m_{ij}^2} \right).$$

Here $L^{(\alpha)}$ is related to the lepton number asymmetry associated with the $\alpha$-flavour active neutrino. If $L^{(\alpha)}$ is not too big then $|a|, |b| \ll 1$ for the temperature range of interest, and hence the matter effect is very tiny and can be neglected.

$$\frac{n_{\nu_{\rm R0}}^{\rm col-osc}}{n_\gamma} \sim 10^{-4}\eta^2\left(\frac{\rm keV}{m_{\nu_{\rm R1}}}\right)^2\left(\frac{T_{\rm RH}}{\rm MeV}\right)^3 . \tag{60}$$

Clearly the number density of the right-handed neutrinos is very low, which is of course expected given that $T_{\rm RH} < T_{\rm F}$. The total contribution of the right-handed neutrinos produced via the collisional mechanisms normalised to the critical energy density of the Universe is easily computed to be

$$\Omega^{\rm col}_{\nu_{\rm R1}+\nu_{\rm R3}}h^2 = \Omega^{\rm dp}_{\nu_{\rm R1}+\nu_{\rm R3}}h^2 + \Omega^{\rm col-osc}_{\nu_{\rm R1}+\nu_{\rm R3}}h^2 \,, \tag{61}$$

where

$$\Omega^{\rm dp}_{\nu_{\rm R1}+\nu_{\rm R3}}h^2 \sim 10^{-5}\left(\frac{m_{\nu_{\rm R1}}}{\rm keV}\right)\left(\frac{\rm TeV}{M_{Z'}}\right)^4\left(\frac{T_{\rm RH}}{\rm MeV}\right)^3 \tag{62}$$

and

$$\Omega^{\rm col-osc}_{\nu_{\rm R1}+\nu_{\rm R3}}h^2 \sim 10^{-2}\eta^2\left(\frac{T_{\rm RH}}{\rm MeV}\right)^3\left(\frac{\rm keV}{m_{\nu_{\rm R1}}}\right) . \tag{63}$$

We see that $\Omega^{\rm col}_{\nu_{\rm R1}+\nu_{\rm R3}}h^2$ is consistent with a value of $\lesssim$ unity for a significant range of parameters[11]

$$T_{\rm RH} \lesssim 50\left[\left(\frac{m_{\nu_{\rm R1}}}{\rm keV}\right)\left(\frac{\rm TeV}{M_{Z'}}\right)^4 + 10^3\eta^2\left(\frac{\rm keV}{m_{\nu_{\rm R1}}}\right)\right]^{-\frac{1}{3}} \,{\rm MeV}\,. \tag{64}$$

We thus conclude that the relic abundance of the $\nu_{\rm R1}, \nu_{\rm R3}$ neutrinos is consistent with the cosmological energy density bound in the low reheating scenario.

Let us summarise what we have done so far for the $\nu_{\rm R1}, \nu_{\rm R3}$ neutrinos. In the alternative 422 model developed to accommodate the LSND and

[11] Technically, Eq. (61) is not complete. In addition to the collisional processes, there is the effect of "oscillations between collisions". Because $\langle\tau_{\rm osc}\rangle \ll \langle\tau_{\rm coll}\rangle$, the number density of $\nu_{\rm R}$ from oscillation of $\nu_{\rm L}$ is simply given by

$$n^{\rm osc}_{\nu_{\rm R}} \approx n_\nu\,\frac{1}{2}\sin^2 2\psi,$$

where $n_\nu$ is the number density of the relic $\nu_{\rm L}$ neutrino background, which is related to $n_\gamma$ via $n_\nu/n_\gamma = 3/11$ at present. Thus, the energy density of the light right-handed neutrinos produced via the "pure oscillation" channel is given by

$$\Omega^{\rm osc}_{\nu_{\rm R1}+\nu_{\rm R3}}h^2 \sim \frac{3n_\gamma}{11}\frac{m_{\nu_{\rm R1}}h^2}{\rho_{\rm c}}\frac{\sin^2 2\psi}{2} \sim 10^{-2}\eta^2\left(\frac{\rm keV}{m_{\nu_{\rm R1}}}\right).$$

Clearly this pure oscillation contribution is within the cosmological bound for all parameter space of interest.

solar neutrino anomalies, the masses $m_{\nu_{\rm R1}} \approx m_{\nu_{\rm R3}}$ lie in the range given by Eq. (32), that is $1\ {\rm keV} \lesssim m_{\nu_{\rm R1}}, m_{\nu_{\rm R3}} \lesssim 10\ {\rm keV}$. In the limit of high reheating temperature, $T_{\rm RH} \to \infty$, the right-handed neutrinos are fully populated and, as shown in Sec. 3, would be inconsistent with the cosmological energy density bound. However, in a low reheating scenario, $n_{\nu_{\rm R}}$ is suppressed and these right-handed neutrino will be consistent with the cosmological energy density bound provided that Eq. (64) holds.

Note that since their existence is not excluded by the cosmological energy density bound in a low reheating Universe, the light right-handed neutrinos $\nu_{\rm R1}, \nu_{\rm R3}$, being effectively stable, could be a viable dark matter candidate. Specifically, the right-handed neutrinos in the range of keV could play a role as warm dark matter [44]. We will leave the details of this possibility for future study.

### *4.2. Heavy right-handed neutrino decay mediated via the $W_{\rm R}$ gauge boson*

Having shown that the two lightest right-handed neutrinos, $\nu_{\rm R1}, \nu_{\rm R3}$, could exist in a low reheating Universe without violating the cosmological energy density bound, we are still left with the heavy right-handed neutrino, $\nu_{\rm R2}$, to worry about. However, because $\nu_{\rm R2}$ has a large mass ($4\ {\rm MeV} \lesssim m_{\nu_{\rm R2}} \lesssim 1\ {\rm GeV}$), it can decay much more rapidly than $\nu_{\rm R1}, \nu_{\rm R3}$. Furthermore, its production rate is highly suppressed if $T_{\rm RH} \ll m_{\nu_{\rm R2}}$. For simplicity, we first consider $\nu_{\rm R2}$ in the standard case of high reheating temperature. Of course, we should keep in mind that in the case of low reheating temperature the constraints will be much weaker.

If $\nu_{\rm R2}$ decays rapidly enough, then it will not lead to any cosmological problems. If kinematically allowed (*i.e.* $m_{\nu_{\rm R2}} > m_\mu + m_e + m_{\nu_{\rm R3}}$), the dominant decay channel of $\nu_{\rm R2}$ is

$$\nu_{\rm R2} \xrightarrow{W_{\rm R}} \mu^- e^+ \nu_{\rm R3}\,. \tag{65}$$

This Feynman diagram is shown in Fig. 4.

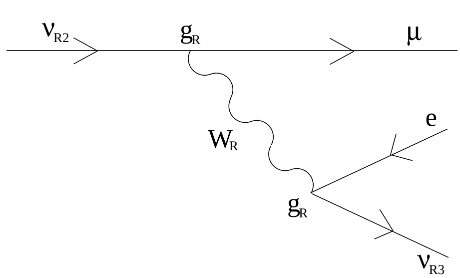


Fig. 4. $\nu_{\rm R2} \to \mu^- e^+ \nu_{\rm R3}$ decay channel dominated by $W_{\rm R}$.

The decay of $\nu_{\rm R2} \xrightarrow{W_{\rm R}} \mu^- e^+ \nu_{\rm R3}$ is similar to that of $\mu^-$ decaying into $e^- \nu_\mu + \bar{\nu}_e$, with $\nu_{\rm R2}$ playing the role of $\mu^-$. In the limit where the masses of the decay products vanish in comparison to $m_{\nu_{\rm R2}}$, the lifetime of the $\nu_{\rm R2} \xrightarrow{W_{\rm R}} \mu^- e^+ \nu_{\rm R3}$ decay can be roughly expressed in terms of the muon lifetime, $\tau_\mu \approx 10^{-6}$ s,

$$\tau_{\nu_{\rm R2}} \approx \tau_\mu \left(\frac{g_{\rm L}}{g_{\rm R}}\right)^4 \left(\frac{M_{W_{\rm R}}}{M_{W_{\rm L}}}\right)^4 \left(\frac{m_\mu}{m_{\nu_{\rm R2}}}\right)^5 = 0.2 \left(\frac{M_{W_{\rm R}}}{\rm TeV}\right)^4 \left(\frac{m_\mu}{m_{\nu_{\rm R2}}}\right)^5 \, {\rm s}$$

$$\text{for} \quad m_{\nu_{\rm R2}} \gtrsim m_\mu + m_e + m_{\nu_{\rm R3}}\,, \tag{66}$$

which is short enough to be consistent with all cosmological and astrophysical bounds.

If $m_{\nu_{\rm R2}} \lesssim m_\mu$ then the decay of $\nu_{\rm R2}$ into muon is not allowed. In this case $\nu_{\rm R2}$ decays via cross generational mixing into $e^- e^+ \nu_{\rm R3}$ which is suppressed by a cross generational mixing angle $|\sin\phi_{12}| \lesssim 10^{-1}$. Explicitly, the lifetime of $\nu_{\rm R2} \to e^- e^+ \nu_{\rm R3}$ is

$$\begin{aligned}\tau_{\nu_{\rm R2} \to e^- e^+ \nu_{\rm R3}} &= \tau_\mu \left(\frac{g_{\rm L}^2}{g_{\rm R}^2 \sin\phi_{12}}\right)^2 \left(\frac{M_{W_{\rm R}}}{M_{W_{\rm L}}}\right)^4 \left(\frac{m_\mu}{m_{\nu_{\rm R2}}}\right)^5 \\ &\approx 3 \times 10^{11} \left(\frac{10^{-1}}{\sin\phi_{12}}\right)^2 \left(\frac{M_{W_{\rm R}}}{\rm TeV}\right)^4 \left(\frac{\rm MeV}{m_{\nu_{\rm R2}}}\right)^5 \, {\rm s}\,. \end{aligned} \tag{67}$$

Because the annihilation of the $e^+e^-$ pair from this decay mode could produce photons that potentially distort the CMBR, the $\nu_{\rm R2} \to e^- e^+ \nu_{\rm R3}$ decay channel is subjected to the stringent CMBR constraint $\tau_{\nu_{\rm R2} \to e^- e^+ \nu_{\rm R3}} \lesssim 10^6$ seconds (see Fig. 5.6 in Ref. [35]). Using Eq. (67), this constraint implies that

$$\sin\phi_{12} \gtrsim 10^{-1} \left(\frac{10{\rm MeV}}{m_{\nu_{\rm R2}}}\right)^{5/2} \left(\frac{M_{W_{\rm R}}}{\rm TeV}\right)^2. \tag{68}$$

In summary, for $m_{\nu_{\rm R2}} \gtrsim m_\mu$ the decay of $\nu_{\rm R2}$ is rapid enough to be consistent with standard cosmology (irrespective of the value of the reheating temperature). For $m_{\nu_{\rm R2}} \lesssim m_\mu$ the decay is rapid enough provided that Eq. (68) holds. Recall that this is only valid in the limit where $T_{\rm RH}$ is high. In the case of low $T_{\rm RH}$, the astrophysical constraint is much weaker (depending on $T_{\rm RH}$).

## 5. Conclusion

The alternative 422 model is an interesting extension to the standard model for many reasons, which include addressing the problem of neutrino masses and their mixings. In its minimal form the model quite naturally accommodates a set of simultaneous solutions (with active–active neutrino oscillations) to the LSND and solar neutrino data without involving any mass scale higher than a few TeV — thereby avoiding the hierarchy problem. It turns out that the masses of the 3 right-handed neutrinos $\nu_{\mathrm{R}j}$ ($j = 1, 2, 3$) are constrained to lie between 1–10 keV (for $\nu_{\mathrm{R1}}, \nu_{\mathrm{R3}}$) and 4 MeV–1 GeV (for $\nu_{\mathrm{R2}}$) (see Eq. (32)).

On the other hand, standard hot Big Bang cosmology imposes stringent bounds on the masses of these right-handed neutrinos. We show that in the framework of standard cosmology their predicted abundance would violate the cosmological energy density bound. This inconsistency between the alternative 422 model and the standard cosmology implicitly assumes that the reheating temperature is much higher than the freeze-out temperature of the right-handed neutrinos, so that the right-handed neutrinos are fully populated during the radiation-dominated era. However, if the reheating temperature is low, $\sim$ MeV (not excluded by BBN or any other observations), then the right-handed neutrino production is highly suppressed. As a result there is a significant range of parameters where the model can be reconciled with cosmology. We conclude that low-scale quark–lepton unification is a viable candidate for at least part of the new physics suggested by the neutrino physics anomalies.

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