THE SEARCH FOR UNIVERSALITY: SMALL x AND THE COLOR GLASS CONDENSATE

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Dedicated to Jan Kwieciński in honour of his 65th birthday

I describe the search for universal properties of strongly interacting matter. The Color Glass Condensate is presented as the universal form of matter from which controls the high energy limit of strong interactions. At high energies, this strongly interacting but weakly coupled matter allows first computations from first principles in QCD.

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1. Introduction

As physicists, we always try to look for simple structure hidden in the sometimes complicated patterns we see around us. When I was a graduate student, the outstanding goals of particle physics were to describe the weak and strong interactions. This involved a combination of experimental and theoretical efforts. With the discovery of the Glashow–Weinberg–Salam model, and the proof of its renormalizability, a plausible and simple theory emerged which unified electric and weak interactions. It was subsequently verified by the discovery of the weak bosons at CERN.

A description of the strong interactions proved more difficult to find. A candidate theory had been written down by Yang and Mills many years before. After one learned how to compute in such theories, and after one understood the remarkable phenomenon of asymptotic freedom, it was possible to test the theory in a variety of different environments. All of these tests involved studying the theory at short distances or at high momentum transfer, and it was verified in much detail that QCD is the correct theory of strong interactions.

At about the same time, attempts were made to study non-perturbative phenomena such as quark confinement and mass generation. In spite of early hope, analytic methods proved intractable. Monte Carlo methods of numerical simulation were successfully employed, and one can describe both qualitatively and semi-quantitatively these phenomena. Precise comparison has been elusive.

Much of the original interest in strong interaction physics arose from attempts to understand the interactions of strongly interacting particles. Later after QCD was proposed and quarks had been discovered at SLAC (in large part due to the theoretical efforts of Bjorken), one tried to understand how these strongly interacting particles are composed in terms of fundamental quark and gluon degrees of freedom. Interest in these problems decreased as one tested quantitative aspects of QCD in short distance processes, and qualitative features of confinement and mass generation in numerical Monte Carlo simulation. It became and still remains somewhat disreputable within the particle physics community to try to develop understanding of strong interactions when these interactions are strong. Nevertheless, interactions of quarks and gluons inside of hadrons are strong, as are the resulting interactions which control the overwhelming bulk of strong interaction processes. Most of the interest in particle physics now centres on physics of inaccessible energies (string theory), speculative issues such as extra dimensions, or on the phenomenology of supersymmetry. Strong interaction physics is considered by most of the particle physics community to be either too complicated to be of interest or solved (one knows the Hamiltonian) with only the uninteresting details to be filled in. Strong interaction physics has become largely an issue for nuclear physics.

Part of the purpose of this paper is to describe some of the remaining unsolved issues in strong interaction physics and make the case that there are simple problems of great generality which are by their very nature fundamental. The issues I concentrate on are those which physicists such as Jan Kwieciński has worked on for most of his career. They reflect the easily accessible and simple structural aspects of strongly interacting particles. Among the issues I shall discuss are:

- What are the sizes of hadrons and how does this size depend on energy? Is this dependence on energy universal and independent of hadron?

- How are hadrons made from fundamental constituents? Is the part of the hadronic wavefunction important for high energy interactions simple and universal?

- What is the relationship between the constituents of high energy hadrons and the distributions of particles produced in high energy collisions? What is the space–time description of the evolution of matter produced in such collisions?

I believe that a partial answer to these questions lies in the existence of a new form of matter universal to hadrons which controls the part of a hadron wavefunction important for high energies. This matter, for reasons which will be explained later is called a Color Glass Condensate. It has simple properties which can be explained from first principles in QCD. I will argue that at this time, among the most compelling evidence for this matter comes from the work of Golec-Biernat, Kwieciński and Staśto [1], and related work by Golec-Biernat and Wüsthoff [2].

I argue that universality is the fundamental underpinning of high energy strong interactions. It means that a few simple properties of the matter which make up hadrons describe a very wide dispersion of phenomena seen at high energies. I will argue that the Color Glass Condensate unifies results seen in lepton–hadron and hadron–hadron interactions for all varieties of hadrons. Such seemingly different phenomena as hadronic total cross sections, deep inelastic scattering and diffraction are included under one unified description.

2. Ancient history

The very early work on strong interactions, before QCD was an accepted theory, tried to use analytic properties of scattering matrices to abstract general features of strong interaction processes. One was able to argue that if one used analytic properties of scattering matrices as a function of complex angular momentum, that many features of strong interaction processes could be understood. In particular, there were poles in the complex angular momentum plane, and one could associate the exchange of a pseudo-particle with such a pole. These are the so called Reggeons. The pole of maximal angular momentum, the Pomeron controls the total cross section at high energy.

A variety of simple structural aspects of strong interactions were understood on the basis of Regge poles. The idea of universality first appeared: The energy dependence of the total cross section should be universal. One could prove an absolute upper bound on the total cross section of hadrons, the Froissart bound

$$\sigma \leq \kappa \frac{1}{m_\pi^2} \ln^2\left(\frac{E}{E_0}\right) , \tag{1}$$

where E is the center of mass energy and κ is a computed constant.

One thing the early Regge description missed was the growth of the total cross section with energy, as discovered in the experiments at Serpukhov. In the pomerons simplest realization, the cross section becomes a constant at high energy. Experimentally measured high energy cross sections seem to have a $\ln^2(E)$ behaviour which saturates the Froissart bound. One could fix the problem within the Regge theory of the pomeron, but a consequence of this was that the cross section grew too rapidly with energy, and so a simple single pomeron could not describe the high energy limit. This was the first indication that the high energy limit is controlled by a high density of matter. Thought about in this old language, this high density matter is composed of pomerons. The most valiant attempt to make sense of the high energy limit was Gribov–Reggeon Calculus. I remember the excitement that people had over this now long forgotten theory. Like so many attempts, it was abandoned not because one had solved the Gribov–Reggeon Calculus but because something new had intervened.

Early experiments at SLAC had revealed tantalising hints of the quark composition of matter. Quarks would have however been seen due to their fractional charge, so quarks had to be confined permanently inside of hadrons. A hint about the resolution to this mystery came from a somewhat obscure theory, QCD. When this theory's coupling constant's dependence on energy was computed, it was found that it became weak at short distances. The SLAC experiments probed the short distance structure of QCD, and this is where the quarks appeared. One had vague hopes that the theory would somehow solve the problem of confinement when the coupling constant became large. In the mid 70's, the J/Ψ meson was discovered. This heavy state had the remarkable property that it was much more stable than ordinary strongly interacting particles, and that it had an excitation spectrum vaguely reminiscent of atomic physics. This suggested the J/Ψ was a somewhat weakly bound state, since the mass was big the interaction strength of the quarks inside it would be weak. This strongly suggested that QCD was the correct theory of strong interactions.

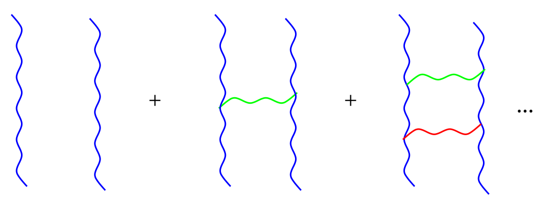
These revolutionary developments changed the entire course of the study of strong interactions. For many years one verified that in fact QCD was the fundamental theory of strong interactions. These tests inevitably involved asking questions about very rare processes, since one had to isolate a short distance component of a particle interaction. In such processes the coupling is weak, and one can do controlled computations.

Only a few brave physicists, such as Jan Kwieciński, remained interested in understanding the strong interactions when they are strong, and how the hadrons and typical interactions as we see them in nature are composed in terms of fundamental degrees of freedom. Jan was among those few who realized the importance of understanding issues such as the origin of the structure of hadrons in terms of quarks and gluons, and was one of the pioneers in developing a description of the pomeron itself in terms of gluons, and conversely what we knew of pomerons to the description of distributions of quarks and gluons inside of hadrons.

3. The pomeron and gluons

Pomerons and reggeons provide a successful phenomenology of intermediate energy hadron collisions. Once one accepts that QCD is the correct theory of strong interactions, one must ask how pomerons and reggeons arise as the effective or pseudo-particle degrees of freedom of QCD. One very successful way of incorporating many of the constraint of the quark degrees of freedom into the reggeon and pomeron description is the dual parton model [3]. This model has the correct physics to automatically incorporate the valence quark coupling to reggeons and to build in many of the constraints of field theory which relate processes in the s and t channel. Any complete theory of strong interactions will have to successfully compete with the successful phenomenology of this theory.

In early work Jan was intrigued with making a connection between the underlying quark–gluon description and the reggeon–pomeron description. He was one of the first to apply the ideas of the dual parton model to nucleus–nucleus collisions, a problem close to my own heart [4]. He did pioneering work on understanding the odderon [5]. To understand what is the odderon, one needs first to understand how to think about the pomeron. The pomeron mediates charge conjugation even processes. Therefore, we can think of it as composed of two interacting gluons. In fact, in the classic BFKL paper, it was shown that one can generate the pomeron by summing up ladders of gluon exchanges between gluons, as shown in Fig. 1 [6].



The Pomeron as a Gluon Ladder

Fig. 1. The pomeron as a pseudo-particle composed of two gluons interacting by the exchange of two gluons.

The odderon is the natural generalisation in QCD to an interacting three gluon exchange as shown in Fig. 2. It has opposite charge conjugation to the pomeron. It is an object of much interest, and remains to be discovered in experiment.

The sum of ladders which generates the pomeron makes for a rapidly rising cross section

$$\sigma \sim \left(\frac{E_{\rm cm}}{\Lambda_{\rm QCD}}\right)^{\kappa\alpha_{\rm s}} . \tag{2}$$

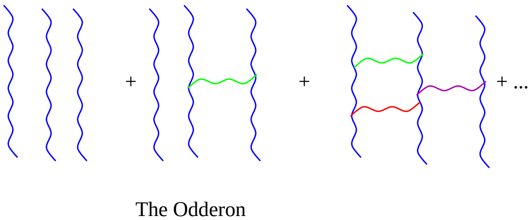


Fig. 2. The odderon as a pseudoparticle composed of two gluons interacting by the exchange of three gluons.

The constant κ is computed by BFKL. Here α_s is the strong coupling constant evaluated at an energy scale typical of the typical transverse momentum of a gluon which composes the pomeron. We naively expect that this scale is of order Λ_{QCD}, so the coupling is big, and the weak coupling techniques implicit in the derivation break down. We shall see later that in fact this coupling should be evaluated at an energy dependent momentum, the saturation momentum, which itself grows with energy. When the saturation momentum becomes big, the coupling is weak, and the BFKL analysis is consistent. The growth of the single pomeron exchange amplitude with energy is inevitable.

This leads to at least two problems:

The first is that because the pomeron itself is made of gluons, and since the low momentum gluons in the center of mass frame join from one hadron to the other to generate an interaction, these low momentum gluons must be increasing in density as the energy increases. If we identify gluons by their fractional longitudinal momentum in a frame where the hadron moves fast, this means the density of gluons grows as

$$xG(x, Q^2) \sim \left(\frac{1}{x}\right)^{\kappa\alpha_s} . \tag{3}$$

We shall later identify this high density very coherent configuration of gluons as the Color Glass Condensate.

The second problem is that it appears the Froissart bound limit is violated. In fact, the Frossart bound is saved by the high density of gluons. When the gluons fill the geometrical area of a hadron, the hadron cross section becomes roughly a constant. As one adds more gluons, they are hidden by the already high density of gluons. Only in cases where one can access a low density of pomerons does one see the naive growth with energy of cross sections. (In the theory we describe later, the gluons have to overpack the volume by a factor of $1/\alpha_s$ in order for the hadron to become a black disk. This is because interactions are proportional to α_s. This happens naturally.)

We can even use these naive consideration to see how the Froissart bound is saturated. Suppose that in impact parameter space, the density of matter has a radial profile falling like $e^{-2m_\pi r}$, which should be the case at large distances. On the other hand suppose the density of partons grows like e^{cy} where $y = \ln(1/x)$ is the rapidity of the hadron. Requiring that $e^{-2m_\pi r} e^{cy} \sim 1$, that is that the hadron becomes opaque, is equivalent to [7] $2\pi R^2 \sim \ln^2(E)/m_\pi^2$. (Whether this simple and transparent argument actually applies to QCD is disputed by at least one group [8].) Therefore, if one understand how this high density gluon configuration arises, the Color Glass Condensate, then one naturally explains the saturation of the Froissart bound.

Jan was one of the first to realize the connection between the rise of the gluon density and the singular behaviour of the pomeron [9]. He was also one of the first to understand the implications of this rise in the gluon distribution for the total cross section as seen concretely through the contribution of mini-jets to the total cross section [10].

I think that at the time the idea of growing gluon distributions seemed to be quite radical. This was before the discovery in HERA of the small x enhancement of gluons distributions, shown in Fig. 3 [11]. The HERA results, although no surprise to Jan, were a great surprise to many of us.

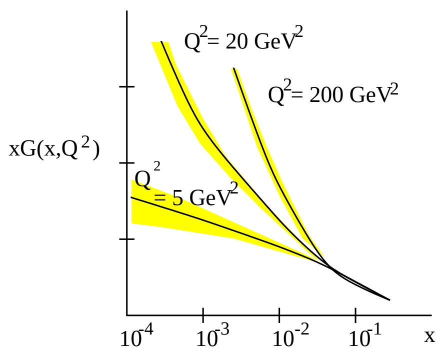


Fig. 3. The small x enhancement seen at HERA

4. Implications of high gluon density

In Fig. 4, I show a slice of a hadron at small x, that is the transverse distribution of the low longitudinal momentum degrees of freedom (wee partons). At small x, this density grows and at very small x satisfies

$$\Lambda^2 = \frac{1}{\pi R^2} \frac{dN}{dy} >> \Lambda_{\rm QCD}^2 \, . \qquad (4)$$

This means the typical separation between partons is small, and $\alpha_{\rm s}$ evaluated at this scale is weak.

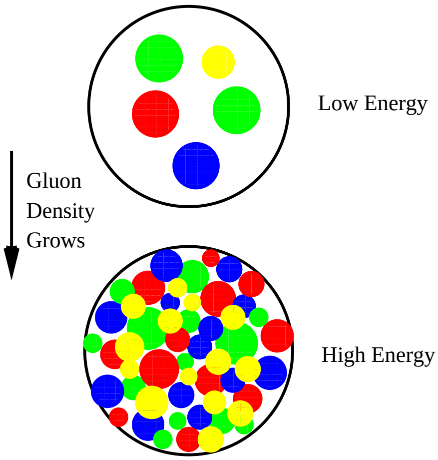


Fig. 4. The transverse distribution of partons inside a hadron.

What can stabilise the density distribution of partons? Instabilities are typically driven by negative mass squared terms proportional to the phase space density ρ. They are typically stabilised by interactions proportional to $\alpha_s\rho^2$. The stable density is $\rho \sim 1/\alpha_s$, and when α_s is weak, this is large. This is closely related to the phenomena of Bose condensation and superconductivity. The quantum occupation numbers for this state are large and it is highly coherent. Hence the name condensate. This implies the gluon density distribution should saturate [12].

The partons which generate this condensate come from higher values of x in the frame where the condensate is at rest. These higher x degrees of freedom are Lorentz time dilated compared to their natural time scales, and this time dilation is transferred to the low x degrees of freedom. Since the color distribution comes from partons at very many different values of x, one expects the distribution of color in the transverse space to be random. These properties are similar to ordinary glasses, and in fact the theory one writes down to describe this is the same type used to describe spin glasses. Hence the name glass.

The gluons which make up this distribution are colored. It is, therefore, a scientifically accurate name that this low x high density gluon matter be called Color Glass Condensate.

In order to parameterise the Color Glass Condensate, we introduce the saturation momentum,

$$Q^2_{\rm sat} \sim \alpha_s \Lambda^2 \,. \tag{5}$$

This is the largest momentum where the phases space density remains of order $1/\alpha_s$.

To compute the dependence of this momentum on x, it turns out we need to understand the gluon distribution in an intermediate range of phase space density, where $1/\alpha_s \gg \rho \gg 1$. One can analyse the BFKL equation in this range of momentum, and compute the dependence of the saturation momentum on x [13]. Over a wide range of energy, one finds [14]

$$Q^2_{\rm sat} \sim 1 \ {\rm GeV}^2 \ \left(\frac{x_0}{x}\right)^{0.3} . \tag{6}$$

This result is quite close to the value found by an analysis of deep inelastic scattering and diffraction by Golec-Biernat–Wusthoff [2].

In fact, Golec-Biernat, Kwieciński and Staśto made a very important discovery based on the HERA data [1]. They discovered that over a very wide range of momentum of the virtual photon, Q^2 in deep inelastic scattering, the cross section

$$\sigma_{\gamma^* p} = F\left(\frac{Q^2}{Q^2_{\rm sat}}\right) . \tag{7}$$

This works for $x < 10^{-2}$ and $Q^2 < 400\,{\rm GeV}^2$. One can understand this result for small $Q^2 \sim Q^2_{\rm sat}$, but it is a surprise it works well at such large Q^2. In fact one finds that another scale appears in the problem $Q^4_{\rm sat}/\Lambda^2_{\rm QCD}$ In this intermediate scaling region, $Q^2_{\rm sat} \ll Q^2 \ll Q^4_{\rm sat}/\Lambda^2$ correlation function scale as powers of Q^2, that is anomalous dimensions which are computable within BFKL dynamics. In this region, the phase space density is not so large, and the time scales for evolution of matter are more or less normal time scales, but the correlations are non-trivial. The description of the matter is quantum, not classical [13]. This region has been called the Quantum Color Fluid by my good friend Dima Kharzeev [15].

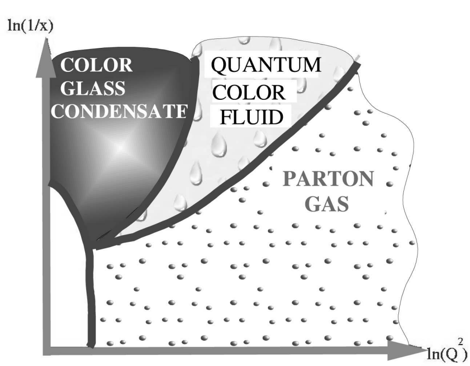


Fig. 5. The phase diagram for high density QCD.

We can now draw a phase diagram of QCD. In Fig. 5, the regions of phase space occupied by the Color Glass Condensate, the Quantum Color Fluid, and the ordinary parton gas is shown in the $\ln(1/x) - \ln(Q^2)$ plane. We believe we have a semi-quantitative theory of this matter, and this theory should become precise at asymptotically small x. This is because the density becomes large at small x and, therefore, the coupling is weak, and we can compute the properties of weakly coupled theories.

5. Universality

The concept of universality is built into our theoretical description of the Color Glass Condensate. The properties of the matter depend only upon the density of partons per unit area, independent of the nature of the original parton. Because all matter is made from CGC at high energies, the properties of hadrons relevant for high energy processes are universal. The parton distributions themselves can be computed as a property of this matter.

There is a deeper sense in which this matter is universal which arises from renormalisation group ideas. To understand this, one needs to know about the property of limiting fragmentation (see Fig. 6). If one plots the distribution of produced particles as a function of rapidity measured from the rapidity of one of the colliding particles, except for a few units of rapidity which corresponding to the slow moving particles (the wee partons) in the center of mass frame, the distributions are the same at different energies.

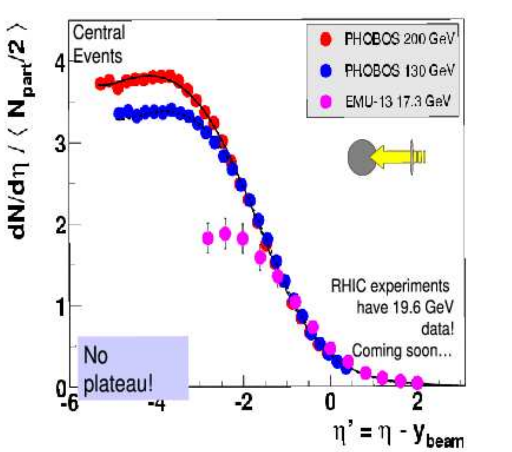


Fig. 6. Limiting fragmentation in heavy ion collisions.

It is as if the only effect of going to higher energy is to add in new low momentum degrees of freedom. The high momentum degrees of freedom are frozen out.

This means that there should be an effective action for the low momentum degrees of freedom, and going to higher energy integrates out these degrees of freedom, and results in a new theory for the yet lower momentum degrees of freedom at the higher energy scale. In fact these low momentum degrees of freedom are integrated out to generate sources for the wee partons at higher energy. This process is a renormalisation group [12, 16].

Unlike the ordinary renormalisation groups we are familiar with from perturbative field theory, this renormalisation group turns out to be a functional differential equation. It is essentially an infinite dimensional diffusion equation for the wavefunction which describes the small x gluons. Because of its diffusive nature, going to smaller values of x implies that the wavefunction spreads, and this spreading is the origin of the non-trivial growth of the gluon distribution function and the saturation momentum. One can write the functional differential equations explicitly, and see that all known renormalisation group equations which include DGLAP and BFKL are reproduced. In addition one gets a complete description of both the Color Glass Condensate and the Quantum Color Fluid regions of the QCD $\ln(1/x) - \ln(Q^2)$ phase diagram. The solutions at small x appear to be universal [17]. For a complete review of these and other topics see the recent review of Iancu and Venugopalan [18].

What I am claiming here is very radical: we have a complete understanding of the high energy limit of QCD in terms of a universal form of matter. At high enough energy, we have the tools at our disposal to explicitly compute the properties of QCD.

6. Summary and conclusions

Our current theoretical understanding of high energy strong interaction processes promises to unify deep inelastic and diffractive processes in lepton hadron interactions. It also allows for the determination of initial conditions for hadron–hadron collisions and in particular for nucleus–nucleus collisions. This latter understanding in my opinion will ultimately prove crucial for determining the properties of a quark–gluon plasma as produced in nuclear collisions. This has already proven to be extremely useful for describing multiplcities and transverse momentum distributions found in RHIC collisions [19–21].

For lack of space, I cannot go into details here [18]. I would like to say that it is very, very exciting to see these separate fields of experimental and theoretical work, that share both a common origin and many intellectual goals, coming back together again.

I thank Andrzej Bialas for asking me to write this contribution for the celebration of the 65-th birthday of Jan Kwieciński. I thank Edmond Iancu, Dima Kharzeev and Raju Venugopalan for advice on the style and text of this manuscript. This manuscript has been authorised under Contract No. DE-AC02-98H10886 with the US Department of Energy.

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