

CAN ONE DISENTANGLE HIGHER TWIST FROM SUDAKOV RESUMMATION AT HERA II?

R. G. ROBERTS

Rutherford Laboratory, Chilton, Didcot, Oxfordshire, OX11 0QX, UK

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Dedicated to Jan Kwieciński in honour of his 65th birthday

Measurements of the proton structure function at large x , low Q^2 display a strong Q^2 dependence which is usually interpreted as evidence for a significant higher twist contribution. Recent progress in understanding the resummation of large logs, $\ln(1-x)$, up to next-to-next-to-leading log order and beyond suggest that the observed enhancement may possibly be due to such terms alone. We study the implications for different theoretical scenarios in the light of novel suggestions for extracting large x information from future measurements at HERA.

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Much of our present understanding of the physics of deep inelastic scattering (DIS) at small x is the result of the pioneering work of Jan Kwiecinski. I personally have much reason to be grateful to Jan, since his deep knowledge and the ability to explain clearly and simply the theoretical issues has given me a basis to develop phenomenological studies with him and other collaborators. Much of that phenomenology was particularly relevant to HERA and here I wish to continue probing the connection between QCD and HERA physics but in the context of resumming potentially large logarithms, $\ln(1-x)$, rather than $\ln(1/x)$, however. That is, we are interested in the situation $W^2 \ll Q^2$ where $W^2 = Q^2(1-x)/x$ is the characteristic scale of the jet in the final state of DIS.

Recently there has been significant progress in our theoretical understanding of DIS as $x \rightarrow 1$ or, in moment space, as $N \rightarrow \infty$. In particular, Gardi *et al.*, [1] have conjectured that, in this limit, the dominant contribution at *each* (higher) twist is that part which mixes with the leading twist. Also in this limit the perturbative corrections factorise in a form describing the production of a single jet. The conjecture of Ref. [1] implies that large- x

factorisation, which is known to hold to all orders in perturbation theory, actually holds beyond the perturbative level. The dominant contributions at large x in any twist can be taken into account through a non-perturbative shape function of Q^2/N which multiplies the leading twist result in moment space.

Furthermore, as $x \rightarrow 1$, the coefficient functions for the twist-2 perturbative corrections describing the production of the single jet are dominated by Sudakov logarithms. In moment space we can expand the coefficient functions as

$$C_i(N, \alpha_s) \sim \sum_m \alpha_s (a_m \ln^{2m} N + b_m \ln^{2m-1} N + \dots), \quad (1)$$

which should be resummed to all orders in α_s to get a reliable estimate.

Varying the factorisation scale μ leads to mixing between twist-2 and twist-4 at the level of quadratic divergences, μ^2 , which introduces an ambiguity in the separation between the twist-2 and twist-4. This ambiguity is cancelled, however, by the contribution from infrared renormalons to the twist-2 coefficient functions, leaving the OPE of the structure function moments free of ambiguity. In Ref. [1], the assumption of “ultraviolet dominance” [2] implies the neglect of contributions to higher twist other than that which mixes with twist-2. This contribution is just that associated with the renormalons and thus any attempt to quantify the higher twist contribution requires resummation of the renormalons. At large x when powers of Λ^2/W^2 are not negligible *both* renormalons and Sudakov logs need to be resummed in the twist-2 coefficient function. The effect of this resummation can be numerically significant since the coefficients of sub-leading Sudakov logarithms are enhanced factorially with m due to infrared renormalons and grow increasingly singular as $W^2 \sim \Lambda^2$. Thus as $x \rightarrow 1$ all these enhanced log terms need to be resummed.

In the kernel of the Sudakov resummation, the first few orders are known from fixed-order calculations. In particular, we can practically compute the complete next-to-next-to-leading logarithmic (NNLL) correction as a result of the latest information of the anomalous dimension and coefficient function [3]. To go beyond this, one can use the “dressed gluon exponentiation” (DGE) approach of Gardi [4] which takes into account some all-order information on the kernel itself.

In Ref. [5], this combined resummation of Sudakov logarithms, renormalon contributions and higher twists was confronted by data on the moments of the structure function extracted from low Q^2 data from SLAC [6] and BCDMS [7]. The conclusion was that quite satisfactory descriptions of the high moments ($N \geq 5$) could be achieved provided the shapes (in x) of the leading order parton distributions are different from those suggested

by a leading twist pure NNLO analysis [8]. The latter excludes such high moment data since their Q^2 dependence clearly conflicts with standard LO, NLO or NNLO evolution.

While these phenomenological descriptions are successful, it is practically impossible to discriminate between solutions where the Q^2 dependence is driven primarily by the Sudakov resummation and solutions where there is a significant higher twist component. The latter require a smaller value of α_s in order to lessen the role of the resummation. In fact the value of α_s favoured by the former solutions is more in line with the value from NNLO analysis of DIS where the large x region is excluded. Therefore, the potential ability for HERA to explore the large x region and to extract reliable estimates of the $N = 5$ to 8 moments at high Q^2 could be a crucial factor in discriminating between such solutions. In any case the moments discussed in Ref. [5] rely, for $Q^2 \geq 20 \text{ GeV}^2$, on combining data from two experiments – a situation which can be avoided if HERA data only are used. Thus we could expect greater precision in estimating the moments even in the region below 100 GeV^2 .

In this context, a recent proposal by Helbich and Caldwell [9] addressing the question of measuring F_2 at HERA II at large x could, in principle, help to unravel the respective roles of the Sudakov resummation and higher twist contributions. Taking 1 fb^{-1} of data with 30% precision on x , when combined with data from HERA I where data exist up to $x = 0.4$ could yield reliable estimates of the $N = 5, 8$ moments. Note that we expect that for Q^2 around 500 GeV^2 , $\langle x \rangle \sim 0.62, 0.66$ for $N = 5, 8$, respectively, and so there should sufficient coverage in x to extract a reliable estimate.

From Ref. [1] the non-perturbative factorisation expression for the N -th moment of F_2 is

$$F_2^N(Q^2) = H(Q^2) J_N(Q^2; \mu^2) q_N(\mu^2) J^{\text{NP}} \left(\frac{N\Lambda^2}{Q^2} \right), \quad (2)$$

where $H(Q^2)$ describes the hard part of the coefficient function, $J_N(Q^2; \mu^2)$ is the Sudakov resummed jet function which depends on the jet mass W^2 , $q_N(\mu^2)$ is just the twist-2 quark matrix element and $J^{\text{NP}}(N\Lambda^2/Q^2)$ is the twist-2 non-perturbative shape function which, making the simplest ansatz that just the leading power appears in the exponent, can be written as

$$J^{\text{NP}} \left(\frac{N\Lambda^2}{Q^2} \right) = \exp \left[\frac{C_{\text{HT}} N}{Q^2} \right], \quad (3)$$

where C_{HT} is expected to be $O(\Lambda^2)$.

The logarithm of the resummed jet function can be written to fixed log accuracy as

$$\ln J_N(Q^2; \mu^2) = \sum_m g_m(\lambda) \alpha_s^{m-1}, \quad (4)$$

where $\lambda = \beta_0 \alpha_s \ln N / \pi$.

Since there are now available explicit expressions for $g_i(\lambda)$ for $i = 1, 3$ as result of fixed order calculations, one option is to stop at this exact NNLL result and use it to as an estimate for the resummed jet function. A more ambitious approach is to use a scheme invariant Borel representation, as discussed in Ref. [5]. In this way we can estimate subleading logs — beyond the NNLL, up to the minimal term in the series.

Figs. 1 and 2 show the comparisons for several types of fit with the data on moments $N = 5, 8$ extracted in Ref. [5] from structure function data of Refs. [6] and [7]. Notice that the data are very precise for $Q^2 < 15 \text{ GeV}^2$

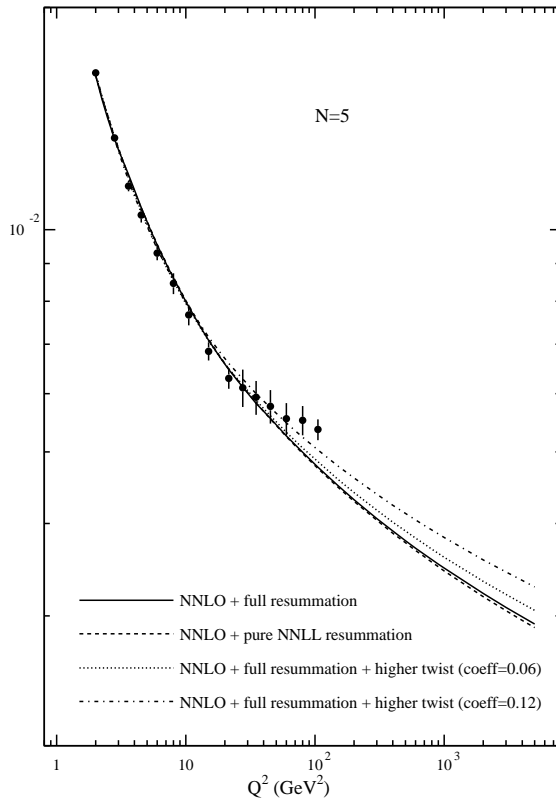


Fig. 1. The $N = 5$ moment of the structure function showing the comparisons with experimental values extracted in Ref. [5] together with four different theoretical descriptions. Note that the values of the parameters α_s , q_N and C_{HT} vary according to the individual fit.

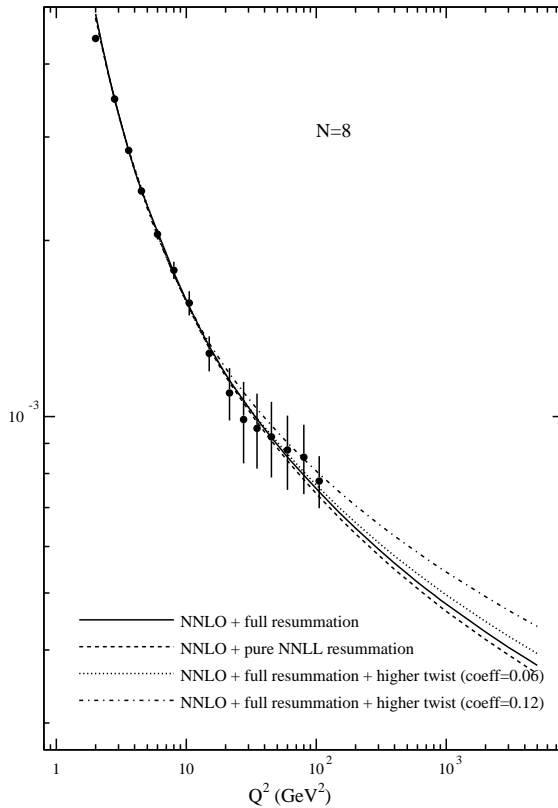


Fig. 2. The $N = 8$ moment of the structure function showing the comparisons with experimental values extracted in Ref. [5] together with four different theoretical descriptions. Note that the values of the parameters α_s , q_N and C_{HT} vary according to the individual fit.

since only the SLAC data are relevant there. For higher values of Q^2 the SLAC and BCDMS data have to be combined and the extrapolation to large x introduces further uncertainty. Thus measuring the large x data even for Q^2 around 100 GeV² at HERA would improve significantly the precision in this region. The curves in Figs. 1 and 2 all include resummation of the Sudakov logs, either just to NNLL or to NNLL and beyond. For two of the latter fits, a higher twist contribution as in Eq. (4) has been included.

In these fits, there are three pertinent parameters. First is the value of the strong coupling $\alpha_s(M_Z)$, second is the quark matrix element $q_N(\mu^2)$ and third the value of C_{HT} in Eq. (4). While each moment is separately fitted, the value of $\alpha_s(M_Z)$ obtained is virtually independent of N , in contrast to trying to fit the data with a pure NNLO description without any resummation of Sudakov logs [5]. For the fits with no higher twist, $\alpha_s(M_Z)$ is close

to 0.115 which is the value of the coupling obtained from fitting DIS and related data at NNLO [8]. For $C_{\text{HT}} = 0.06$, $\alpha_s(M_Z)$ drops to 0.111 and for $C_{\text{HT}} = 0.12$ drops even lower to 0.105. Therefore, it is harder to reconcile these latter fits into an overall description of DIS over the full range of x . Suppose one disregards any attempt to include resummation of Sudakov logs and tries to accommodate the data with NNLO evolution (with the accepted value of $\alpha_s(M_Z) = 0.115$) multiplied by a conventional higher twist correction term $(1 + C_{\text{HT}}N/Q^2)$ then the resulting curves are close to the uppermost curves in Figs. 1 and 2.

While the lower two curves are phenomenologically more consistent, it would be nice to have this confirmed by experiment. There is perhaps a hint of a “flattening off” in the high Q^2 $N = 5$ moments, which if true would be unexpected. As stated above there is a proposal to measure the structure function F_2 at large x at HERA II [9] and from Figs. 1 and 2 we can read off the typical precision needed to differentiate between the different descriptions. Unfortunately we see that the variation around $Q^2 = 500 \text{ GeV}^2$ is only of order 10% whereas the quoted precision in Ref. [9] is more like 30%. On the other hand we can say that the curves in Figs. 1 and 2 represent the spread of uncertainty of our understanding at large x and any sizeable deviation from those curves would indicate some new source of physics.

An extra source of information is recent data on the proton structure function in the resonance region. These data are from Jefferson Laboratory and allow the computation of moments for $Q^2 < 4.5 \text{ GeV}^2$ [10] giving even more precision in the low Q^2 region than from the SLAC experiments.

I am especially grateful to Einar Gardi, the work described here being the result of our joint collaboration. His patience in explaining the physics of the recent theoretical developments by him and his collaborators is deeply appreciated. I am grateful to the Leverhulme Trust for an Emeritus Fellowship.

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