ON THE DIPOLE PICTURE IN THE NONFORWARD DIRECTION

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We calculate, for nonzero momentum transfer, the dipole formula for the high energy behaviour of elastic and quasielastic scattering of a virtual photon. We obtain an expression of the nonforward photon impact factor and of the nonforward photon wave function, and we give a physical interpretation.

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1. Introduction

The colour dipole picture [1, 2] of deep inelastic scattering at small x has turned out to be very useful in describing, at low Q^2, the transition from pQCD to nonperturbative strong interactions. In a first sequence of attempts models for the dipole cross section have been formulated in order to describe the interaction between the quark–antiquark pair and the proton in the forward direction [3–7]. Equivalently, in these models the interaction is integrated over the full region of impact parameter b. More recently, attention has been given to the t-dependence of diffractive vector production in deep inelastic scattering. HERA data show significant differences compared to hadron–hadron scattering. For example, in J/Ψ production, the t-slope is smaller, indicating a smaller transverse extension of the scattering system. Also, shrinkage is considerably smaller, hinting at a quite different picture in impact parameter space. A phenomenological analysis in [2] investigates

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the beginning of saturation at small impact parameter and low Q^2. It therefore seems natural to understand the dipole picture at nonzero momentum transfer, and then to search for models for the dipole cross section which depend upon the impact parameter b.

First calculations of the photon impact factor for real photons in the nonforward direction have been presented rather long time ago [8,9]. The results have been obtained in momentum space, but at that time no attempt has been made to find an interpretation in terms of the photon wave function and a dipole cross section. More recently, the nonforward photon impact factor to an off-shell incoming and real outgoing photon with massless quarks has been given in [11,12], and in [10,13] the nonforward diffractive production of a vector particle has been analysed. Finally, in [14] the nonforward diffractive production of a (massive) quark–antiquark pair has been studied, but the result has not been presented in a form which is convenient for the colour dipole picture analysis mentioned above.

In this paper we present an analysis of the nonforward photon impact factor for all photon helicities, using massive quarks and the general kinematic $\gamma^*(Q_+^2) \to \gamma^*(Q_-^2)$, and we present our result in the form of the colour dipole picture: (photon wave function)·(colour dipole cross section)·(photon wave function). We find the form of the (nonforward) photon wave function and of the dipole cross section, and we give a physical interpretation in the infinite momentum frame. Transforming to impact parameter, the dipole cross section will be shown to depend upon the distance between one of the quarks of the colour dipole and the target. For open quark antiquark production we show that the integrated diffractive cross section can be expressed in terms of the same nonforward dipole cross section as the elastic $\gamma^* \to \gamma^*$ scattering amplitude.

Our paper will be organised as follows. We first (Section 2) calculate, in momentum space, the high energy behaviour of the elastic scattering process $\gamma^* + \gamma^* \to \gamma^* + \gamma^*$ in lowest order QCD. The resulting formula leads to the nonforward photon impact factor which, when transformed to transverse coordinate space, leads to the photon wave function and to the nonforward dipole scattering amplitude (Section 3). The transformation to impact parameter is done in Section 4, and a physical interpretation is given in the infinite momentum frame. Section 5 briefly describes the nonforward dipole formula for $\gamma^* p$ scattering, and in Section 6 we present the formula for open quark–antiquark production.

2. Nonforward impact factors

To be definitive, let us consider elastic $\gamma^*\gamma^*$ scattering at high energies and small (but nonzero) momentum transfer. In order to have a non-falling cross section we consider the lowest order diagrams with two-gluon exchange.

The scattering amplitude takes the form:

$$A(s,\boldsymbol{l}) \;=\; \frac{is}{2}\int \frac{d^2\boldsymbol{k}}{(2\pi)^3}\,\frac{\Phi(q,k,l)}{(\boldsymbol{k}+\boldsymbol{l})^2}\,\frac{\Phi(p,-k,-l)}{(\boldsymbol{k}-\boldsymbol{l})^2}\,. \tag{1}$$

The four diagrams which contribute to the photon impact factor Φ are shown in Fig. 1. As usual, the integration of the two longitudinal components of the loop momenta are absorbed into the definition of the impact factors, and the intermediate quark lines inside the impact factor are taken on-shell. Our task here is the calculation of the photon impact factor, and its interpretation in transverse coordinate space.

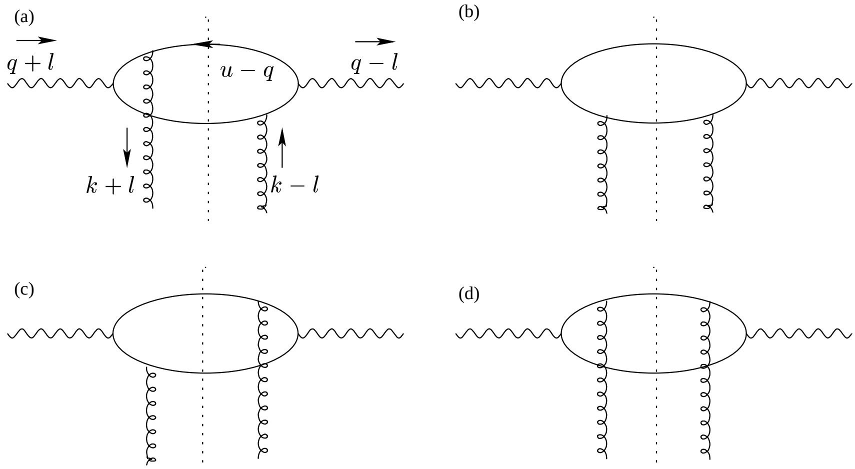

Fig. 1. The diagrams contributing to the impact factor.

We first do the calculation in momentum space, and we compute the discontinuity of the scattering amplitude. The assignment of the loop momentum is illustrated in Fig. 1; in all four graphs, the momenta of the cut quark lines are identical, *e.g.* $u-q$ for the upper quark line. We use Sudakov variable with the light cone vectors $p' = p + yq$, $y \simeq -p^2/2p\cdot q$ and $q' = q + xp$, $x \simeq -q^2/2p\cdot q$. For the incoming and outgoing photon we introduce the invariants

$$Q^2_{\pm} \;=\; -(q\pm l)^2, \qquad s\equiv(p+q)^2 \;\simeq\; 2\,p'\cdot q', \qquad t \;=\; (2\,l)^2. \tag{2}$$

The other photons involved in the process have similar expressions with the momentum p instead of q. We write the momenta:

$$\begin{aligned} u &= \alpha\, q' \;+\; \beta\, p' \;+\; u_{\perp}\,,\\ k &= \alpha_k\, q' \;+\; \beta_k\, p' \;+\; k_{\perp}\,,\\ l &= \alpha_l\, q' \;+\; \beta_l\, p' \;+\; l_{\perp}\,. \end{aligned} \tag{3}$$

In terms of these parameters the photon virtualities are

$$Q^2_{\pm} \;\simeq\; (x \mp \beta_l)s - l^2_{\perp}\,, \tag{4}$$

where we used the fact that $\alpha_l \sim |t|/s \ll 1$ in the high energy limit. In the following we use the Euclidian form of the transverse momenta marked in boldface, *i.e.* $k_{\perp}^2 = -\boldsymbol{k}^2 < 0$. In the high energy limit, we also approximate the numerator of the exchanged gluons by the first term in the decomposition

$$g_{\mu\nu} \;=\; \frac{2}{s}(p'_\mu q'_\nu + p'_\nu q'_\mu) + g^{\perp}_{\mu\nu}\,. \tag{5}$$

With these simplifications, the integral of the first diagram takes the following form:

$$\begin{aligned}
&\alpha_{\rm s}\, e^2 \sum_f q_f^2\, \frac{s}{2} \int \frac{d\beta_k}{2\pi} \int \frac{d\alpha\, d\beta\, d^2u_\perp}{(2\pi)^4}\\
&\times \frac{\delta[(\alpha-1)(\beta-x)s + u_\perp^2 - m_f^2]\,\delta[\alpha(\beta-\beta_k)s + (u-k)_\perp^2 - m_f^2]}{[\alpha(\beta-\beta_l)s + (u-l)_\perp^2 - m_f^2][(\alpha-1)(\beta-\beta_k+x-\beta_l)s + (u-k-l)_\perp^2 - m_f^2]}\\
&\times {\rm Tr}[\not\varepsilon(q+l)(\not u - \not k - \not q - \not l + m_f^2)\not p'(\not u - \not q + m_f^2)\not\varepsilon(q-l)(\not u - \not l + m_f^2)\not p'(\not u - \not k + m_f^2)]\,.
\end{aligned} \tag{6}$$

Performing the two β-integration with the delta functions leads to the following relations

$$\beta_k = \frac{(u-k)_\perp^2 - m_f^2}{\alpha s} + \frac{u_\perp^2 - m_f^2}{(1-\alpha)s} - x\,, \tag{7}$$

$$\beta = \frac{u_\perp^2 - m_f^2}{(1-\alpha)s} - x\,. \tag{8}$$

For longitudinally polarized photons, the polarization vectors read:

$$\varepsilon_{\rm L}(q \pm l) \;=\; \frac{1}{|Q_\pm|}\left[(1 \pm \alpha_l)\, q' + \left(x \mp \beta_l - \frac{2l_\perp^2}{s}\right) p' \pm l_\perp\right], \tag{9}$$

The calculation of the trace in Eq. (6) greatly simplifies if we make use of the Ward identity, *i.e.* the addition of a vector proportional to $q \pm l$ to the polarization vector does not change the result. In this way the q' dependence of $\varepsilon_{\rm L}$ can be eliminated. The trace is computed in terms of Sudakov variables using the fact that p' and q' are light cone vectors. After summing the contributions corresponding to the four diagrams in Fig. 1, given by the integrals of the type (6), the impact factor for the longitudinally polarized photons takes the form:

$$\Phi_{\rm LL}(q,k,l) = \alpha_{\rm s}\sqrt{N_c^2-1}\,e^2\sum_f q_f^2\,4\sqrt{4\pi}\,|Q_+||Q_-|\int_0^1 d\alpha\int\frac{d^2\boldsymbol{u}}{(2\pi)^2}$$
$$\times\,\alpha^2(1-\alpha)^2\left(\frac{1}{D_1^+}-\frac{1}{D_2^+}\right)\left(\frac{1}{D_1^-}-\frac{1}{D_2^-}\right). \tag{10}$$

The four terms have been written in a factorised form, each of them corresponds to one Feynman graph in Fig. 1, where we have used:

$$D_1^{\pm} = (\boldsymbol{u}\pm(1-\alpha)\boldsymbol{l})^2\;+\;\alpha(1-\alpha)\,Q_{\pm}^2\;+\;m_f^2\,, \tag{11}$$
$$D_2^{\pm} = (\boldsymbol{u}-\boldsymbol{k}\mp\alpha\boldsymbol{l})^2\;+\;\alpha(1-\alpha)\,Q_{\pm}^2\;+\;m_f^2\,. \tag{12}$$

Setting $t=0$ one gets the well known expression in the forward direction. In the same way we calculate the transverse impact factor where the transverse polarization vector reads:

$$\varepsilon_{\rm T}^{(h)}(q\pm l) = \varepsilon_{\perp}^{(h)}\;\pm\;\frac{2l_{\perp}\cdot\varepsilon_{\perp}^{(h)}}{s}\,(q'-p'\pm l_{\perp})\,, \tag{13}$$

where $h=\pm$ denotes two helicity states and

$$\varepsilon_{\perp}^{(h)}\;=\;\frac{1}{\sqrt{2}}\,(0,1,\pm i,0)\,. \tag{14}$$

After some algebra and the use of the Ward identity, the expression for the transverse impact factor becomes:

$$\begin{aligned}\Phi_{\rm TT}^{(ij)}(q,k,l) \;=\;& \alpha_{\rm s}\sqrt{N_c^2-1}e^2\sum_f q_f^2\sqrt{4\pi}\int_0^1 d\alpha\int\frac{d^2\boldsymbol{u}}{(2\pi)^2}\\ &\times\Bigg\{-4\alpha(1-\alpha)\;\boldsymbol{\varepsilon}_i\cdot\left(\frac{\boldsymbol{N_1^+}}{D_1^+}-\frac{\boldsymbol{N_2^+}}{D_2^+}\right)\left(\frac{\boldsymbol{N_1^-}}{D_1^-}-\frac{\boldsymbol{N_2^-}}{D_2^-}\right)\cdot\boldsymbol{\varepsilon}_j^*\\ &+\boldsymbol{\varepsilon}_i\cdot\boldsymbol{\varepsilon}_j^*\Bigg[\left(\frac{\boldsymbol{N_1^+}}{D_1^+}-\frac{\boldsymbol{N_2^+}}{D_2^+}\right)\cdot\left(\frac{\boldsymbol{N_1^-}}{D_1^-}-\frac{\boldsymbol{N_2^-}}{D_2^-}\right)\\ &+m_f^2\left(\frac{1}{D_1^+}-\frac{1}{D_2^+}\right)\left(\frac{1}{D_1^-}-\frac{1}{D_2^-}\right)\Bigg]\Bigg\}\,,\end{aligned} \tag{15}$$

where

$$\boldsymbol{\varepsilon}_j = 1/\sqrt{2}(1,\pm i) \tag{16}$$

are two-dimensional polarization vectors corresponding to the two transverse polarizations $j = \pm$. Again, it was possible to write the result in a factorised form where we used the following definitions:

$$\boldsymbol{N_1^\pm} = \boldsymbol{u} \pm (1-\alpha)\boldsymbol{l} \qquad\qquad \boldsymbol{N_2^\pm} = \boldsymbol{u} - \boldsymbol{k} \mp \alpha \boldsymbol{l}\,. \tag{17}$$

Setting $t = 0$ and summing over the two helicity states, one gets the well-known expressions in the forward direction, see *e.g.* [15]. This result also agrees with the one for real photons obtained in [8].

Finally, with the same procedure one can study the impact factor for the incoming photon longitudinally polarized while the outgoing photon is transversely polarized.

$$\begin{aligned}\Phi_{\rm LT}^{(j)}(q,k,l) &= \alpha_{\rm s}\sqrt{N_c^2-1}\, e^2 \sum_f q_f^2\, 2\,|Q_+|\sqrt{4\pi}\int_0^1 d\alpha \int \frac{d^2\boldsymbol{u}}{(2\pi)^2} \\ &\times \alpha(1-\alpha)(1-2\alpha)\left(\frac{1}{D_1^+}-\frac{1}{D_2^+}\right)\left(\frac{\boldsymbol{N_1^-}}{D_1^-}-\frac{\boldsymbol{N_2^-}}{D_2^-}\right)\cdot \boldsymbol{\varepsilon}_j^*\,. \end{aligned} \tag{18}$$

It is important to realize that the impact factor $\Phi_{\rm LT}$ is nonzero in the nonforward direction in contrast to the forward case. This corrects the statement made in the analysis [12] in which $\Phi_{\rm LT} = 0$ in the nonforward case was claimed. The reason for $\Phi_{\rm LT} \neq 0$ is the symmetry property of the integrand of the integral over α in Eq. (18). While in the forward case ($\boldsymbol{l} = 0$) the integrand is antisymmetric with respect to the transformation $\alpha \to (1-\alpha)$, thus giving zero after the integration over α, it becomes symmetric in the nonforward case ($\boldsymbol{l} \neq 0$), leading to the nonzero result.

3. Formulation in the coordinate space

It is instructive to transform these results to coordinate space. We start from the general formulae:

$$\begin{aligned}\frac{1}{\boldsymbol{k}^2+\delta^2} &= \int \frac{d^2\boldsymbol{r}}{2\pi}\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} K_0(\delta r)\,, \\ \frac{\boldsymbol{k}}{\boldsymbol{k}^2+\delta^2} &= \int \frac{d^2\boldsymbol{r}}{2\pi}\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\, i\,\delta\,\frac{\boldsymbol{r}}{r}\, K_1(\delta r)\,, \end{aligned} \tag{19}$$

where $K_{0,1}(z)$ are modified Bessel functions. Let us concentrate first on the longitudinal impact factor. After using the first relation from the above, Eq. (10) takes the form

$$\Phi_{\rm LL}(q,k,l) = \alpha_{\rm s}\sqrt{N_c^2-1}e^2\sum_f q_f^2 4\sqrt{4\pi}|Q_+||Q_-|\int_0^1 d\alpha\,\alpha^2(1-\alpha)^2\int\frac{d^2\boldsymbol{u}}{(2\pi)^2}$$
$$\times\int\frac{d^2\boldsymbol{r}}{2\pi}\mathrm{e}^{i\boldsymbol{u}\cdot\boldsymbol{r}}\left(\mathrm{e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}-\mathrm{e}^{-i(\boldsymbol{k}+\alpha\boldsymbol{l})\cdot\boldsymbol{r}}\right)K_0(\delta_+ r)$$
$$\times\left(\int\frac{d^2\boldsymbol{r'}}{2\pi}\mathrm{e}^{i\boldsymbol{u}\cdot\boldsymbol{r}'}\left(\mathrm{e}^{-i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}'}-\mathrm{e}^{-i(\boldsymbol{k}-\alpha\boldsymbol{l})\cdot\boldsymbol{r}'}\right)K_0(\delta_- r')\right)^*,\quad (20)$$

where $\delta_\pm^2 = \alpha(1-\alpha)Q_\pm^2 + m_f^2$. The integration over $\boldsymbol{u}$ leads to the delta function,

$$\int\frac{d^2\boldsymbol{u}}{(2\pi)^2}\,\mathrm{e}^{i\boldsymbol{u}\cdot(\boldsymbol{r}-\boldsymbol{r}')} = \delta^2(\boldsymbol{r}-\boldsymbol{r}')\,,$$

which allows to perform the integration over $\boldsymbol{r'}$. Thus we obtain

$$\Phi_{\rm LL}(q,k,l) = \alpha_{\rm s}\sqrt{N_c^2-1}\,e^2\sum_f q_f^2 4\sqrt{4\pi}\,|Q_+||Q_-|\int_0^1 d\alpha\int\frac{d^2\boldsymbol{r}}{(2\pi)^2}$$
$$\times\,\alpha^2(1-\alpha)^2\,\underbrace{\mathrm{e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\,K_0(\delta_- r)}\left(1-\mathrm{e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}}\right)\left(1-\mathrm{e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}}\right)\underbrace{\mathrm{e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\,K_0(\delta_+ r)}\,.\quad (21)$$

It is important to note that the integrand (the second line) in (21) is invariant under the transformation: $\alpha\to 1-\alpha$ and $\boldsymbol{r}\to-\boldsymbol{r}$. This reflects the symmetry of the dipole formula under the interchange of quark and antiquark.

The underlined elements in (21) are proportional to the light-cone wave functions of the longitudinal incoming and outgoing photons. To be precise, in the nonforward case the *longitudinal* photon wave function $\Psi^0_{\lambda'\lambda}$ (where $\lambda'(\lambda) = \pm$ denotes the helicity of the (anti)quark) for a given flavour f is given by

$$\Psi^0_{+-}(q\pm l,\boldsymbol{r},\alpha) = \Psi^0_{-+} = \frac{e\,q_f}{2\,\pi^{3/2}}\,\sqrt{N_c}\,\alpha(1-\alpha)\,|Q_\pm|\,K_0(\delta_\pm r)\,\mathrm{e}^{\pm i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}$$
$$\Psi^0_{++} = \Psi^0_{--} = 0\,.\quad (22)$$

The above wave function differs from the photon wave function with only longitudinal momentum, known from the total cross section for the $\gamma^*\gamma^*$ scattering [1], by the exponential factor involving transverse momentum $\boldsymbol{l}$

and the longitudinal momentum fraction α (this factor has also been found in [10, 11]). It has important consequences for the discussion of the impact parameter representation presented in the next section. Summarising, the impact factor (21) in the coordinate space becomes

$$\Phi_{\rm LL}(q,k,l) = 4\pi^{3/2}\,\alpha_{\rm s}\,\frac{\sqrt{N_c^2-1}}{N_c}\int_0^1 d\alpha\int d^2\boldsymbol{r}\left(1-{\rm e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}}\right)\left(1-{\rm e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}}\right)$$
$$\times\sum_f\sum_{\lambda'\lambda}\left\{\overline{\Psi}^0_{\lambda'\lambda}(q-l,\boldsymbol{r},\alpha)\,\Psi^0_{\lambda'\lambda}(q+l,\boldsymbol{r},\alpha)\right\}\,, \tag{23}$$

where $\overline{\Psi}$ is complex conjugate to Ψ. Inserting this relation into the formula (1) for the $\gamma^*\gamma^*$ scattering amplitude, we find for the longitudinally polarized external photons

$$\begin{aligned} A_{\rm LL}(s,\boldsymbol{l}) \;=\; & is\int d^2\boldsymbol{r}_1\int d^2\boldsymbol{r}_1\int_0^1 d\alpha_1\int_0^1 d\alpha_2 \\ & \times\sum_{f_1}\sum_{\lambda'\lambda}\left\{\overline{\Psi}^0_{\lambda'\lambda}(q-l,\boldsymbol{r}_1,\alpha_1)\,\Psi^0_{\lambda'\lambda}(q+l,\boldsymbol{r}_1,\alpha_1)\right\} \\ & \times N(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{l}) \\ & \times\sum_{f_2}\sum_{\lambda'\lambda}\left\{\overline{\Psi}^0_{\lambda'\lambda}(p+l,\boldsymbol{r}_2,\alpha_2)\,\Psi^0_{\lambda'\lambda}(p-l,\boldsymbol{r}_2,\alpha_2)\right\}\,, \end{aligned} \tag{24}$$

where $N(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{l})$ is the scattering amplitude of two dipoles of the transverse size $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ with the momentum transfer $2\boldsymbol{l}$, in the two gluon exchange approximation

$$\begin{aligned} N(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{l}) \;=\; & \alpha_{\rm s}^2\,\frac{(N_c^2-1)}{N_c^2}\int\frac{d^2\boldsymbol{k}}{(\boldsymbol{k}+\boldsymbol{l})^2(\boldsymbol{k}-\boldsymbol{l})^2} \\ \times & \left(1-{\rm e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}_1}\right)\left(1-{\rm e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}_1}\right)\left(1-{\rm e}^{i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}_2}\right)\left(1-{\rm e}^{-i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}_2}\right). \end{aligned} \tag{25}$$

Let us concentrate now on the impact factor for *transverse* photons $\Phi^{(ij)}_{\rm TT}$, Eq. (15). Using formulas (19) and repeating the steps presented in the longitudinal case, we obtain

$$\begin{aligned}
&\Phi^{(ij)}_{\rm TT}(q,k,l) = \\
&\alpha_{\rm s}\sqrt{N_c^2-1}e^2\sum_f q_f^2\sqrt{4\pi}\int_0^1 d\alpha\int\frac{d^2\boldsymbol{r}}{(2\pi)^2}\left(1-{\rm e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}}\right)\left(1-{\rm e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}}\right) \\
&\times\Bigg\{-4\alpha(1-\alpha)\left(\frac{\boldsymbol{\varepsilon}_j\cdot\boldsymbol{r}}{r}\delta_- K_1(\delta_- r){\rm e}^{-i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\right)^*\left(\frac{\boldsymbol{\varepsilon}_i\cdot\boldsymbol{r}}{r}\delta_+ K_1(\delta_+ r){\rm e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\right) \\
&+\delta_{ij}\left(\delta_- K_1(\delta_- r)\,{\rm e}^{-i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\right)^*\left(\delta_+ K_1(\delta_+ r)\,{\rm e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\right) \\
&+\delta_{ij}\,m_f^2\left(K_0(\delta_- r)\,{\rm e}^{-i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\right)^*\left(K_0(\delta_+ r)\,{\rm e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\right)\Bigg\}\,, \qquad (26)
\end{aligned}$$

where we used the fact that $\boldsymbol{\varepsilon}_i\cdot\boldsymbol{\varepsilon}_j^* = \delta_{ij}$. The above formula can be written in a similar form as Eq. (23) using helicity wave functions

$$\begin{aligned}
&\Phi^{(ij)}_{\rm TT}(q,k,l) = \\
&4\pi^{3/2}\alpha_{\rm s}\frac{\sqrt{N_c^2-1}}{N_c}\int_0^1 d\alpha\int d^2\boldsymbol{r}\left(1-{\rm e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}}\right)\left(1-{\rm e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}}\right) \\
&\times\sum_f\sum_{\lambda'\lambda}\left\{\overline{\Psi^j_{\lambda'\lambda}}(q-l,\boldsymbol{r},\alpha)\,\Psi^i_{\lambda'\lambda}(q+l,\boldsymbol{r},\alpha)\right\} \qquad (27)
\end{aligned}$$

where i,j are the helicities of the incoming and outgoing photon, respectively, and $\lambda'(\lambda)$ denotes the helicity of the (anti)quark. The photon wave functions are defined in terms of these helicities as:

$$\Psi^+_{+-}(q\pm l,\boldsymbol{r},\alpha) = \frac{i\,e\,q_f}{2\,\pi^{3/2}}\sqrt{N_c}\,\alpha\,\frac{\boldsymbol{\varepsilon}_+\cdot\boldsymbol{r}}{r}\,\delta_\pm\,K_1(\delta_\pm r){\rm e}^{\pm i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}, \qquad (28)$$

$$\Psi^-_{+-}(q\pm l,\boldsymbol{r},\alpha) = -\frac{ieq_f}{2\pi^{3/2}}\sqrt{N_c}(1-\alpha)\frac{\boldsymbol{\varepsilon}_-\cdot\boldsymbol{r}}{r}\delta_\pm K_1(\delta_\pm r){\rm e}^{\pm i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}, \qquad (29)$$

$$\Psi^+_{-+}(q\pm l,\boldsymbol{r},\alpha) = -\frac{ieq_f}{2\pi^{3/2}}\sqrt{N_c}(1-\alpha)\frac{\boldsymbol{\varepsilon}_+\cdot\boldsymbol{r}}{r}\delta_\pm K_1(\delta_\pm r){\rm e}^{\pm i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}, \qquad (30)$$

$$\Psi^-_{-+}(q\pm l,\boldsymbol{r},\alpha) = \frac{i\,e\,q_f}{2\,\pi^{3/2}}\sqrt{N_c}\,\alpha\,\frac{\boldsymbol{\varepsilon}_-\cdot\boldsymbol{r}}{r}\,\delta_\pm\,K_1(\delta_\pm r)\,{\rm e}^{\pm i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}, \qquad (31)$$

$$\Psi^+_{++} = \Psi^-_{--}(q\pm l,\boldsymbol{r},\alpha) = \frac{e\,q_f}{(2\pi)^{3/2}}\sqrt{N_c}\,m_f\,K_0(\delta_\pm r)\,{\rm e}^{\pm i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}, \qquad (32)$$

$$\Psi^+_{--} = \Psi^-_{++} = 0\,. \qquad (33)$$

Again, the exponentials in the above are due to the transverse momentum of the photons. Similar wave functions were defined in [11] and [14]. The difference between reference [11] and our results lies in the minus sign in (29) and (30), and we found a different relative normalisation between (32) and the rest of the nonzero wave functions (28)–(31) compared to [14]. It is important to realize that in the forward case, when $\boldsymbol{l} = 0$ and $i = j$, after the summation over the two transverse polarizations, the expression in the curly brackets in Eq. (26) becomes a well known expression for the square of the photon wave function in forward kinematics

$$|\Psi_{\rm T}(q,\boldsymbol{r},\alpha)|^2 \sim \sum_f e_f^2 \left\{[\alpha^2+(1-\alpha)^2]\,\delta^2\,K_1^2(\delta r) + m_f^2\,K_0^2(\delta r)\right\} ,$$

where $\delta^2 = \alpha(1-\alpha)Q^2 + m_f^2$. The structure of the full $\gamma^*\gamma^*$ amplitude for transverse photons is the same as in Eq. (24) with the wave function factors replaced by those in the curly brackets in Eq. (27).

For completeness we also present the formula for the impact factor with mixed polarization (18)

$$\Phi_{\rm LT}^{(j)}(k,l) = 4\,\pi^{3/2}\alpha_{\rm s}\,\frac{\sqrt{N_c^2-1}}{N_c}\int\limits_0^1 d\alpha\int d^2\boldsymbol{r}\left(1-{\rm e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}}\right)\left(1-{\rm e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}}\right) \times\sum_f\sum_{\lambda'\lambda}\left\{\overline{\Psi^j_{\lambda'\lambda}}(q-l,\boldsymbol{r},\alpha)\Psi^0_{\lambda'\lambda}(q+l,\boldsymbol{r},\alpha)\right\} . \tag{34}$$

In each presented case the $\gamma^*\gamma^*$ scattering amplitude $A(s,\boldsymbol{l})$ has the form of Eq. (24) with the wave function replacement which takes into account helicities of the external photons.

4. Impact parameter

In our notation, the four-momentum transfer squared t between the two $q\bar{q}$ systems is given by the transverse two-dimensional vector $\boldsymbol{\Delta} = 2\boldsymbol{l}$, *i.e.* $t = -\boldsymbol{\Delta}^2$. The impact parameter $\boldsymbol{b}$ is defined as a Fourier conjugate variable to $\boldsymbol{\Delta}$, and the $\gamma^*\gamma^*$ scattering amplitude in the impact parameter space is given by

$$\tilde{A}(s,\boldsymbol{b}) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{\Delta}}\, A(s,\boldsymbol{\Delta})\,. \tag{35}$$

Performing this integral, we keep the total energy $\sqrt{s}$ as well as virtualities of the external photons, $Q_\pm^2$ and $P_\pm^2$, fixed. Let us concentrate on the amplitude for longitudinal photons (24) with the wave function (22). The dependence on $\boldsymbol{\Delta}$ (or $\boldsymbol{l}$) resides in the exponential factors in the wave functions and the dipole–dipole scattering amplitude $N(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{\Delta})$, Eq. (25). Thus performing the transformation (35) the amplitude (24) we find

$$\begin{aligned}
\tilde{A}(s, \boldsymbol{b}) &= is \int d^2\boldsymbol{r}_1 \int d^2\boldsymbol{r}_1 \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \\
&\times \sum_{f_1} \sum_{\lambda'\lambda} \left\{ \overline{\Psi}_{\lambda'\lambda}(Q_-, \boldsymbol{r}_1, \alpha_1) \, \Psi_{\lambda'\lambda}(Q_+, \boldsymbol{r}_1, \alpha_1) \right\} \\
&\times \tilde{N}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{b} + (1-\alpha_1)\boldsymbol{r}_1 + (1-\alpha_2)\boldsymbol{r}_2) \\
&\times \sum_{f_2} \sum_{\lambda'\lambda} \left\{ \overline{\Psi}_{\lambda'\lambda}(P_+, \boldsymbol{r}_2, \alpha_2) \, \Psi_{\lambda'\lambda}(P_-, \boldsymbol{r}_2, \alpha_2) \right\} . \qquad (36)
\end{aligned}$$

The wave functions above are the forward photon wave functions with the indicated photon virtualities, given by Eqs. (28) with the exponential factors being removed. This is due to the fact that, when transforming to impact parameter, we have to include the momentum transfer dependence of these factors into the definition of the dipole–dipole scattering amplitude $\tilde{N}$:

$$\begin{aligned}
&\tilde{N}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{b} + (1-\alpha_1)\boldsymbol{r}_1 + (1-\alpha_2)\boldsymbol{r}_2) \\
&= \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} \, e^{i(\boldsymbol{b}+(1-\alpha_1)\boldsymbol{r}_1+(1-\alpha_2)\boldsymbol{r}_2)\cdot\boldsymbol{\Delta}} \, N(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{\Delta}) \, . \qquad (37)
\end{aligned}$$

From (37) we see that, in the dipole–dipole scattering amplitude, the dependence upon the impact parameter contains an α-dependent shift relative to the impact parameter $\boldsymbol{b}$ which refers to the external photon Eq. (35): $\boldsymbol{b} \to \boldsymbol{b} + (1-\alpha_1)\boldsymbol{r}_1 + (1-\alpha_2)\boldsymbol{r}_2$. Furthermore, as we have already remarked after Eq. (21), the dipole scattering amplitude (for fixed transverse dipole sizes and longitudinal momenta) is invariant under the interchanges $\alpha_1 \to (1-\alpha_1)$, $\boldsymbol{r}_1 \to -\boldsymbol{r}_1$ or/and $\alpha_2 \to (1-\alpha_2)$, $\boldsymbol{r}_2 \to -\boldsymbol{r}_2$.

Let us give a physical interpretation of this result. In the high energy limit, the incoming photon and the quark–antiquark pair are conveniently described in an infinite momentum frame. In this frame, as it has been shown many years ago [16,17], there exists a subgroup of the Poincare group which is isomorphic to the symmetry group of Galilei transformations in nonrelativistic two-dimensional quantum mechanics. This two dimensional motion takes place in the transverse plane, and the longitudinal momentum plays the role of the nonrelativistic mass. In our case of dipole–dipole scattering we choose the upper incoming photon to move in the positive z-direction, *i.e.*

in light cone variables q has a large q_+ component. Inside the upper impact factor, the upper quark line with longitudinal momentum fraction $1-\alpha_1$ carries the 'mass' $m_u = (1-\alpha_1)q_+$, whereas the lower quark line has the 'mass' $m_l = \alpha_1 q_+$. The vector $\boldsymbol{r}_1$ denotes the transverse distance between the upper and lower quark line, and the impact parameter $\boldsymbol{b}$ is the transverse distance between the two incoming photons. If we interpret the upper incoming photon as being in the center of mass of the upper quark–antiquark system, see Fig. 2, the two vectors

$$\boldsymbol{b} + \frac{m_u}{m_u+m_l}\boldsymbol{r}_1 \;=\; \boldsymbol{b} + (1-\alpha_1)\boldsymbol{r}_1 \tag{38}$$

and

$$\boldsymbol{b} - \frac{m_l}{m_u+m_l}\boldsymbol{r}_1 \;=\; \boldsymbol{b} - \alpha_1\boldsymbol{r}_1 \tag{39}$$

denote the position of the upper and lower quark, respectively. A similar argument holds for the lower dipole, *i.e.* the vector

$$\boldsymbol{b} + (1-\alpha_1)\,\boldsymbol{r}_1 + (1-\alpha_2)\,\boldsymbol{r}_2 \tag{40}$$

denotes the distance between the upper quark line in the upper dipole and the lower quark line in the lower dipole. The peculiar $\boldsymbol{b}$-dependence of Eq. (37) then says that the interaction between the upper and lower colour dipoles depends upon the distance between one of the two quarks of the upper dipole and one of the quarks in the lower dipole (and not upon the distance $\boldsymbol{b}$ between the center of mass coordinates of the two dipoles). Moreover, because of the symmetry under $\alpha \to 1-\alpha$, $\boldsymbol{r} \to -\boldsymbol{r}$ we are free to interchange the quarks inside one of the two colour dipoles.

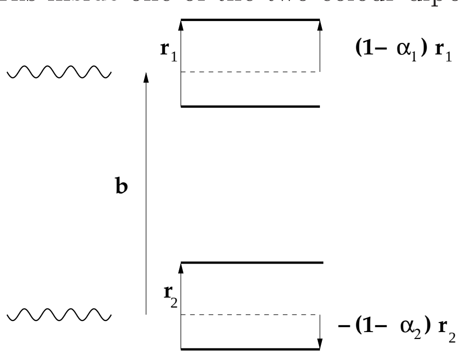

Fig. 2. The interpretation of relation (40) in the transverse coordinate plane: the wavy lines denote the incoming photons, the full lines the quark antiquarks of the upper and the lower colour dipoles.

Finally, the appearance of the phase factors $\exp\{\pm i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}\}$ in (28) can be understood as follows. Inverting (19) we find, for example, for the upper incoming longitudinal photon

$$K_0(\delta_+ r)\,\mathrm{e}^{i(1-\alpha)\boldsymbol{l}\boldsymbol{r}} = \int \frac{d^2\boldsymbol{k}}{2\pi}\,\mathrm{e}^{-i(\boldsymbol{k}-(1-\alpha)\boldsymbol{l})\cdot\boldsymbol{r}}\,\frac{1}{\boldsymbol{k}^2+\delta_+^2}\,, \tag{41}$$

i.e. the momentum conjugate to the transverse distance $\boldsymbol{r}$ is $(\boldsymbol{k}-(1-\alpha)\boldsymbol{l})$ rather than $\boldsymbol{k}$. The incoming photon carries transverse momentum $\boldsymbol{l}$ and splits into quark and antiquark with longitudinal and transverse momenta $(1-\alpha,\,\boldsymbol{k})$ and $(\alpha,\,-\boldsymbol{k}+\boldsymbol{l})$, respectively. In the infinite momentum frame this corresponds to the masses $m_1=(1-\alpha)q_+$ and $m_2=\alpha q_+$. From nonrelativistic mechanics we know that the relative momentum of the quark–antiquark pair (*i.e.* the momentum conjugate to the relative coordinate $\boldsymbol{r}$) is given by

$$\frac{m_2}{m_1+m_2}\boldsymbol{p}_1-\frac{m_1}{m_1+m_2}\boldsymbol{p}_2=\alpha\boldsymbol{k}-(1-\alpha)(-\boldsymbol{k}+\boldsymbol{l})=\boldsymbol{k}-(1-\alpha)\boldsymbol{l}\,. \tag{42}$$

So the appearance of the phase factor looks very natural.

5. Nonforward dipole–proton scattering amplitude

In a case when the lower photon is replaced by the proton, an unintegrated gluon distribution $f(x_\pm,,\boldsymbol{k},\boldsymbol{l})$ appears [18], where $x_\pm=Q_\pm^2/s$ and the adopted notation means that f depends on both the variables. Thus, strictly speaking, f is a nondiagonal (skewed or generalised) unintegrated gluon distribution since both the longitudinal and transverse momenta of the exchanged gluons, $k\pm l$, are not equal, see Fig. 2. In the proton case, the amplitude (1) reads

$$A(s,\boldsymbol{l}=\boldsymbol{\Delta}/2) = \frac{is}{2}\int\frac{d^2\boldsymbol{k}}{(2\pi)^3}\,\frac{\Phi(q,\boldsymbol{k},\boldsymbol{l})}{(\boldsymbol{k}+\boldsymbol{l})^2}\,\frac{f(x_\pm,\boldsymbol{k},\boldsymbol{l})}{(\boldsymbol{k}-\boldsymbol{l})^2}\,. \tag{43}$$

Repeating the steps from Section 3, we find in the coordinate space the analogue of Eq. (24)

$$\begin{aligned}&A(s,\boldsymbol{l}=\boldsymbol{\Delta}/2)\\&= is\int d^2\boldsymbol{r}\int_0^1 d\alpha\sum_f\sum_{\lambda'\lambda}\left\{\overline{\Psi}_{\lambda'\lambda}(q-l,\boldsymbol{r},\alpha)N(x_\pm,\boldsymbol{r},\boldsymbol{l})\Psi_{\lambda'\lambda}(q+l,\boldsymbol{r},\alpha)\right\}\,,\end{aligned} \tag{44}$$

where now $N(x_\pm, \boldsymbol{r}, \boldsymbol{l})$ is a nonforward dipole–proton scattering amplitude

$$N(x_\pm, \boldsymbol{r}, \boldsymbol{l}) = \frac{\alpha_s}{4\pi^{3/2}} \frac{\sqrt{N_c^2 - 1}}{N_c} \times \int \frac{d^2\boldsymbol{k}}{(\boldsymbol{k}+\boldsymbol{l})^2(\boldsymbol{k}-\boldsymbol{l})^2} \left(1 - \mathrm{e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}}\right)\left(1 - \mathrm{e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}}\right) f(x_\pm, \boldsymbol{k}, \boldsymbol{l}) \,. \tag{45}$$

In the impact parameter representation we have:

$$\tilde{A}(s, \boldsymbol{b}) = is \int d^2\boldsymbol{r} \int_0^1 d\alpha \sum_f \sum_{\lambda'\lambda} \left\{ \overline{\Psi}_{\lambda'\lambda}(Q_-, \boldsymbol{r}, \alpha) \tilde{N}(x_\pm, \boldsymbol{r}, \boldsymbol{b} + (1-\alpha)\boldsymbol{r}) \Psi_{\lambda'\lambda}(Q_+, \boldsymbol{r}, \alpha) \right\} \tag{46}$$

with

$$\tilde{N}(x_\pm, \boldsymbol{r}, \boldsymbol{b} + (1-\alpha)\boldsymbol{r})1 = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} e^{i(\boldsymbol{b}+(1-\alpha)\boldsymbol{r})\cdot\boldsymbol{\Delta}} N(x_\pm, \boldsymbol{r}, \boldsymbol{l} = \boldsymbol{\Delta}/2). \tag{47}$$

Again, there are forward photon wave functions in (46) since the nonforward exponentials are incorporated into (47). As we have mentioned before, this expression is invariant under the replacement: $\alpha \to 1-\alpha$ and $\boldsymbol{r} \to -\boldsymbol{r}$, which can be easily seen by inserting (45) into (47) and performing the symmetry transformation.

For completeness we also quote the formula for diffractive vector production in the nonforward direction [10], *e.g.* for the production of longitudinal ρ-mesons. In our kinematics, the impact factor has the form (44), with the wave function for the outgoing photon, $\Psi(q-l, \boldsymbol{r}, \alpha)$, being replaced by the meson wave function:

$$\Psi(q-l, \boldsymbol{r}, \alpha) \quad \to \quad \Psi_\rho(\boldsymbol{r}, \alpha)\, \mathrm{e}^{-i(1-\alpha)\boldsymbol{l}\boldsymbol{r}} \,. \tag{48}$$

As in the case of $\gamma^*\gamma^*$ scattering, the extra phase factor accounts for the nonzero transverse momentum of the vector particle.

6. Open quark–antiquark production

In the final step of our discussion we remove, in the upper dipole system, the wave function of the outgoing photon, *i.e.* we consider the diffractive production of an open quark–antiquark pair in the nonforward direction, $\gamma^* + p \to (q\bar{q}) + p$. We follow our previous notation: the incoming photon carries the momentum $q + l$, and the outgoing antiquark and quark have the momenta $q-u$ and $u-l$, respectively. The scattering amplitude has the form (43) with the replacement: $\Phi(q,k,l) \to \Phi_{\lambda'\lambda}^{\gamma^*\to(q\bar{q})}(q,u,k,l)$. For the longitudinal photon, the new impact factor for open quark–antiquark production is given by:

$$\Phi_{+-}^{0,\gamma^*\to(q\bar{q})}(q,\boldsymbol{u},\boldsymbol{k},\boldsymbol{l}) \;=\; 2\,(2\pi)^{3/2}\alpha_{\rm s}\sqrt{\frac{N_c^2-1}{N_c}}\,e_f\,[\alpha(1-\alpha)]^{3/2}\,|Q_+|$$
$$\times\left(\frac{1}{D(\boldsymbol{u}+(1-\alpha)\boldsymbol{l})}+\frac{1}{D(\boldsymbol{u}-(1+\alpha)\boldsymbol{l})}-\frac{1}{D(\boldsymbol{u}+\boldsymbol{k}-\alpha\boldsymbol{l})}-\frac{1}{D(\boldsymbol{u}-\boldsymbol{k}-\alpha\boldsymbol{l})}\right) \tag{49}$$

with $D(\boldsymbol{k})=\boldsymbol{k}^2+\alpha(1-\alpha)Q_+^2=\boldsymbol{k}^2+\delta_+^2$. Inserting (19), we arrive at:

$$\Phi_{+-}^{0,\gamma^*\to(q\bar{q})}(q,\boldsymbol{u},\boldsymbol{k},\boldsymbol{l}) = 2\,(2\pi)^{3/2}\alpha_{\rm s}\sqrt{\frac{N_c^2-1}{N_c}}e_f\,[\alpha(1-\alpha)]^{3/2}\,|Q_+|$$
$$\times\!\int\!\frac{d^2\boldsymbol{r}'}{2\pi}\,{\rm e}^{i\boldsymbol{u}\cdot\boldsymbol{r}'}\left(1-{\rm e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}'}\right)\left(1-{\rm e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}'}\right){\rm e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}'}K_0(\delta_+r')\,. \tag{50}$$

From the discussion at the end of Section 3 we know that the transverse distance between the outgoing quark an antiquark, $\boldsymbol{r}$, is conjugate to the transverse momentum $\boldsymbol{u}-(1-\alpha)\boldsymbol{l}$. We therefore define the scattering amplitude for the production of a dipole of the size $\boldsymbol{r}$ by taking the Fourier transform

$$\Phi_{+-}^{0,\gamma^*\to(q\bar{q})}(q,\boldsymbol{r},\boldsymbol{k},\boldsymbol{l}) \;\equiv\; \int\frac{d^2\boldsymbol{u}}{(2\pi)^2}\,{\rm e}^{-i(\boldsymbol{u}-(1-\alpha)\boldsymbol{l})\cdot\boldsymbol{r}}\,\Phi_{+-}^{0,\gamma^*\to(q\bar{q})}(q,\boldsymbol{u},\boldsymbol{k},\boldsymbol{l})\,, \tag{51}$$

and

$$\Phi_{+-}^{0,\gamma^*\to(q\bar{q})}(q,\boldsymbol{r},\boldsymbol{k},\boldsymbol{l}) = 2\sqrt{2\pi}\,\alpha_{\rm s}\sqrt{\frac{N_c^2-1}{N_c}}\,e_f\,[\alpha(1-\alpha)]^{3/2}\,|Q_+|$$
$$\times{\rm e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\left(1-{\rm e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}}\right)\left(1-{\rm e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}}\right){\rm e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\,K_0(\delta_+r)$$
$$=\sqrt{2}\,(2\pi)^2\alpha_{\rm s}\frac{\sqrt{N_c^2-1}}{N_c}\sqrt{\alpha(1-\alpha)}$$
$$\times{\rm e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}\left(1-{\rm e}^{-i(\boldsymbol{k}+\boldsymbol{l})\cdot\boldsymbol{r}}\right)\left(1-{\rm e}^{i(\boldsymbol{k}-\boldsymbol{l})\cdot\boldsymbol{r}}\right)\Psi_{+-}^0(q+l,\boldsymbol{r},\alpha). \tag{52}$$

Finally, the scattering amplitude of the dipole on a proton our result takes the form:

$$A_{+-}^{\gamma^*\to(q\bar{q})}(s,\alpha,\boldsymbol{r},\boldsymbol{l}) = is\sqrt{2\pi}\sqrt{\alpha(1-\alpha)}{\rm e}^{i(1-\alpha)\boldsymbol{l}\cdot\boldsymbol{r}}N(x_\pm,\boldsymbol{r},\boldsymbol{l})\Psi_{+-}^0(q+l,\boldsymbol{r},\alpha) \tag{53}$$

with the dipole scattering amplitude from (45). When transforming to impact parameter $\boldsymbol{b}$, again the shifted variable $\boldsymbol{b}+(1-\alpha)\boldsymbol{l}$ appears, the distance from the target center to the quark. For the integrated and spin summed cross section

$$\frac{d\sigma}{d^2\boldsymbol{\Delta}} = \frac{1}{16\pi^2 s^2}\frac{1}{4\pi}\frac{1}{\alpha(1-\alpha)}\int_0^1 d\alpha \int \frac{d^2\boldsymbol{u}}{(2\pi)^2}\sum_{\lambda'\lambda}|A_{\lambda'\lambda}^{\gamma^*\to(q\bar{q})}(s,\alpha,\boldsymbol{u},\boldsymbol{l}=\boldsymbol{\Delta}/2)|^2\,, \tag{54}$$

we obtain after transforming to transverse coordinates:

$$\frac{d\sigma}{d\boldsymbol{\Delta}^2} = \frac{1}{16\pi}\int_0^1 d\alpha \int d^2\boldsymbol{r}\sum_{\lambda'\lambda}\overline{\Psi^0_{\lambda'\lambda}}(q+l,\boldsymbol{r},\alpha)\,|N(x_\pm,\boldsymbol{r},\boldsymbol{l})|^2\,\Psi^0_{\lambda'\lambda}(q+l,\boldsymbol{r},\alpha). \tag{55}$$

This generalises the well-known formula for the diffractive quark–antiquark production in the forward direction [1]. In particular, the integrated cross section for diffractive quark–antiquark production contains the same dipole scattering amplitude as the elastic $\gamma^* p$ process (45).

7. Conclusions

In this paper we have discussed the generalisation of the colour dipole picture to the nonforward direction. For the elastic scattering of two colour dipoles we found that the dipole cross section depends upon the transverse distance between the ends of the two dipoles, and not between the centres of the two quark–antiquark pairs. This result also holds for diffractive vector production. For open $q\bar{q}$-production, we confirm the well-known result of the forward direction: the integrated (over the transverse size of the produced quark–antiquark pair) cross section is described by the square of the elastic dipole cross section.

We believe that our findings might be useful in several respects. First, it may provide some guidance in modelling the b-dependence of the dipole cross section. As we have indicated in our introduction, the b-dependence represents the mayor challenge in understanding the transition from the region of perturbative to nonperturbative QCD, and our formula may provide a starting point along these lines. Secondly, our general expression for the nonforward impact factor will be of use also in electroweak physics where higher order contributions to vector boson scattering are of interest [19].

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