HOW IMPORTANT ARE THE LONGITUDINAL VIRTUAL PHOTONS IN THE SEMI-INCLUSIVE *ep* PROCESSES?

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Dedicated to Jan Kwieciński in honour of his 65th birthday

We study an importance of the longitudinally polarized virtual photons $\gamma^*_{\rm L}$ in the semi-inclusive ep processes in the ep center-of-mass and Breit frames, and for various distributions. We used the factorization formulae for the unpolarized inclusive and semi-inclusive ep processes which hold in an arbitrary reference frame. The numerical studies were performed for the ep HERA collider for a process with a large-$p_{\rm T}$ (prompt) photons production, *i.e.*, the unpolarized Compton process $ep \to e\gamma X$, in the Born approximation. In the ep center-of-mass frame we found that the differential cross section for the longitudinally polarized intermediate photon, $d\sigma_{\rm L}$, and the term due to the interference between the longitudinal- and transverse-polarization states of the photon, $d\tau_{\rm LT}$, are small, *i.e.* below 10% of the cross section. Moreover, these two contributions almost cancel one another, leading to a stronger domination of the transversely polarized virtual photon, even for its large virtuality Q^2. We found that in this frame the interference term gives non-vanishing contributions even for the cross sections integrated over the azimuthal angle, contrary to a naive expectation. Relevance of the contributions of the longitudinal photon in a jet production in DIS events at the HERA collider is commented. A relatively large ($\sim$ 30%) effect due to the interference term $d\tau_{\rm LT}$ was found for the considered process at the HERA collider in the azimuthal-angle distribution in the Breit frame. Here this contribution vanishes in the cross section integrated over ϕ.

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1. Introduction

Assuming that the one-photon exchange dominates in the deep inelastic lepton–nucleon collisions (DIS), a cross section for such a process can be described in terms of two transverse (T) and one longitudinal (L) polarization states of the intermediate virtual photon γ^*. The differential cross section for the unpolarized $lN \to lX$ process can always be decomposed into two differential cross sections, $d\sigma_{\rm T}$ and $d\sigma_{\rm L}$, describing the processes with the transversely and the longitudinally polarized γ^*, $\gamma^*_{\rm T}N \to X$ and $\gamma^*_{\rm L}N \to X$, respectively [1–6]. When the initial particles in the discussed process $lN \to lX$ are polarized, or when for the unpolarized particles a semi-inclusive process $lN \to laX$ is considered, there appear in addition terms coming from the interference between the longitudinally and transversely polarized virtual photons, and between two different transverse-polarization states of $\gamma^*_{\rm T}$, $d\tau_{\rm LT}$ and $d\tau_{\rm TT}$, respectively [6].

It is well known that for the two-photon exchange processes, for instance in the e^+e^- collisions, the interference terms occur in the cross sections, as discussed in [6,9][1]. The detailed study of relevance of various contributions, especially of the interference terms, has been performed in [10, 11] for the process $e^+e^- \to e^+e^-\mu^+\mu^-$, for the kinematical range of the PLUTO and LEP experiments. For a corresponding subprocess $\gamma^*\gamma^* \to \mu^+\mu^-$ large cross sections for the contribution involving at least one longitudinally polarized photon were found. Moreover, the interference terms were found to give a large negative contribution. Both contributions vary strongly as a function of the kinematical variables, and for some kinematical regions a cancellation between the cross sections for processes with one or two $\gamma^*_{\rm L}$ and the interference contributions occurs. The conclusion from this analysis was that both types of contributions have to be taken into account in extracting from the data the leptonic (mionic) structure functions of the virtual photon (see also [12]). However, in some of the measurements of the structure functions $F_2^{\gamma^*}$ and $F_{\rm L}^{\gamma^*}$, the interference terms and cross section for two longitudinally polarized virtual photons were neglected, see [13, 14], and [15].

The contributions due to the longitudinally polarized virtual photons occur also in electroproduction and the question is how significant such contributions are. Moreover, this question is related to the ongoing discussion on the relevance of the resolved-$\gamma^*_{\rm L}$ contributions in the hard processes [16–22]. For example, it was pointed out in [22] that, in the case of the di-jet production in the *ep* HERA collider, the contributions coming from the longitudinally polarized photon are sizeable and the partonic content of the $\gamma^*_{\rm L}$ should be taken into account in describing the data. However, it is clear

[1] In e^+e^- collisions, the interference terms are also important for the Higgs boson production via WW or ZZ fusion as was shown in [7, 8].

that the study of cross sections with $\gamma^*_{\rm L}$ for the large-$p_{\rm T}$ processes should be accompanied by a consideration of the corresponding interference terms containing $\gamma^*_{\rm L}$. Unfortunately, there is a lack of such studies in the literature. In this paper we would like to initiate a discussion on the relevance of such terms in the semi-inclusive ep processes.

It is well known that to get access to the interference between $\gamma^*_{\rm L}$ and $\gamma^*_{\rm T}$, or between two different transverse-polarization states of γ^*, it is natural to consider the azimuthal-angle dependence in a special reference frame called the Breit frame [28–35]. Recently, the azimuthal asymmetries for the charged-hadrons production in the neutral-current deep inelastic e^+p scattering have been measured with the ZEUS detector at HERA [41], and indeed effects due to the corresponding interference terms were observed.

In this article we study the contributions due to $\gamma^*_{\rm L}$ and $\gamma^*_{\rm T}$ in unpolarized ep collisions at HERA, taking as an example the process with a production of high-$p_{\rm T}$ (prompt) photon: $ep \to e\gamma X$ (Compton process). In Section 2 a short derivation of the factorization formulae for the inclusive and semi-inclusive ep processes is presented in an arbitrary frame (some details are given also in Appendix A). In the case of the semi-inclusive collision, two cross sections, $d\sigma_{\rm T}$ and $d\sigma_{\rm L}$, and two additional terms due to the interference between different polarization states of γ^*, $d\tau_{\rm LT}$ and $d\tau_{\rm TT}$, appear. The relation of these interference terms to the contributions proportional to $\cos\phi$ and $\cos 2\phi$ in the azimuthal-angle distribution for a final γ in the Breit frame is discussed in Section 3 and Appendix B. Section 4 is devoted to the numerical studies of the different contributions for the process $ep \to e\gamma X$ in the Born approximation. Conclusions are presented in Section 5. In the Appendices the explicit form of the polarization vectors of the γ^* and factorization formulae for the semi-inclusive process are presented both in a frame-independent form and for the Breit frame.

2. Factorization in the inclusive and semi-inclusive unpolarized lepton–nucleon scattering

2.1. Inclusive process $ep \to eX$ (DIS)

We start with a short description of the standard DIS process for unpolarized ep collisions (Fig. 1),

$$ep \to eX\,, \tag{1}$$

assuming that the one-photon exchange dominates. The corresponding differential cross section is denoted by $d\sigma^{ep\to eX}$, and we use the following notation for the kinematical variables: k^μ ($k'^{\,\mu}$) denotes the four-momentum of the initial (final) electron, p_p^μ the four-momentum of the initial proton, q^μ the four-momentum of the intermediate photon, and $Q^2 = -q^2 > 0$ is the photon's virtuality. We denote by ε_i^μ ($i = T_1, T_2, L$) the two transverse-and one longitudinal-polarization vectors of the exchanged virtual photon γ^*. The standard scaling variables are $x = Q^2/2p_p q$ and $y = p_p q/p_p k$.

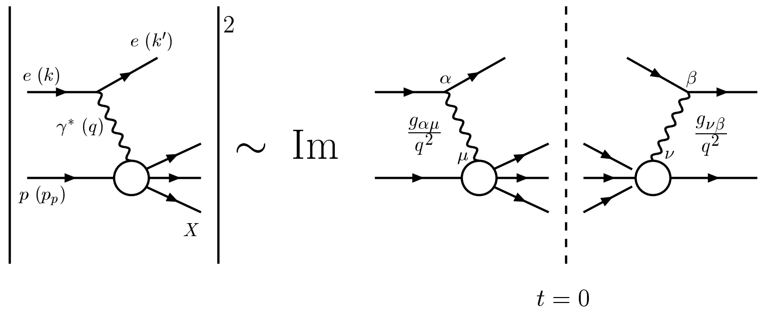

Fig. 1. Kinematics and notation for the process $ep \to eX$ with the one-photon exchange. The optical theorem relation of the squared matrix element for $ep \to eX$ and the imaginary part of the amplitude for $ep \to ep$ at $t = 0$.

It is well known that for the considered process (1) there is a factorization of the differential cross section onto the lepton and hadron parts and a separation between the contributions of the longitudinal- and transverse-polarization states of the intermediate photon [1–6]. So, we have here:

$$d\sigma^{ep\to eX} \;=\; \Gamma_{\mathrm{T}}\, d\sigma_{\mathrm{T}}^{\gamma^* p\to X} \;+\; \Gamma_{\mathrm{L}}\, d\sigma_{\mathrm{L}}^{\gamma^* p\to X} \;\equiv\; d\sigma_{\mathrm{T}} \;+\; d\sigma_{\mathrm{L}}\;, \tag{2}$$

where $d\sigma_{\mathrm{T}}^{\gamma^* p\to X}$ and $d\sigma_{\mathrm{L}}^{\gamma^* p\to X}$ are the cross sections for the $\gamma^* p$ collision with the virtual photon polarized transversely and longitudinally, respectively. The functions Γ_{T} and Γ_{L} describe the probabilities of the emission, by the initial electron, of a virtual photon in the transverse- and the longitudinal-polarization states, respectively.

The above factorization and separation formula can be obtained in various ways [1–6]. For example, the cross section for the process $ep \to eX$ (see Fig. 1) can be expressed as a convolution of the lowest order leptonic tensor $L_{\mathrm{e}}^{\mu\nu}(k,q)$ and the hadronic tensor $W_p^{\mu\nu}(p_p,q)$, both symmetric in the indices μ and ν. Namely we have (for $k^2 = k'^{\,2} = 0$, $p_p^2 = M^2$)

$$d\sigma^{ep\to eX} \sim \frac{1}{q^4} L_{e\mu\nu}\, W_p^{\mu\nu}\,, \tag{3}$$

where:

$$L_e^{\mu\nu}(k,q) \;=\; 2\left(2k^\mu k^\nu - q^\mu k^\nu - k^\mu q^\nu + \tfrac{1}{2}q^2 g^{\mu\nu}\right), \tag{4}$$

$$W_p^{\mu\nu}(p_p,q) \;=\; W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + \frac{W_2}{M^2}\left(p_p^\mu - \frac{p_p q}{q^2}q^\mu\right)\left(p_p^\nu - \frac{p_p q}{q^2}q^\nu\right). \tag{5}$$

The gauge invariance leads to the conditions:

$$q_\mu L_e^{\mu\nu} = q_\nu L_e^{\mu\nu} = 0\,, \qquad q_\mu W_p^{\mu\nu} = q_\nu W_p^{\mu\nu} = 0\,. \tag{6}$$

On the other hand, one can express the hadronic tensor in terms of the polarization states of the exchanged photon. Using the explicit form of the longitudinal-polarization vector and the completeness relation given in Appendix A, we obtain

$$W_p^{\mu\nu} = W_1 \sum_{T=T1}^{T2} \varepsilon_{\rm T}^{*\mu}\varepsilon_{\rm T}^{\nu} + (\bar{W}_2 - W_1)\varepsilon_{\rm L}^{*\mu}\varepsilon_{\rm L}^{\nu}\,, \qquad \bar{W}_2 = \frac{(p_p q)^2 - q^2 p_p^2}{-q^2}\frac{W_2}{M^2}\,. \tag{7}$$

From the above form of $W_p^{\mu\nu}$ (7) one can easily derive formula (2).

Another way of obtaining the considered formula (2) is "the propagator decomposition method" [26,27]. In this method the cross section for process (1) is represented in the following form:

$$d\sigma^{ep\to eX} \;\sim\; L_{\rm e}^{\alpha\beta}\,\frac{g_{\alpha\mu}}{q^2}\,\frac{g_{\nu\beta}}{q^2}\,W_p^{\mu\nu}\,, \tag{8}$$

where $\frac{g_{\alpha\mu}}{q^2}\,\frac{g_{\nu\beta}}{q^2}$ represents the propagators of the exchanged photon in the Feynman gauge. One can decompose two propagators occurring in Eq. (8) by using the completeness relation (A.3). This leads straightforwardly to the factorization of the cross section for the considered process and, after some calculations, to the separation into two parts related to $\gamma_{\rm L}^*$ and $\gamma_{\rm T}^*$. This method is especially useful in analysing the semi-inclusive processes[2], which we will discuss below.

2.2. Semi-inclusive process $ep \to e\gamma X$ (Compton process)

Let us now consider the semi-inclusive process $ep \to eaX$, assuming that all particles are unpolarized. Here, in comparison to the DIS process $ep \to eX$, one additional particle a is produced. In the following we choose as particular final state a prompt photon (*i.e.* $a = \gamma$), with four-momentum p.

We will study the factorization of the cross section for this process, limiting ourselves to the case in which the γ is emitted from the hadronic part of the diagram only (Fig. 2). Of course, the final photon can be emitted also from the electron line – a typical bremsstrahlung process also called the Bethe–Heitler process; a relevance of this contribution is discussed at the beginning of Section 4.

[2] In the case of the semi-inclusive processes, one can also use the first method, but then the explicit form of the hadronic vertex has to be known (see [3]).

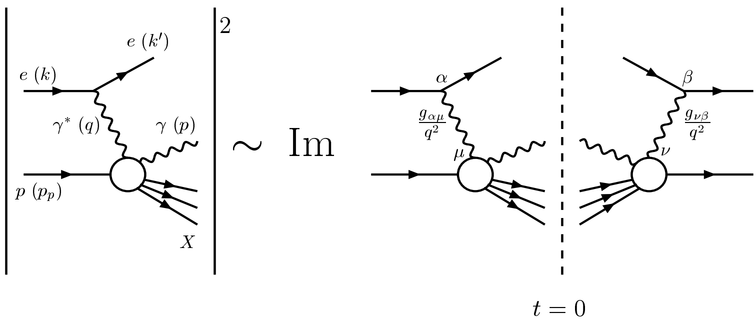

Fig. 2. Kinematics and notation for the semi-inclusive process $ep \to e\gamma X$ with a one-photon exchange. The optical theorem which relates $\sigma(ep \to e\gamma X)$ to the imaginary part of the forward amplitude for the process $ep \xrightarrow{\gamma} ep$ ($t = 0$) is also shown.

The differential cross section for the unpolarized process

$$ep \to e\gamma X \tag{9}$$

can be written as for process (1), namely

$$d\sigma^{ep\to e\gamma X} \sim L_{\mathrm{e}}^{\alpha\beta}\frac{g_{\alpha\mu}}{q^2}\frac{g_{\nu\beta}}{q^2}T^{\mu\nu}\,, \tag{10}$$

where the corresponding hadronic tensor $T^{\mu\nu}(p_p, q, p)$ is introduced (*cf.* (8)). Here the hadronic tensor $T^{\mu\nu}(p_p, q, p)$ depends not only on the four-momenta of the intermediate photon q and of the proton p_p, but also on the four-momentum of the final photon p. New scaling variables appear here, *e.g.* $z_\gamma = p_p p/p_p q$. Using "the propagator decomposition method" one can obtain the factorization formula in which the interference between two different transverse-, and between the transverse- and the longitudinal-polarization states of the exchanged photon, denoted by TT and LT (or TL), may appear. We obtain:

$$d\sigma^{ep\to e\gamma X} = \sum_{T=T1}^{T2} \Gamma_{\mathrm{T}} d\sigma_{\mathrm{T}}^{\gamma^* p\to\gamma X} + \sum_{i,j=T1,T2;i\neq j} \Gamma_{ij} d\tau_{ij}^{\gamma^* p\to\gamma X} \tag{11}$$

$$+\Gamma_{\mathrm{L}} d\sigma_{\mathrm{L}}^{\gamma^* p\to\gamma X} + \sum_{T=T1}^{T2}\left(\Gamma_{\mathrm{TL}} d\tau_{\mathrm{TL}}^{\gamma^* p\to\gamma X} + \Gamma_{\mathrm{LT}} d\tau_{\mathrm{LT}}^{\gamma^* p\to\gamma X}\right)\,. \tag{12}$$

Below we will use the following short notation for the groups of contributions which appear in (11) and (12)[3]:

$$d\sigma^{ep\to e\gamma X} \equiv d\sigma_{\rm T} + d\tau_{\rm TT} + d\sigma_{\rm L} + d\tau_{\rm LT}\,. \tag{13}$$

We see that the cross section (13) for the considered process (9) contains $d\sigma_{\rm T}$, $d\sigma_{\rm L}$ and in addition two interference terms, $d\tau_{\rm TT}$ and $d\tau_{\rm LT}$ (see also [6]). These four terms are related by the optical theorem (see Fig. 2) to the corresponding amplitudes:

$$d\sigma_{\rm T} \sim \frac{1}{q^4}\sum_{T=T1}^{T2}\left[(\varepsilon_{\rm T}^*)_\mu L_{\rm e}^{\mu\nu}(\varepsilon_{\rm T})_\nu\right]\left[(\varepsilon_{\rm T})_\mu\, T^{\mu\nu}(\varepsilon_{\rm T}^*)_\nu\right], \tag{14}$$

$$d\sigma_{\rm L} \sim \frac{1}{q^4}\left[(\varepsilon_{\rm L}^*)_\mu L_{\rm e}^{\mu\nu}(\varepsilon_{\rm L})_\nu\right]\left[(\varepsilon_{\rm L})_\mu T^{\mu\nu}(\varepsilon_{\rm L}^*)_\nu\right], \tag{15}$$

$$d\tau_{\rm TT} \sim \frac{1}{q^4}\sum_{i,j=T1,T2;i\neq j}\left[(\varepsilon_i^*)_\mu L_{\rm e}^{\mu\nu}(\varepsilon_j)_\nu\right]\left[\,(\varepsilon_i)_\mu T^{\mu\nu}(\varepsilon_j^*)_\nu\right], \tag{16}$$

$$d\tau_{\rm LT} \sim \frac{1}{q^4}\left\{\sum_{T=T1,T2}\left[(\varepsilon_{\rm L}^*)_\mu L_{\rm e}^{\mu\nu}(\varepsilon_{\rm T})_\nu\right]\left[(\varepsilon_{\rm L})_\mu T^{\mu\nu}(\varepsilon_{\rm T}^*)_\nu\right]\right.$$
$$\left. + \sum_{T=T_1,T_2}\left[(\varepsilon_{\rm T}^*)_\mu L_{\rm e}^{\mu\nu}(\varepsilon_{\rm L})_\nu\right]\left[(\varepsilon_{\rm T})_\mu T^{\mu\nu}(\varepsilon_{\rm L}^*)_\nu\right]\right\}. \tag{17}$$

It is worth noticing that the decomposition of the differential cross section $d\sigma^{ep\to e\gamma X}$ into three components: $d\hat{\sigma}_{\rm T} = d\sigma_{\rm T} + d\tau_{\rm TT}$, $d\sigma_{\rm L}$ and $d\tau_{\rm LT}$, does not depend on the choice of the reference frame or of the basis for the polarization vectors. Note that in the differential cross section $d\sigma^{ep\to e\gamma X}$ there are two independent terms related to the longitudinal-polarization state of the virtual photon: $d\sigma_{\rm L}$ and $d\tau_{\rm LT}$.

Obviously the above factorization formula (13) holds for the semi-inclusive process with arbitrary final-state particle a.

3. Azimuthal-angle distribution for $ep \to e\gamma X$

In studies of the process $ep \to e\gamma X$ it is useful to consider the azimuthal-angle (ϕ) distribution. This angle is defined as the difference between the azimuthal angle of the final electron (ϕ_e) and that of the final photon γ (ϕ_γ):

$$\phi = \phi_e - \phi_\gamma\,. \tag{18}$$

[3] Although the symbols $\sigma_{\rm T}$ and $\sigma_{\rm L}$ have appeared already in (2) for other process (DIS), this should not lead to any confusion, as in the rest of the paper we consider only the semi-inclusive process (9).

In the special reference frames in which the momenta of the virtual photon and of the proton are antiparallel (for example in the Breit frame or in the $\gamma^* p$ center-of-mass frame) ϕ is the angle between the electron scattering-plane and the plane defined by the momenta of the exchanged γ^* and the final γ (Fig. 3). In such reference frames, the cross section

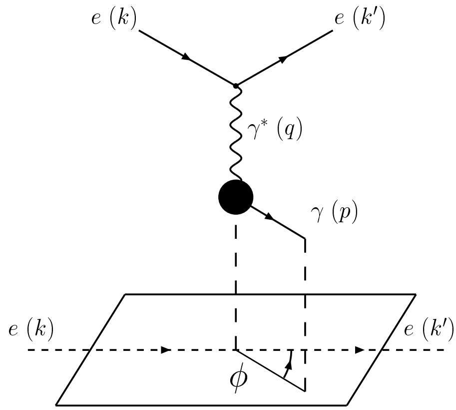

Fig. 3. The azimuthal angle ϕ defined in the Breit frame for the Compton process $ep \to e\gamma X$.

$d\sigma^{ep\to e\gamma X}/d\phi$ is linear in $\cos\phi$, $\cos 2\phi$, $\sin\phi$ and $\sin 2\phi$. In the Born approximation the terms containing $\sin\phi$ and $\sin 2\phi$ vanish as a consequence of a time-reversal invariance, so the azimuthal-angle distribution reduces to the following form [28–35] (see also Appendix B):

$$\frac{d\sigma^{ep\to e\gamma X}}{d\phi} = \sigma_0 + \sigma_1 \cos\phi + \sigma_2 \cos 2\phi\,, \tag{19}$$

with independent on ϕ coefficients σ_0, σ_1 and σ_2. These coefficients calculated for instance in the Breit frame, are related to the terms $d\sigma_{\rm T}/d\phi$, $d\sigma_{\rm L}/d\phi$, $d\tau_{\rm LT}/d\phi$ and $d\tau_{\rm TT}/d\phi$, introduced in Sect. 2.2, calculated in the same reference frame. In the circular-polarization basis, where the polarization vectors correspond to the definite helicity states, the term $\sigma_2 \cos 2\phi$ arises from the interference between two different transverse-polarization states of the exchanged photon ($\sim d\tau_{\rm TT}$). The longitudinal-transverse interference ($\sim d\tau_{\rm LT}$) gives rise to the term $\sigma_1 \cos\phi$. The σ_0 consists of the sum of the cross sections with the intermediate photon polarized transversely and longitudinally ($\sim (d\sigma_{\rm L} + d\sigma_{\rm T})$). Obviously, since σ_1 and σ_2 (and σ_0) do not depend on ϕ, the contributions due to the interferences will disappear after integrating over ϕ.

The interference terms can be extracted in a straightforward way from the data for the azimuthal-angle distribution $d\sigma/d\phi$ in the frames with antiparallel virtual photon's and proton's momenta, for example in the Breit frame.

In other frames, *i.e.* in frames in which the momenta of the virtual photon and proton are not antiparallel, the dependence on the azimuthal angle is more complicated. Each of the four terms contributing to the differential cross section (see Eqs. (13)–(17)) consists of the part that does not depend on the azimuthal angle. Consequently the interference terms integrated over a ϕ give non-zero result. This important fact was mentioned in papers [7,8], where the Higgs boson production via WW or ZZ fusion in e^+e^- collisions has been studied.

4. Numerical results

We perform calculations of the cross sections for the unpolarized Compton process $ep \to e\gamma X$. We consider the one-photon exchange only; this approximation is correct for the virtuality of the intermediate photon $Q^2 \ll M_Z^2$, while for larger values of Q^2 the Z/W boson exchange should be included. Below we present predictions for HERA for the beam energies equal to: $E_e = 27.5$ GeV and $E_p = 920$ GeV.

We analyse, at the Born level, the emission of the prompt photon from the hadronic vertex, *i.e.* we consider the Compton subprocess: $\gamma^* q \to \gamma q$. In principle, the Bethe–Heitler (BH) process, where the prompt photons can be emitted from the electron line and the interference between the Compton and BH amplitudes, should also be included in the calculation [37–39]. Both latter contributions dominate the cross section in the electron–proton center-of-mass frame for the rapidity range of the final photon's $Y < 0$, while for greater rapidities the Compton process dominates [40]. We expect that our results should be reliable for positive values of the Y in the CM_{ep}.

Results presented below were obtained for two frames: the electron–proton center-of-mass frame (CM_{ep}) and the Breit frame. The transverse momentum p_{T} of the final photon is defined as perpendicular to the direction of the electron in the ep collision in the CM_{ep} frame or to the direction of the $\gamma^* p$ collision in the Breit frame.

For the quark density in the proton $q^p(x, \tilde{Q}^2)$ we use the CTEQ5L parton parametrization [36] with a fixed number of flavors: $N_f = 4$. This parametrization imposes the limit on a range of a hard scale $\tilde{Q}^2$: it has to be greater than 1 GeV and less than 10^4 GeV. In our calculations we use as a hard scale $\tilde{Q}$ the transverse momentum of the final photon p_{T}; effects of other choices of the hard scale were studied by us, and are briefly discussed below.

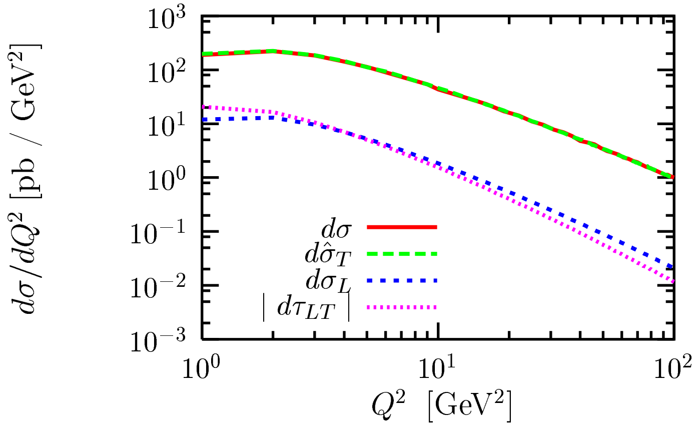

Fig. 4. Various contributions to the differential cross section $d\sigma/dQ^2$ in the CM_{ep} frame for the prompt-photon production ($(p_\mathrm{T})_\mathrm{min} = 1$ GeV) at HERA. The contribution due to the transverse-polarization states: $d\hat{\sigma}_\mathrm{T} = d\sigma_\mathrm{T} + d\tau_\mathrm{TT}$ (long-dashed line), the cross section for the longitudinally polarized intermediate photon: $d\sigma_\mathrm{L}$ (short-dashed line) and the absolute value of the interference term: $|d\tau_\mathrm{LT}|$ (dotted line), together with the sum of all terms $d\sigma$ (solid line), are shown.

First we consider the differential cross section $d\sigma/dQ^2$ in the CM_{ep} frame, integrated over p_T from $(p_\mathrm{T})_\mathrm{min} = 1$ GeV to $(p_\mathrm{T})_\mathrm{max} = \sqrt{S_{ep}}/2$, as a function of Q^2. We consider separately four frame-independent contributions: $d\hat{\sigma}_\mathrm{T} = d\sigma_\mathrm{T} + d\tau_\mathrm{TT}$, $d\sigma_\mathrm{L}$ and $|d\tau_\mathrm{LT}|$ as a function of Q^2. As it is presented in Fig. 4, the Q^2 dependence is roughly similar for all contributions. The differential cross section is dominated by the contribution due to the transversely polarized intermediate photon, *i.e.* $d\hat{\sigma}_\mathrm{T} = d\sigma_\mathrm{T} + d\tau_\mathrm{TT}$. The $d\sigma_\mathrm{L}$ contributes at the few per cent level to the whole cross section, similarly to the interference term $d\tau_\mathrm{LT}$. Moreover, $d\sigma_\mathrm{L}$ is positive while $d\tau_\mathrm{LT}$ negative, and these two contributions almost cancel one another. The resulting cross section $d\sigma/dQ^2$, even for large values of Q^2 (~ 100 GeV^2), is described with a high accuracy by the $d\hat{\sigma}_\mathrm{T}$ term only.

The ratio $[d\sigma_\mathrm{L}/dQ^2]$ / $[d\sigma_\mathrm{T}/dQ^2]$ as a function of virtuality Q^2 for the CM_{ep} frame and the Breit frame[4] is presented in Fig. 5. In the CM_{ep} frame it grows slowly with the virtuality of the intermediate photon up to $Q^2 \sim (p_\mathrm{T})^2_\mathrm{min}$, then this ratio slowly decreases. In the Breit frame the corresponding growth is much faster, the ratio reaches its maximum at much larger Q^2 (10 times $(p_\mathrm{T})_\mathrm{min}$). Also the maximum is higher (two times) in the Breit frame than in the CM_{ep} one. At $Q^2 = 100$ GeV^2 the considered ratio $[d\sigma_\mathrm{L}/dQ^2]$ / $[d\sigma_\mathrm{T}/dQ^2]$ is equal to about 0.02 for the CM_{ep} frame, while it is about four times larger for the Breit frame.

[4] The value of p_T changes when we are going from the CM_{ep} frame to the Breit frame.

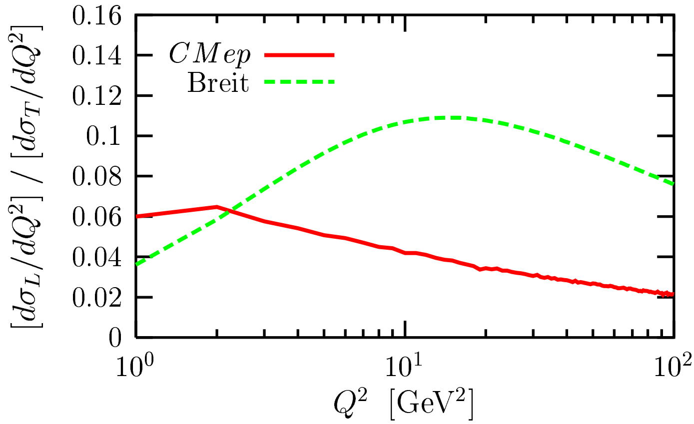

Fig. 5. The ratio $[d\sigma_{\rm L}/dQ^2]$ / $[d\sigma_{\rm T}/dQ^2]$ for prompt-photon production in the ep collision at HERA ($(p_{\rm T})_{\rm min}$ = 1 GeV), as a function of Q^2, in the CM$_{ep}$ frame (solid line) and in the Breit frame (dashed line), is shown.

Figure 6 shows results for the differential cross section $d\sigma/dY\,dp_{\rm T}$ in the CM$_{ep}$ frame. The $p_{\rm T}$ distribution for a fixed value of the photon rapidity, $Y = 0$, is shown in Fig. 6 (left), while the Y distribution at $p_{\rm T} = 5$ GeV is shown in Fig. 6 (right). We see that these distributions are described with a very good accuracy by the $d\hat{\sigma}_{\rm T}$ terms only. Contributions $d\sigma_{\rm L}$ and $d\tau_{\rm LT}$ are small and there is almost a cancellation between them. In other words, the measurements of such cross sections in the CM$_{ep}$ frame are not sensitive to the individual contributions involving the longitudinally polarized virtual photon: $d\sigma_{\rm L}$ and/or $d\tau_{\rm LT}$.

Another interesting and important fact is that, in the CM$_{ep}$ frame the interference term gives non-vanishing contributions even for the cross sections integrated over the azimuthal angle (defined by Eq. (18)), as it was supposed. This can be seen in the azimuthal-angle distribution for the interference term in this frame (Fig. 7). It is clear that the $d\tau_{\rm LT}$ integrated over ϕ gives a non-vanishing contribution. This is the main difference between the CM$_{ep}$ frame and the Breit frame (or other frames with antiparallel virtual photon and proton). In the latter frame the contributions due to the interference terms disappear in the cross sections integrated over ϕ, see below.

Finally, we study the azimuthal-angle distribution of the final photon in the Breit frame (19). As discussed in Sect. 3, in this frame the coefficients σ_1, σ_2 and σ_0 are independent of ϕ. They are directly related to the interference terms $d\tau_{\rm TT}/d\phi$, $d\tau_{\rm LT}/d\phi$ and to the sum: $d\sigma_{\rm L}/d\phi + d\sigma_{\rm T}/d\phi$, respectively. (For more details see also Appendix B.)

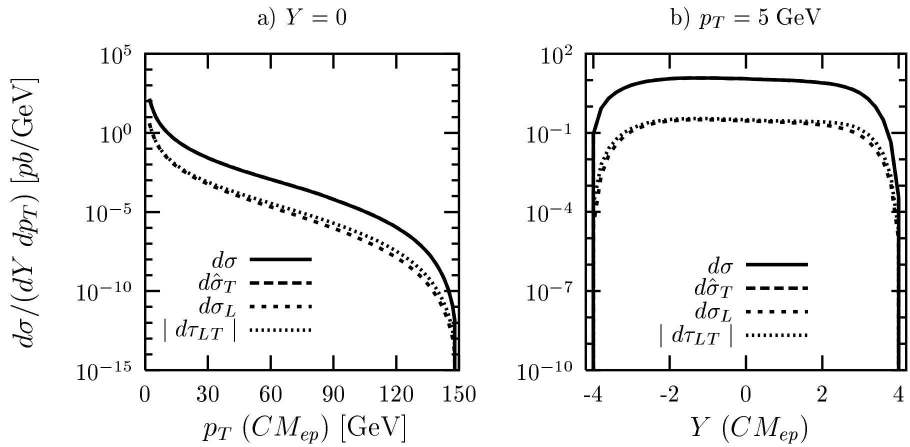

Fig. 6. Various contributions to the $d\sigma/(dY dp_{\mathrm{T}})$ in the CM_{ep} frame for high-p_{T} photon production at HERA ($(p_{\mathrm{T}})_{\min} = 1$ GeV) as a function of (a) p_{T} at $Y = 0$ and (b) Y at $p_{\mathrm{T}} = 5$ GeV. Same notation as in Fig. 4.

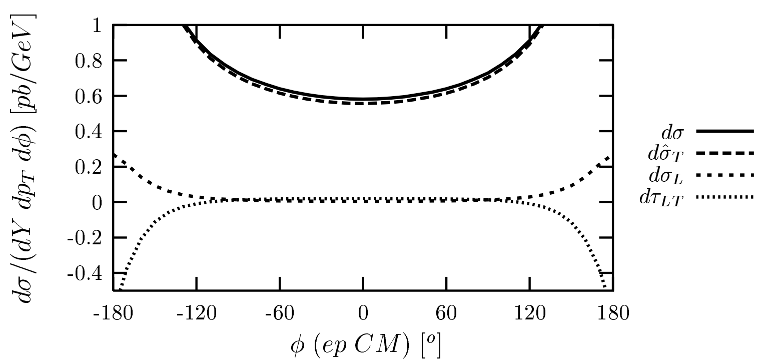

Fig. 7. The $d\hat{\sigma}_{\mathrm{T}} = d\sigma_{\mathrm{T}} + d\tau_{\mathrm{TT}}$, $d\sigma_{\mathrm{L}}$ and $d\tau_{\mathrm{LT}}$ contributions to the high-p_{T} photon production at HERA. Results for the azimuthal-angle distributions $d\sigma/(dY dp_{\mathrm{T}}\, d\phi)$ in the CM_{ep} frame, for $Y = 0$ and $p_{\mathrm{T}} = 5$ GeV.

The numerical calculations for the azimuthal-angle distribution for the Compton process in the Breit frame are performed for the same kinematical region in which the charged-hadrons production was measured in the ZEUS experiment at the HERA collider [41]: 180 $\mathrm{GeV}^2 < Q^2 < 7220\ \mathrm{GeV}^2$, $0.2 < y < 0.8$, $0.2 < z_\gamma < 1.0$ and the $(p_{\mathrm{T}})_{\min} = 2$ GeV. Results for the Compton process are presented in Fig. 8. The contribution related to the interference between two transverse polarization states of γ^* (the term proportional to $\sigma_2 \cos 2\phi$) gives a negligible effect, while the interference between the virtual photon polarized longitudinally and transversely (the term proportional to

$\sigma_1 \cos\phi$) leads to a visible effect (about 30%). Clearly, both interference contributions being symmetric under the $\phi \to -\phi$ transformation disappear after the integration over the ϕ angle over $-\pi$ to $+\pi$.

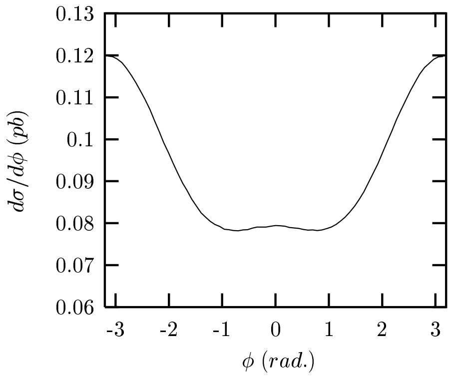

Fig. 8. The ϕ distributions in the Breit frame (see text) obtained for: 180 GeV^2 $< Q^2 <$ 7220 GeV^2, $0.2 < y < 0.8$, $0.2 < z_\gamma < 1.0$ and the $(p_{\rm T})_{\rm min} = 2$ GeV, for $ep \to e\gamma X$ at HERA.

Now we compare our results for the prompt-photon production with those obtained in the ZEUS experiment for the charged hadrons [41]. In the ZEUS data the term $\sigma_1 \cos\phi$ is clearly seen for all four considered values of $p_{\rm T}$ cut: $(p_{\rm T})_{\rm min}$ = 0.5, 1, 1.5 and 2 GeV, while the term $\sigma_2 \cos 2\phi$ gives a negligible effect for lower values of $(p_{\rm T})_{\rm min}$, becoming visible for a larger $(p_{\rm T})_{\rm min}$. This is different from the Compton process discussed above, where the corresponding σ_2 term is very small, even for $(p_{\rm T})_{\rm min} = 2$ GeV (see Fig. 8). This difference arises from the following fact: in the case of the Compton process $ep \to e\gamma X$, calculated in the Born approximation, there is only one subprocess $\gamma^* q \to \gamma q$ that contributes to the cross section. For the charged-hadrons production there are, at the Born level, two subprocesses, $\gamma^* q \to gq$ and $\gamma^* g \to q\bar{q}$. The second process $\gamma^* g \to q\bar{q}$ dominates in the $\sigma_2 \cos 2\phi$ term, while the contribution coming from the process $\gamma^* q \to gq$ (analogous to our $\gamma^* q \to \gamma q$ process) dominates in the $\sigma_1 \cos\phi$ term. Therefore both contributions can (and are) visible in the azimuthal-angle distribution of the charged-hadrons production, while in our case, based on one subprocess, the visible effect is expected only in $\sigma_1 \cos\phi$.

Finally, let us comment on the dependence of our results on a choice of the hard scale. Although all the cross sections presented above are computed for a hard scale equal to $p_{\rm T}$, one can choose for the considered process also other scales: *e.g.* Q^2 or $\sqrt{p_{\rm T}^2 + Q^2}$. We have checked by an explicit calculation that the cross sections obtained when the hard scale is equal to Q^2 or $\sqrt{p_T^2 + Q^2}$ are slightly larger (not more than 5%) than the ones obtained when the hard scale is equal to p_T. This difference is not significant and does not change our main conclusions.

5. Conclusions

In this paper we investigated the importance of the contributions due to the longitudinal virtual photon in unpolarized, semi-inclusive ep collisions at HERA. In general there are two such contributions: the cross section for $\gamma_L^* p$ collision $d\sigma_L$, and the interference term $d\tau_{LT}$. As a particular semi-inclusive process we chose the Compton process, where the prompt photon is emitted from the hadronic system. For the semi-inclusive process $ep \to e\gamma X$ at HERA energies we analysed three contributions: the cross section $d\sigma_L$, $d\tau_{LT}$ coming from the interference between the longitudinal- and the transverse-polarization states of γ^* and the contribution due to γ_T^*: $d\hat{\sigma}_T = d\sigma_T + d\tau_{TT}$. The calculations were performed in the Born approximation. In the ep center-of-mass frame we studied various distributions, the p_T and the rapidity distributions, as well as the Q^2 distribution, were the contributions mentioned simply add up. We found that both $d\sigma_L$ and $d\tau_{LT}$ are small and of similar size, below 10% of the cross section. This suggests that the contribution $d\sigma_L$ and the interference terms need be included on the same footing. Additionally, because of their opposite signs, they almost cancel in the cross section. This leads to a strong domination of the considered cross section by a contribution due to transversely polarized virtual photon, even for large values of its virtuality Q^2.

Although our results are based on the Compton process in a Born approximation only, we think that they already shed some light on the importance of contributions due to γ_L^* in other semi-inclusive processes, like the jet production in the DIS events at the HERA collider. It is clear that in order to reach a final conclusion on the importance of the contributions related to the γ_L^*, as advocated in some analysis, further studies are needed, with the incorporation of the relevant interference terms for consistency.

The studies of the azimuthal-angle dependence of $d\sigma^{ep\to e\gamma X}/d\phi$ in the Breit frame give access to the interference term $d\tau_{LT}$, as expected. Its effect is about 30%, showing in this case the importance of the γ_L^* contributions in the form of the interference $d\tau_{TL}$ term.

Finally, it is worth noticing that in the CM_{ep} frame the interference term gives non-vanishing contributions even for the cross sections integrated over the azimuthal angle, in contrast to the Breit-frame, where such contribution vanishes.

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Appendix A

Polarization vectors of γ^* and factorization formula for the Compton process

Below we present the polarization vectors of a virtual photon γ^* (with the four-momentum q^μ: $q^2 < 0$) in a form that is independent of the reference frame for the two types of basis for the polarization vectors: linear and circular.

A.1 Polarization vectors

The Lorentz condition for the polarization vectors gives rise to the physical observables

$$\varepsilon_\mu \, q^\mu = 0 \, . \tag{A.1}$$

Consequently, the scalar-polarization state of the γ^*, described by the following vector

$$\varepsilon^\mu_{\mathrm{S}} = \frac{q^\mu}{\sqrt{-q^2}} \, , \tag{A.2}$$

does not contribute to the physical observables. We can write the completeness relation as follows:

$$-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} = \sum_{T=T1}^{T2} (\varepsilon^*_{\mathrm{T}})^\mu \, (\varepsilon_{\mathrm{T}})^\nu \; - \; (\varepsilon^*_{\mathrm{L}})^\mu \, (\varepsilon_{\mathrm{L}})^\nu \, . \tag{A.3}$$

The four polarization vectors of the virtual photon satisfy also the orthonormality relation:

$$(\varepsilon^*_m)_\mu \, (\varepsilon_n)^\mu = \zeta_m \delta_{mn} \, , \qquad \text{where} \quad \zeta_{\mathrm{L}} = 1 \, , \quad \zeta_{\mathrm{S}} = \zeta_{\mathrm{T}} = -1 \, . \tag{A.4}$$

A.2 Linear polarization

The linear-polarization states are represented by the real polarization vectors. The longitudinal-polarization vector ($\varepsilon_{\rm L}^2 = 1$) for the electron–proton collisions depends on the four-momenta of the virtual photon (q^μ) and proton (p_p^μ) or quark ($p_q^\mu = x p_p^\mu$), (see also [6]):

$$\varepsilon_{\rm L}^\mu = \frac{(p_p q) q^\mu - q^2 p_p^\mu}{\sqrt{-q^2}\,(p_p q)}\,. \tag{A.5}$$

For the semi-inclusive processes we need to construct the two transverse-polarization vectors ($\varepsilon_{\rm T}^2 = -1$). In order to construct them we have to introduce the third four-momentum; for the Compton process we use the momentum of the final photon (p^μ) and obtain:

$$\varepsilon_1^\mu = \frac{-2}{\sqrt{stu}} \left[(p_p p) q^\mu - (p_p q) p^\mu + \frac{(qp)(p_p q) - q^2 (p_p p)}{(p_p q)}\, p_p^\mu \right] \tag{A.6}$$

and

$$\varepsilon_2^\mu = \frac{2}{\sqrt{stu}}\, \epsilon^{\mu\nu\alpha\beta}\, q_\nu (p_p)_\alpha p_\beta\,, \tag{A.7}$$

where $stu = 4[2(p_p q)(p_p p)(qp) - q^2 (p_p p)^2]$. This transverse-polarization vectors satisfy in addition the following equations:

$$\varepsilon_1 p_p = 0, \quad \varepsilon_1 p = 0 \qquad \text{and} \qquad \varepsilon_2 p_p = 0\,.$$

A.3 Circular polarization

Using the above forms of the linear-polarization vectors (Eqs. (A.5), (A.6), (A.7)), one can define the circular-polarization vectors of the γ^*, namely the longitudinal-polarization vector denoted by ε_0^μ and the transverse ones denoted by ε_+^μ and ε_-^μ. These vectors correspond to the helicities of the intermediate photon equal to $\lambda = 0, +$, and $-$, respectively. We get

$$\varepsilon_0^\mu = i\,\varepsilon_{\rm L}^\mu\,, \tag{A.8}$$

$$\varepsilon_+^\mu = \frac{1}{\sqrt{2}} (\varepsilon_1^\mu + i \varepsilon_2^\mu)\,, \tag{A.9}$$

and

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}} (\varepsilon_1^\mu - i \varepsilon_2^\mu)\,. \tag{A.10}$$

A.4 Factorization formula for the Compton process

The general factorization formula for the electron–quark scattering with the photon in the final state has the following form for massless quarks:

$$d\sigma^{eq\to eq\gamma}\sim$$
$$\mathrm{e}^6Q_q^4\mathrm{Re}\left[L_{e\alpha\beta}\frac{g^{\alpha\mu}g^{\nu\beta}}{Q^4}T_{\mu\nu}\right]=\frac{\mathrm{e}^6Q_q^4}{Q^4}\,\mathrm{Re}\left[\sum_{i,j}\zeta_{ij}\underbrace{\left(L_{e\,\alpha\beta}\varepsilon_i^{*\alpha}\varepsilon_j^{\beta}\right)}_{\equiv L_{ij}}\underbrace{\left(\varepsilon_j^{*\nu}\varepsilon_i^{\mu}T_{\mu\nu}\right)}_{\equiv T_{ji}}\right], \tag{A.11}$$

where i,j denote the polarization states of the virtual photon ($i,j = L,T1,T2$ for the linear polarization and $i,j=0,+,-$ for the circular polarization), $\zeta_{\rm LT}=\zeta_{\rm TL}=-1$, while in the remaining cases $\zeta_{ij}=1$. The tensor $L_{e\,\mu\nu}(q,k)$[5] is related to the emission of the virtual photon by the electron, while the tensor $T_{\mu\nu}(q,p_q,p)$ describes the absorption of the virtual photon by the quark (from proton), with the subsequent production of a prompt photon.

Two of the contributions to the cross section, $d\sigma_{\rm T}$ (14) and $d\tau_{\rm TT}$ (16), depend on the choice of the basis of the polarization states of γ^*. But the propagator decomposition method assures that their sum: $d\hat{\sigma}_{\rm T}=d\sigma_{\rm T}+d\tau_{\rm TT}$, as well as the individual terms $d\sigma_{\rm L}$ (15) and $d\tau_{\rm LT}$ (17) are the same in any basis. This fact can also be checked by using the relations between the linear and circular transverse-polarization vectors (Eqs. (A.9), (A.10)).

The calculations presented in this paper rely on the lowest order subprocess $\gamma^*q\to\gamma q$ (the Born approximation). The corresponding partonic tensor has the following form[6]:

$$\begin{aligned}
&T^{\mu\nu}(q,p_q,p)\\
&=\frac{2}{su}\{g^{\mu\nu}(u^2+s^2+2q^2t)+2u[2q^\mu q^\nu+q^\mu(p_q^\nu-p^\nu)+(p_q^\mu-p^\mu)q^\nu]\\
&-2s(p_q^\mu q^\nu+q^\mu p_q^\nu)+2q^2[4p_q^\mu p_q^\nu+2p^\mu p^\nu+q^\mu(2p_q^\nu-p^\nu)\\
&+(2p_q^\mu-p^\mu)q^\nu-2p_q^\mu p^\nu-2p^\mu p_q^\nu]\}\,,
\end{aligned}\tag{A.12}$$

where s, t, u are the Mandelstam variables for the process $\gamma^*q\to\gamma q$:

$$s=(q+p_q)^2\,,\qquad t=(q-p_\gamma)^2\,,\qquad u=(p_q-p_\gamma)^2\,. \tag{A.13}$$

[5] The leptonic tensor $L_{e\,\mu\nu}(q,k)$ is given by Eq. (4).

[6] The tensor for the process: $e\gamma^*\to e\gamma$ at the Born level with massive fermions was calculated in [42]. The hadronic tensor $T^{\mu\nu}$ obtained by us is in agreement with this tensor for the massless fermions.

Appendix B

The Breit frame

B.1 Kinematical variables

The special reference frame, called the Breit frame (see also [31]), is defined by choosing the momenta of the exchanged photon and the electrons in the following form:

$$q^\mu = \sqrt{Q^2}\,(0,\ 0,\ 0,\ 1)\,, \qquad -q^2 = Q^2\,; \tag{B.1}$$

$$k^\mu = \frac{1}{2}\sqrt{Q^2}(\cosh\psi, \sinh\psi\cos\phi_e, \sinh\psi\sin\phi_e,\ 1)\,, \qquad k^2 = 0\,; \tag{B.2}$$

$$k'^\mu = \frac{1}{2}\sqrt{Q^2}(\cosh\psi,\ \sinh\psi\cos\phi_e, \sinh\psi\sin\phi_e, -1)\,,\ k'^{\,2} = 0\,. \tag{B.3}$$

ϕ_e is the azimuthal angle of the scattered electron. The hyperbolic functions of the angle ψ are related to the variables $y = qp_p/kp_p$ as follows:

$$\cosh\psi = \frac{1}{y}\,(2-y)\,, \qquad \sinh\psi = \frac{2}{y}\sqrt{1-y}\,. \tag{B.4}$$

The momenta of the initial proton (p_p^μ), the initial quark (p_q^μ) and the final photon (p^μ) are given by:

$$p_p^\mu = (E_p,\ 0,\ 0,\ -E_p)\ , \qquad p_q^\mu = x\ p_p^\mu \tag{B.5}$$

and

$$p^\mu = p_{\mathrm{T}}\ \left(\frac{1}{\sin\theta_\gamma},\ \cos\phi_\gamma,\ \sin\phi_\gamma,\ \frac{\cos\theta_\gamma}{\sin\theta_\gamma}\right)\ . \tag{B.6}$$

where E_p is the energy of the initial proton[7], x the Bjorken scaling variable, p_{T} the transverse momentum of the prompt photon (perpendicular to the momenta of the initial proton), ϕ_γ the azimuthal angle of the γ and θ_γ — the polar angle of the final photon.

B.2 Polarization vectors of the γ^* in the Breit frame

The longitudinal polarization vector in the Breit frame has a very simple form:

$$\varepsilon_{\mathrm{L}}^\mu = -i\ \varepsilon_0^\mu = (\ 1,\ 0,\ 0,\ 0\ )\,. \tag{B.8}$$

[7] This is limited by the energy of the initial electron and S_{ep} in the ep centre-of-mass frame:

$$S_{ep} = (k+p_p)^2\,. \tag{B.7}$$

The transverse-polarization vectors also simplified in this frame, but they still depend on a choice of the basis. For the linear polarization we obtain:

$$\varepsilon_1^\mu = (0, \cos\phi_\gamma, \quad \sin\phi_\gamma, 0)\,, \tag{B.9}$$

$$\varepsilon_2^\mu = (0, -\sin\phi_\gamma, \quad \cos\phi_\gamma, 0)\,. \tag{B.10}$$

while for the circular polarization we have:

$$\varepsilon_+^\mu = \frac{1}{\sqrt{2}}(0, \cos\phi_\gamma - i\sin\phi_\gamma, \quad \sin\phi_\gamma + i\cos\phi_\gamma, 0)\,, \tag{B.11}$$

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0, \cos\phi_\gamma + i\sin\phi_\gamma, \quad \sin\phi_\gamma - i\cos\phi_\gamma, 0)\,. \tag{B.12}$$

B.3 Factorization formula

From the momenta and polarization vectors defined above one can obtain the explicit form of the coefficients L_{ij} and T_{ij} (defined in Eq. (A.11)). They can be treated as the elements of the matrices L and T, respectively (the ordering of the rows and columns is $T1, L, T2$ for the linear-polarization basis and $+, 0, -$ for the circular-polarization basis). These matrices calculated in the Born approximation (in the Breit frame) depend on the base considered:

- Linear polarization

$$\boldsymbol{L} = \frac{Q^2}{2}\begin{pmatrix} 2\sinh^2\psi\cos^2\phi + 2 & -\sinh 2\psi\cos\phi & \sinh^2\psi\sin 2\phi \\ -\sinh 2\psi\cos\phi & 2\sinh^2\psi & -\sinh 2\psi\sin\phi \\ \sinh^2\psi\sin 2\phi & -\sinh 2\psi\sin\phi & 2\sinh^2\psi\sin^2\phi + 2 \end{pmatrix} \tag{B.13}$$

and

$$\boldsymbol{T} = 2\frac{Q^2}{su}\begin{pmatrix} -U - 4p_{\mathrm{T}}^2 & -4p_{\mathrm{T}}\hat{E} & 0 \\ -4p_{\mathrm{T}}\hat{E} & U - 4(\hat{E}^2 + E_q^2) & 0 \\ 0 & 0 & -U \end{pmatrix}, \tag{B.14}$$

- Circular polarization

$$\boldsymbol{L} = \frac{Q^2}{2}\begin{pmatrix} \cosh^2\psi + 1 & -\frac{i}{\sqrt{2}}\sinh 2\psi\, \mathrm{e}^{-i\phi} & \sinh^2\psi\, \mathrm{e}^{-2i\phi} \\ \frac{i}{\sqrt{2}}\sinh 2\psi\, \mathrm{e}^{i\phi} & 2\sinh^2\psi & \frac{i}{\sqrt{2}}\sinh 2\psi\, \mathrm{e}^{-i\phi} \\ \sinh^2\psi\, \mathrm{e}^{2i\phi} & -\frac{i}{\sqrt{2}}\sinh 2\psi\, \mathrm{e}^{i\phi} & \cosh^2\psi + 1 \end{pmatrix} \tag{B.15}$$

and

$$\boldsymbol{T} = 2\frac{Q^2}{su}\begin{pmatrix} -U - 2p_{\mathrm{T}}^2 & -i2\sqrt{2}p_{\mathrm{T}}\hat{E} & -2p_{\mathrm{T}}^2 \\ i2\sqrt{2}p_{\mathrm{T}}\hat{E} & U - 4(\hat{E}^2 + E_q^2) & i2\sqrt{2}p_{\mathrm{T}}\hat{E} \\ -2p_{\mathrm{T}}^2 & -i2\sqrt{2}p_{\mathrm{T}}\hat{E} & -U - 2p_{\mathrm{T}}^2 \end{pmatrix}, \tag{B.16}$$

where $\hat{E} = E_q - E_\gamma$ ($E_q = xE_p$, $E_\gamma = \frac{p_{\mathrm{T}}}{\sin\theta\gamma}$), $U = \frac{u^2+s^2}{Q^2} - 2t$ and $\phi = \phi_e - \phi_\gamma$.

B.4 The azimuthal-angle distribution

In both bases the azimuthal-angle distribution for the Compton process (in the Born approximation) depends only on $\cos\phi$ and $\cos 2\phi$, namely:

$$\frac{d\sigma^{eq\to eq\gamma}}{d\phi} = \sigma_0 + \sigma_1 \cos\phi + \sigma_2 \cos 2\phi \,. \tag{B.17}$$

It is only for the circular-polarization basis that the coefficients σ_i, $(i = 0, 1, 2)$ in formula (B.17) are strictly related to the corresponding four contributions $d\sigma_{\mathrm{T}}$, $d\sigma_{\mathrm{L}}$, $d\tau_{\mathrm{LT}}$ and $d\tau_{\mathrm{TT}}$, calculated in the Breit frame:

$$\sigma_0 = \frac{d\sigma_{\mathrm{T}}}{d\phi} + \frac{d\sigma_{\mathrm{L}}}{d\phi} \sim -4\frac{1}{su}[(\cosh^2\psi + 1)\, p_{\mathrm{T}}^2 + 2\sinh^2\psi[\hat{E}^2 + E_q^2] + U]\,, \tag{B.18}$$

$$\sigma_1 \cos\phi = \frac{d\tau_{\mathrm{LT}}}{d\phi} \sim \frac{8}{su}\sinh 2\psi p_{\mathrm{T}} \hat{E}\cos\phi\,, \tag{B.19}$$

$$\sigma_2 \cos 2\phi = \frac{d\tau_{\mathrm{TT}}}{d\phi} \sim \frac{-4}{su}\sinh^2\psi p_{\mathrm{T}}^2 \cos 2\phi\,. \tag{B.20}$$

The longitudinal–transverse interference does not depend on the choice of the basis for the polarizaton vectors and consequently, it is related to the $\cos\phi$ term also for the linear-polarization vectors.

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