DOUBLE LOGARITHMIC TERMS $\ln^2 x$ IN THE HEAVY QUARK PRODUCTION $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})$ CROSS SECTIONS

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Dedicated to Jan Kwieciński in honour of his 65th birthday

Predictions for the difference of proton–antiproton and proton–proton cross sections in the heavy quark production within LO DGLAP analysis together with the $\ln^2 x$ terms resummation are presented. An important role of the double logarithmic $\ln^2 x$ corrections in a case of the large rapidity gap between a quark and an antiquark is discussed.

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1. Introduction

Tevatron, LHC and SSC experiments enable to investigate a new kinematical region of hadron collisions, where the hadron center-of-mass energy $\sqrt{s}$ is much larger than the parton momentum transfer Q. Large enough scale Q^2 justifies the use of perturbative QCD (PQCD) and therefore hadron scattering data could be a test not only of the Regge theory (soft physics) but of different PQCD approaches *e.g.* DGLAP or BFKL too. The high energy hadronic processes depending upon the values of the momentum transfer squared Q^2 can be divided on the soft (with small Q^2 and small x involved) and the hard ones (at large $Q^2 > 1$ GeV^2). The high energy behaviour of the total hadronic cross sections in the Regge theory is described by two contributions: one is close $s^{0.1}$, the other is close $s^{-0.5}$. The first term $\sim s^{0.1}$

results from soft Pomeron exchange while the second one $\sim s^{-0.5}$ is identified as corresponding to ρ, ω, f_2 or a_2 exchange. Thus the Regge poles theory predicts a decrease of $\sigma^{\rm tot}(s)$ as $s \to \infty$. Experimental data show however that at $s \sim 100$ GeV^2 the total hadronic cross sections have rather a weak energy dependence *i.e.* they slowly (logarithmically $\sim \ln^2 s$) increase with energy at larger energies. On the other side PQCD predicts very rapid increase of parton distribution functions (PDFs) at small x (*i.e.* large s) region. In the leading $\ln(1/x)$ BFKL approach one obtains bare Pomeron, which causes that cross sections increase as a power of energy, what is the evident unitarity violation. Very interesting both experimentally and theoretically are so called semi-hard processes (SHP), which involve relatively large transverse momenta Q^2 ($Q^2 \gg \Lambda^2 \sim 0.04$ GeV^2) and soft parton collisions with low momentum fraction x ($x < 10^{-2}$) as well. In this way SHP open the description area in which the Regge theory and PQCD adjoin. Unfortunately, in this semihard regime there is a uncertainty in determination of the small x dependence of parton distributions. In order to avoid this uncertainty in predictions of jet production cross sections it is necessary to measure the two-jet inclusive cross section in hadron collisions with a rapidity gap between these two jets. In such processes the dominant part is an exchange of BFKL Pomeron between partons. Events with a rapidity gap *i.e.* a region of rapidity in which no particles are found occur if at the parton level the colour singlet ($q\bar{q}$ or gg) is exchanged in the t-channel. Though the Pomeron (gg) exchange mechanism is the leading one, the second contribution ($q\bar{q}$) coming from the mesonic Reggeons is interesting too. Analysis of the mesonic Reggeon exchange enable to test double logarithmic effects in PQCD. Elimination of the background originating from Pomeron, reggeized gluon and odderon in QCD analysis of hadron collisions gives possibility to determine the appropriate cross sections via valence quark–antiquark elastic scattering terms. The difference of the cross sections of $P\bar{P}$ and PP collisions incorporating $\ln^2 x$ approximation in valence quark distribution functions is not too small (≥ 100 pb) and can be measured *e.g.* in Tevatron and LHC.

In our paper we show predictions for the heavy quark production cross section in hadron collisions. Taking into account the difference $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})$ we are able to investigate the pure mesonic exchange picture modified by PQCD $\ln^2 x$ effects. In the next section we shall briefly recall the cross section formula for the production of heavy quarks in hadron–hadron collisions. We shall show how to eliminate all contributions except one originating from the mesonic Reggeon exchange. Then in point 3 we shall present PQCD modification of the Regge model approach in which the $\ln^2 x$ effects in valence quark distributions are taken into account. We calculate cross sections $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})$ and $d\sigma(P\bar{P} \to h\bar{h})/d\Delta y -$

$d\sigma(PP \to h\bar{h})/d\Delta y$ for heavy quarks charm and bottom production using GRV input parametrizations of valence quarks and including the $\ln^2 x$ effects into the DGLAP evolution of these distribution functions. All our numerical results we shall present in Section 4. We shall emphasize the role of the rapidity gap in our approach. Finally in conclusions we shall roughly discuss future experimental hopes for observation of $\ln^2 x$ effects in hadron–hadron collisions.

2. Production of heavy quarks in hadron–hadron collisions. Valence quark–antiquark elastic scattering as the only contribution to $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})$ cross section

The total cross section for heavy quark production in hadron–hadron collisions via factorization theorem can be written in the form [1]:

$$\sigma(AB \to h\bar{h}) = \sum_{i,j} \int\limits_{\frac{4m_h^2}{s}}^{1} dx_1 p_{i/A}(x_1, Q^2) \int\limits_{\frac{4m_h^2}{sx_1}}^{1} dx_2 p_{j/B}(x_2, Q^2) \int\limits_{t_{\min}}^{t_{\max}} \frac{d\hat{\sigma}}{d\hat{t}} d\hat{t}\,, \tag{2.1}$$

where A, B denote colliding hadrons, $h(\bar{h})$ is a heavy quark (antiquark) with mass m_h, i,j run over all partons *i.e.* quarks and gluons: $(i,j) = (g,g), (q_\alpha, \bar{q}_\alpha), (\bar{q}_\alpha, q_\alpha), \alpha = u,d,s$. $p_{i/A}(x, Q^2)$ is the parton of i sort (quark, antiquark or gluon) distribution in hadron A, $x_{1,2}$ are the longitudinal momenta fraction of partons with respect to their parent hadrons. Q^2 is the parton momentum transfer squared ($Q^2 \approx 4m_h^2$). $d\hat{\sigma}/d\hat{t}$ is the differential partonic cross section, which in a case of quark–antiquark subprocess in LO approximation has a form [2]:

$$\frac{d\hat{\sigma}(q\bar{q} \to h\bar{h})}{d\hat{t}} = \frac{4\pi\alpha_{\rm s}^2(Q^2)}{9\hat{s}^4}[(\hat{t} - m_h^2)^2 + (\hat{u} - m_h^2)^2 + 2m_h^2\hat{s}] \tag{2.2}$$

$\alpha_{\rm s}(Q^2)$ is the strong coupling constant; $\hat{s}, \hat{t}, \hat{u}$ are the well known internal Mandelstam kinematic variables [2] and

$$\hat{s} + \hat{t} + \hat{u} = 2m_h^2\,. \tag{2.3}$$

Hence (2.2) takes a form:

$$\frac{d\hat{\sigma}(q\bar{q} \to h\bar{h})}{d\hat{t}} = \frac{4\pi\alpha_{\rm s}^2(Q^2)}{9\hat{s}^4}(2m_h^4 + \hat{s}^2 + 2\hat{t}^2 + 2\hat{s}\hat{t} - 4m_h^2\hat{t})\,. \tag{2.4}$$

After integration with respect to $\hat{t}$ one can obtain the total partonic cross section, which in a case $q\bar{q}$ part has a form:

$$\hat{\sigma}^{(q\bar{q}\to h\bar{h})}(s,Q^2) = \int_{t_{\min}}^{t_{\max}} \frac{d\hat{\sigma}}{d\hat{t}} d\hat{t}\,. \tag{2.5}$$

The lower and upper limits in the above integral are:

$$t_{\min} = \frac{-\hat{s}}{4}(1+\beta)^2\,, \tag{2.6}$$

$$t_{\min} = \frac{-\hat{s}}{4}(1-\beta)^2\,, \tag{2.7}$$

with

$$\beta^2 = 1 - \frac{4m_h^2}{\hat{s}}\,, \tag{2.8}$$

and hence one can simply obtain:

$$\hat{\sigma}^{(q\bar{q}\to h\bar{h})}(s,Q^2) = \frac{4\pi\alpha_{\rm s}^2(Q^2)}{27\hat{s}}\beta(3-\beta^2)\,. \tag{2.9}$$

The total cross section for heavy quark production is of course dominated by the gluon-gluon $gg \to h\bar{h}$ subprocess and if we want to deal only with a part coming from the quark–antiquark scattering we should consider suitable difference of cross sections. As it has been exactly described in [3], the simplest way to eliminate the strong background due to the exchange of Pomeron, reggeized gluon and oderon is to regard $P\bar{P}$ (proton–antiproton) and PP (proton–proton) collisions. All these mentioned above backgrounds cancel if one takes the difference of the cross sections $\sigma(P\bar{P})-\sigma(PP)$. Then the difference of the $P\bar{P}$ and PP cross sections for the heavy quark production is purely represented by valence quark–antiquark elastic scattering (with mesonic Reggeon exchange in the Regge theory). One can write the differential cross section as [3]:

$$\frac{d\sigma(P\bar{P}\to h\bar{h})}{dx_1dx_2d\hat{t}} - \frac{d\sigma(PP\to h\bar{h})}{dx_1dx_2d\hat{t}} = \frac{1}{2}\sum_f \left[q_{f/P}(x_1,Q^2)\Delta\bar{q}_{f/P}(x_2,Q^2) + \bar{q}_{f/P}(x_1,Q^2)\Delta q_{f/P}(x_2,Q^2)\right]\frac{d\hat{\sigma}}{d\hat{t}} \tag{2.10}$$

with

$$\Delta q_{f/P}(x,Q^2) = q_{f/\bar{P}}(x,Q^2) - q_{f/P}(x,Q^2) \tag{2.11}$$

$$\Delta\bar{q}_{f/P}(x,Q^2) = \bar{q}_{f/\bar{P}}(x,Q^2) - \bar{q}_{f/P}(x,Q^2)\,. \tag{2.12}$$

By summation over flavours f in parent hadrons (P or $\bar{P}$) in (2.10) one gets:

$$\frac{d\sigma(P\bar{P}\to h\bar{h})}{dx_1dx_2d\hat{t}} - \frac{d\sigma(PP\to h\bar{h})}{dx_1dx_2d\hat{t}} = \frac{1}{2}\sum_{f=\mathrm{val}} q_{f/P}(x_1,Q^2)\bar{q}_{f/\bar{P}}(x_2,Q^2)\frac{d\hat{\sigma}}{d\hat{t}}\,. \tag{2.13}$$

Because

$$q_{\mathrm{val}/P}(x,Q^2) = \bar{q}_{\mathrm{val}/\bar{P}}(x,Q^2) \tag{2.14}$$

one has:

$$\frac{d\sigma(P\bar{P}\to h\bar{h})}{dx_1dx_2d\hat{t}} - \frac{d\sigma(PP\to h\bar{h})}{dx_1dx_2d\hat{t}} = \frac{1}{2}[u_{\mathrm{val}}(x_1,Q^2)u_{\mathrm{val}}(x_2,Q^2) + d_{\mathrm{val}}(x_1,Q^2)d_{\mathrm{val}}(x_2,Q^2)]\frac{d\hat{\sigma}}{d\hat{t}} \tag{2.15}$$

or after integration with respect to $\hat{t}$:

$$\frac{d\sigma(P\bar{P}\to h\bar{h})}{dx_1dx_2} - \frac{d\sigma(PP\to h\bar{h})}{dx_1dx_2} = \frac{2\pi\alpha_{\mathrm{s}}^2(Q^2)}{27\hat{s}}\beta(3-\beta^2) \times[u_{\mathrm{val}}(x_1,Q^2)u_{\mathrm{val}}(x_2,Q^2) + d_{\mathrm{val}}(x_1,Q^2)d_{\mathrm{val}}(x_2,Q^2)]\,, \tag{2.16}$$

where β^2 is defined by (2.8) and

$$Q^2 \approx 4m_h^2\,. \tag{2.17}$$

Then the difference of the total cross sections is:

$$[\sigma(P\bar{P}\to h\bar{h}) - \sigma(PP\to h\bar{h})](s) = \frac{2\pi\alpha_{\mathrm{s}}^2(Q^2)}{27}\int\limits_{\frac{4m_h^2}{s}}^{1} dx_1 \int\limits_{\frac{4m_h^2}{sx_1}}^{1} dx_2 \frac{\beta(3-\beta^2)}{\hat{s}} \times[u_{\mathrm{val}}(x_1,Q^2)u_{\mathrm{val}}(x_2,Q^2) + d_{\mathrm{val}}(x_1,Q^2)d_{\mathrm{val}}(x_2,Q^2)] \tag{2.18}$$

with internal kinematical variable $\hat{s}$:

$$\hat{s} = sx_1x_2\,. \tag{2.19}$$

In the formula (2.18) we have contributions from all possible rapidity gaps Δy between a quark and an antiquark:

$$\Delta y = y(h) - y(\bar{h}) \approx \ln\frac{\hat{s}}{4m_h^2}\,, \tag{2.20}$$

and hence

$$0 \le \Delta y \le \ln \frac{s}{4m_h^2}\,. \tag{2.21}$$

In a case of Tevatron with $\sqrt{s} = 1.8$TeV and for $c\bar{c}$ production with $m_c =$ 1.43 GeV theoretically

$$0 \le \Delta y \le 13\,. \tag{2.22}$$

However the interesting double logarithmic $\ln^2 x$ effects are important when the rapidity gap Δy is large:

$$(\Delta y)^2 = \ln^2 \frac{\hat{s}}{4m_h^2} \sim \ln^2 x\,. \tag{2.23}$$

To see this one should take into consideration the differential cross section:

$$\frac{d\sigma(P\bar{P} \to h\bar{h})}{d\Delta y} - \frac{d\sigma(PP \to h\bar{h})}{d\Delta y} = \frac{2\pi\alpha_{\mathrm{s}}^2(Q^2)}{27s} \int\limits_{\frac{4m_h^2 \mathrm{e}^{\Delta y}}{s}}^{1} \frac{dx_1}{x_1}\beta(3-\beta^2)$$
$$\times \left[u_{\mathrm{val}}(x_1,Q^2)u_{\mathrm{val}}\left(\frac{4m_h^2\mathrm{e}^{\Delta y}}{s},Q^2\right) + d_{\mathrm{val}}(x_1,Q^2)d_{\mathrm{val}}\left(\frac{4m_h^2\mathrm{e}^{\Delta y}}{s},Q^2\right)\right] \tag{2.24}$$

with β^2 as a function of Δy:

$$\beta^2 = 1 - e^{-\Delta y}\,. \tag{2.25}$$

From (2.25) one can simply notice that for a large Δy β^2 is close 1.0:

$$\beta^2 = 1 \quad as \quad \Delta y \longrightarrow \infty \tag{2.26}$$

and hence (2.24) for a large Δy can be expressed as:

$$\frac{d\sigma(P\bar{P} \to h\bar{h})}{d\Delta y} - \frac{d\sigma(PP \to h\bar{h})}{d\Delta y} = \frac{4\pi\alpha_{\mathrm{s}}^2(Q^2)}{27s} \int\limits_{\frac{4m_h^2 \mathrm{e}^{\Delta y}}{s}}^{1} \frac{dx_1}{x_1}$$
$$\times \left[u_{\mathrm{val}}(x_1,Q^2)u_{\mathrm{val}}\left(\frac{4m_h^2\mathrm{e}^{\Delta y}}{s},Q^2\right) + d_{\mathrm{val}}(x_1,Q^2)d_{\mathrm{val}}\left(\frac{4m_h^2\mathrm{e}^{\Delta y}}{s},Q^2\right)\right]. \tag{2.27}$$

According to (2.23), with larger rapidity gap Δy the $\ln^2 x$ effects in the $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})$ would be better visible. These effects we introduce via suitable modification in DGLAP LO evolution equations for valence quarks u_{val} and d_{val}. The perturbative QCD approach with an appearance of the double logarithmic terms $\ln^2 x$ can modify the Regge model. We discuss this in the next section.

3. Double logarithmic effects $\ln^2 x$ for flavour nonsinglet parton distribution functions

The high energy s limit corresponds by definition to the Regge limit. In the Regge pole model [4] an amplitude of high energy elastic scattering $T(s,t) \sim s^{\alpha(t)}$, where $\alpha(t)$ is the Regge trajectory, which is equal to spin J of the corresponding particle at $t = M^2$ (M is the mass of the exchanged particle). Hence the total interaction cross section, which by the optical theorem is connected to $\mathrm{Im}\, T(s,0)$ can be written as a sum of the Regge poles contributions:

$$\sigma^{\mathrm{tot}}(s) = \sum_k b_k(0) s^{\alpha_k(0)-1}\,, \tag{3.1}$$

where $\alpha_k(0)$ are the intercepts of the Regge trajectories and b_k denote the couplings. The high energy behaviour of the total hadronic cross section can be described by two parts: first contribution is so called soft Pomeron with intercept $\sim$ 1.08 and the second one are the leading meson Regge trajectories with intercept $\alpha_{\mathrm{R}}(0) \approx 0.5$. These Reggeons correspond to ρ, ω, f or a_2 mesons exchanges. The flavour nonsinglet part *e.g.* valence quark distribution functions or $F_2^{\mathrm{NS}} = F_2^P - F_2^N$ is governed at small x (*i.e.* at high energy $s = Q^2(1/x - 1)$) by a_2 Reggeon:

$$\text{Regge}: \quad q_{\mathrm{val}}(x) \sim x^{-\alpha_{a_2}(0)} \tag{3.2}$$

with $\alpha_{a_2}(0) \approx 0.5$. This behaviour is stable against the leading order DGLAP QCD evolution [5]. In other words the PQCD Q^2 evolution behaviour of q_{val} at high energy is screened by the leading Regge contribution (3.2). But there is a novel effect concerning the flavour nonsinglet functions which is an appearance of the double logarithmic terms $\ln^2 x$ [6,7]. For not too small values of the QCD coupling α_{s} this contribution is approximately the same (or even greater) in comparison to the contribution of the a_2 Regge pole:

$$\ln^2 x: \quad q_{\mathrm{val}}(x) \sim x^{-\bar{\omega}} \tag{3.3}$$

and

$$\bar{\omega} = 2\sqrt{\bar{\alpha}_{\mathrm{s}}}\,, \tag{3.4}$$

where

$$\bar{\alpha}_{\mathrm{s}} = \frac{2\alpha_{\mathrm{s}}}{3\pi}\,. \tag{3.5}$$

At a typical value of $\alpha_{\mathrm{s}}(Q^2 = 4m_h^2)$ for heavy quark production:

$$\alpha_{\mathrm{s}}(Q^2 = 4m_c^2) = 0.30\,, \tag{3.6}$$

$$\alpha_{\rm s}(Q^2 = 4m_b^2) = 0.21\,, \tag{3.7}$$

$\bar{\omega}$ becomes very close to α_{a_2}:

$$\bar{\omega}(\alpha_{\rm s}(Q^2 = 4m_c^2) \approx 0.5\,, \tag{3.8}$$

$$\bar{\omega}(\alpha_{\rm s}(Q^2 = 4m_b^2) \approx 0.4\,. \tag{3.9}$$

It must be however emphasized that even in the case of the PQCD singularity generated by the double logarithmic $\ln^2 x$ resummation, this $\ln^2 x$ behaviour of flavour nonsinglet parton distributions is usually hidden behind leading a_2 Reggeon exchange contribution. Only in the case of large rapidity gap Δy, where the intercept of the Regge trajectory $\alpha_{a_2}(0)$ becomes close to 0 or even negative, it is possible to separate from the Regge background the $\ln^2 x$ behaviour of the considered cross sections. In our analysis we calculate the difference of cross sections (2.18) and (2.24) using $u_{\rm val}(x,Q^2)$ and $d_{\rm val}(x,Q^2)$ obtained from DGLAP LO evolution equations, incorporating the double logarithmic effects $\ln^2 x$ as well. This method combining DGLAP LO and $\ln^2 x$ approaches in the case of polarized flavour nonsinglet functions was presented in [8–10]. For unpolarized ones, which we now investigate, the suitable equations are the same, only the input parametrizations $q_{\rm val}(x,k_0^2)$ are different. In our approach we expect that the cross sections (2.18) and particularly this for the large rapidity gap (2.24) are governed by $\ln^2 x$ terms *i.e.* by the valence quark ladder, shown in Fig. 1.

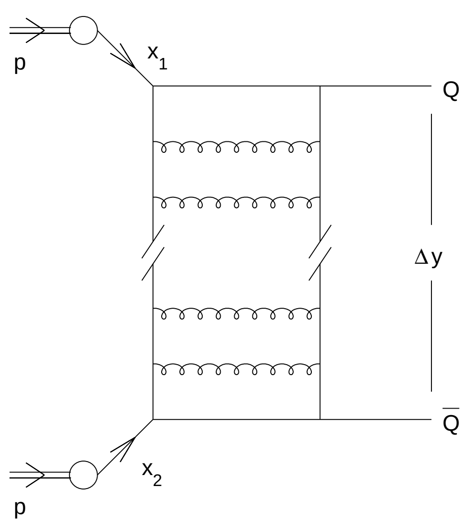

Fig. 1. A ladder diagram generating double logarithmic $\ln^2(1/x)$ terms in the flavour nonsinglet part of heavy quark production cross section with large rapidity gap Δy.

The equation taking into account both DGLAP LO evolution and $\ln^2 x$ effects for $q_{\rm val}(x,Q^2)$ functions has a form [9]:

$$
\begin{aligned}
f(x,k^2) \;=\; & f^{(0)}(x,k^2) + \frac{2\alpha_{\rm s}(k^2)}{3\pi}\int\limits_x^1 \frac{dz}{z}\int\limits_{k_0^2}^{k^2/z}\frac{dk'^2}{k'^2} f\left(\frac{x}{z},k'^2\right) \\
& + \frac{\alpha_{\rm s}(k^2)}{2\pi}\int\limits_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}\left[\frac{4}{3}\int\limits_x^1\frac{dz}{z}\frac{(z+z^2)f(x/z,k'^2)-2zf(x,k'^2)}{1-z}\right. \\
& \left. + \left(\frac{1}{2}+\frac{8}{3}\ln(1-x)\right) f(x,k'^2)\right] , \qquad (3.10)
\end{aligned}
$$

where

$$
\begin{aligned}
f^{(0)}(x,k^2) \;=\; & \frac{\alpha_{\rm s}(k^2)}{2\pi}\left[\frac{4}{3}\int\limits_x^1\frac{dz}{z}\frac{(1+z^2)q_{\rm val}^{(0)}(x/z)-2zq_{\rm val}^{(0)}(x)}{1-z}\right. \\
& \left. + \left(\frac{1}{2}+\frac{8}{3}\ln(1-x)\right) q_{\rm val}^{(0)}(x)\right] . \qquad (3.11)
\end{aligned}
$$

The unintegrated distribution f in the equation (3.10) are related to the $q_{\rm val}(x,Q^2)$ via

$$
q_{\rm val}(x,Q^2) = q_{\rm val}^{(0)}(x) + \int\limits_{k_0^2}^{Q^2(1/x-1)}\frac{dk^2}{k^2} f\left(x\left(1+\frac{k^2}{Q^2}\right),k^2\right) , \qquad (3.12)
$$

where

$$
q_{\rm val}^{(0)}(x) \equiv q_{\rm val}(x,k_0^2) = \int\limits_0^{k_0^2}\frac{dk^2}{k^2} f(x,k^2) \, . \qquad (3.13)
$$

In above equations $q_{\rm val}(x,Q^2)$ denotes $u_{\rm val}(x,Q^2)$ and $d_{\rm val}(x,Q^2)$ distributions as well. In our calculations we use LO fitted GRV [11] input parametrizations $q_{\rm val}(x,k_0^2)$:

$$
k_0^2 = 1 \text{ GeV}^2, \quad \Lambda(n_f=4) = 232 \text{ MeV} , \qquad (3.14)
$$

$$
\begin{aligned}
\text{GRV} \; : \; & u_{\rm val}(x,k_0^2=1 \text{ GeV}^2) = 2.872x^{-0.427} \\
& \times \; (1-0.583x^{0.175}+1.723x+3.435x^{3/2})(1-x)^{3.486} , \quad (3.15)
\end{aligned}
$$

$$\begin{aligned}\text{GRV} \; : \; & d_{\text{val}}(x, k_0^2 = 1 \text{ GeV}^2) = 0.448 x^{-0.624} \\ & \times \; (1 + 1.195 x^{0.529} + 6.164 x + 2.726 x^{3/2})(1-x)^{4.215} \, . \end{aligned} \quad (3.16)$$

The numerical results for (2.18) and (2.24) cross sections we present in the next point.

4. Numerical predictions for $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})(s)$ and $\frac{d\sigma(P\bar{P}\to h\bar{h})}{d\Delta y} - \frac{d\sigma(PP\to h\bar{h})}{d\Delta y}$ incorporating DGLAP LO evolution and the $\ln^2 x$ effects in q_{val} functions

Our numerical results based on GRV (3.14)–(3.16) input parametrizations are presented in Figs. 2–7 and in Table I. In Fig. 2 we plot xq_{val} input parametrizations for u and d valence quarks in proton at $k_0^2 = 1$ GeV^2 together with their LO DGLAP + $\ln^2 x$ predictions at $Q^2 = 10$ GeV^2. Figs. 3, 4 show the difference of total cross sections (2.18) which incorporates $\ln^2 x$ terms in $q_{\text{val}}(x, Q^2)$ functions. Our calculations concern the charm quark (Fig. 3) and the bottom quark (Fig. 4) production. In Fig. 5 we compare LO DGLAP + $\ln^2 x$ prediction for the cross section (2.24) with pure LO DGLAP one at large $\Delta y = 4.0$. In Figs. 6, 7 we plot the difference of differential cross sections (2.24) with LO DGLAP + $\ln^2 x$ $q_{\text{val}}(x, Q^2)$ for both charm and bottom production. The plots in Figs. 6, 7 are performed for different large rapidity gap $\Delta y \geq 2.0$. All plots in Figs. 3–7 are presented as a function of the center mass energy $\sqrt{s}$. Finally in Table I we present the differential cross section

$$\begin{aligned} x_1 x_2 \left[\frac{d\sigma(P\bar{P} \to h\bar{h})}{dx_1 dx_2} - \frac{d\sigma(PP \to h\bar{h})}{dx_1 dx_2} \right] = \frac{4\pi\alpha_s^2(Q^2)}{27s} \\ \times \left[u_{\text{val}}(x_1, Q^2) u_{\text{val}}(x_2, Q^2) + d_{\text{val}}(x_1, Q^2) d_{\text{val}}(x_2, Q^2) \right] \end{aligned} \quad (4.1)$$

for large rapidity gaps at different $\sqrt{s}$ and suitable $x_1 = x_2$:

$$x_1 = x_2 = \frac{2m_h}{\sqrt{s}} \mathrm{e}^{\frac{\Delta y}{2}} \, . \quad (4.2)$$

From Figs. 3, 4 one can read the shape of the cross sections difference $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})(s)$ as a function of $\sqrt{s}$. An initial increase of its value with the increasing value of $\sqrt{s}$ results from the energy threshold for heavy quark production:

$$s \geq \hat{s} \geq 4m_h^2 \, , \quad (4.3)$$

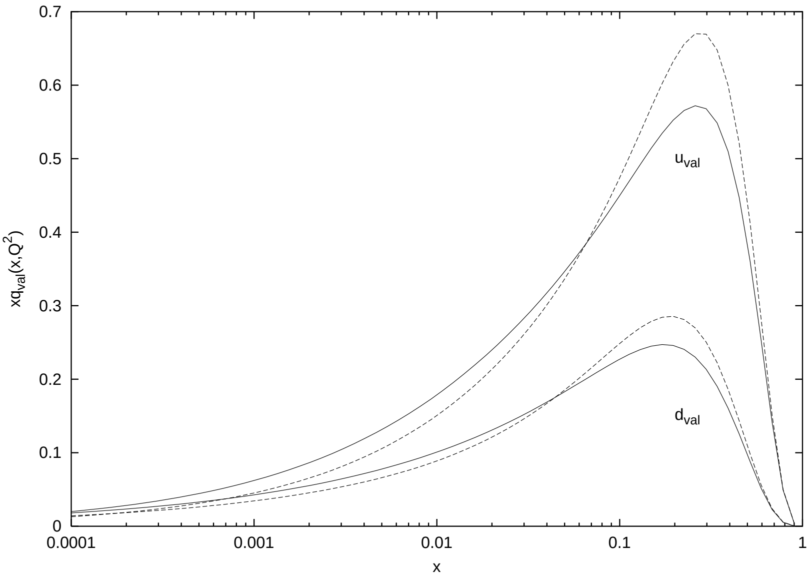

Fig. 2. Valence quark distribution functions $xu_{\rm val}$ and $xd_{\rm val}$ at input scale $k_0^2 = 1$ GeV^2 — dashed and at $Q^2 = 10$ GeV^2 — solid.

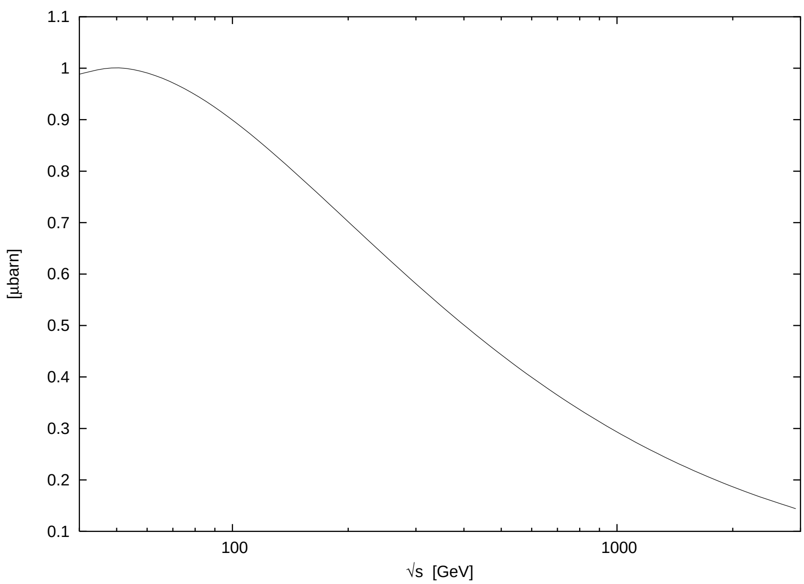

Fig. 3. Predictions for $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})(s)$ incorporating LO DGLAP $+ \ln^2 x \; q_{\rm val}(x, Q^2)$ for the charm quark production.

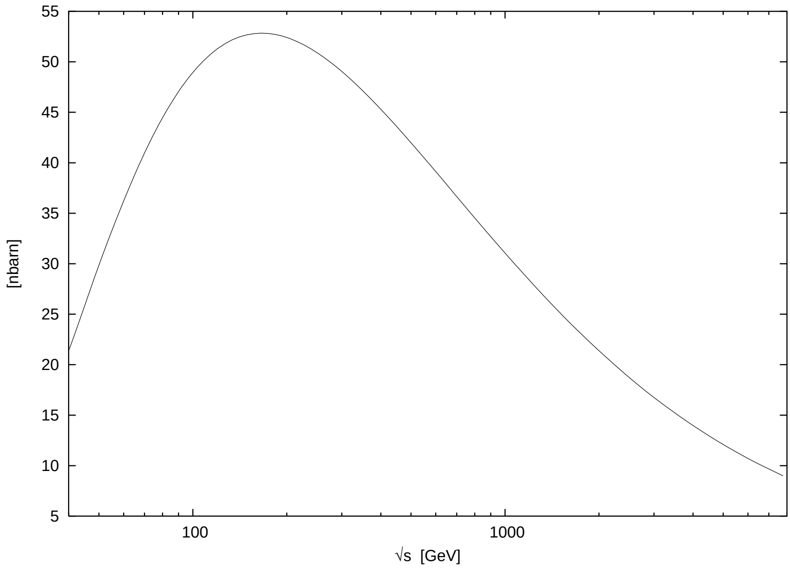

Fig. 4. Predictions for $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})(s)$ incorporating LO DGLAP + $\ln^2 x\; q_{\rm val}(x,Q^2)$ for the bottom quark production.

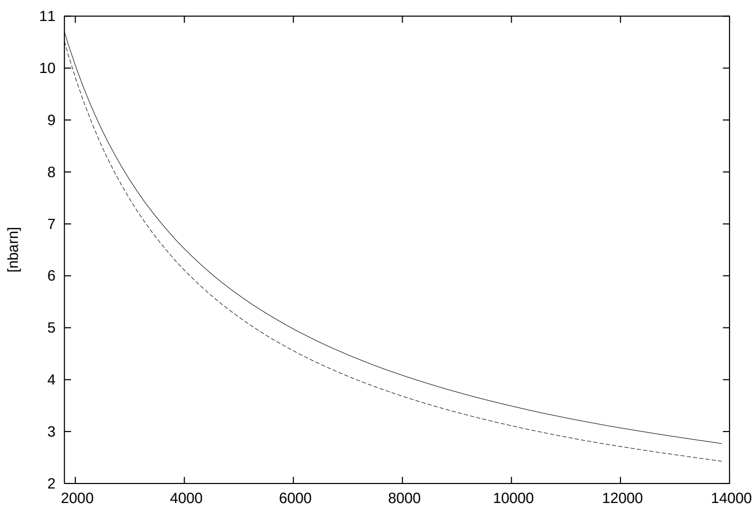

Fig. 5. Comparison of pure LO DGLAP predictions for $\frac{d}{d\Delta y}[\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})]$ — dashed with LO DGLAP + $\ln^2 x$ ones — solid at $\Delta y = 4.0$ for the charm quark production.

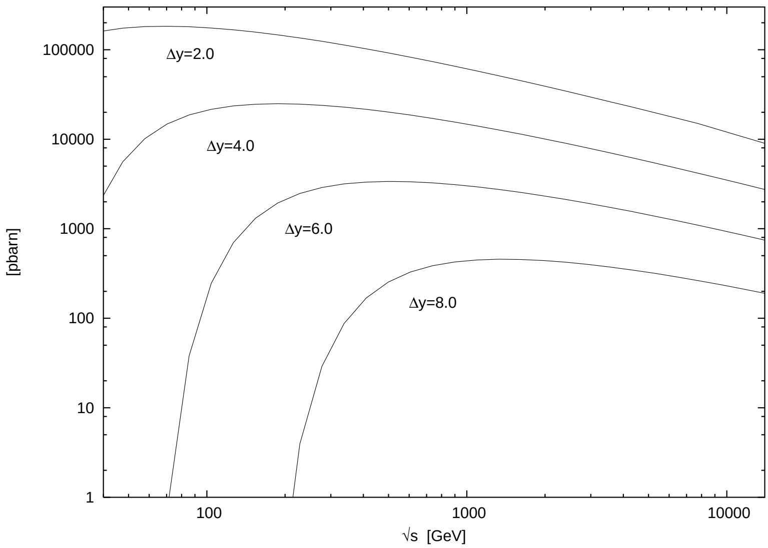

Fig. 6. Differential cross section $\frac{d}{d\Delta y}[\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})]$ incorporating LO DGLAP + $\ln^2 x\ q_{\rm val}(x, Q^2)$ for the charm quark production at different large Δy.

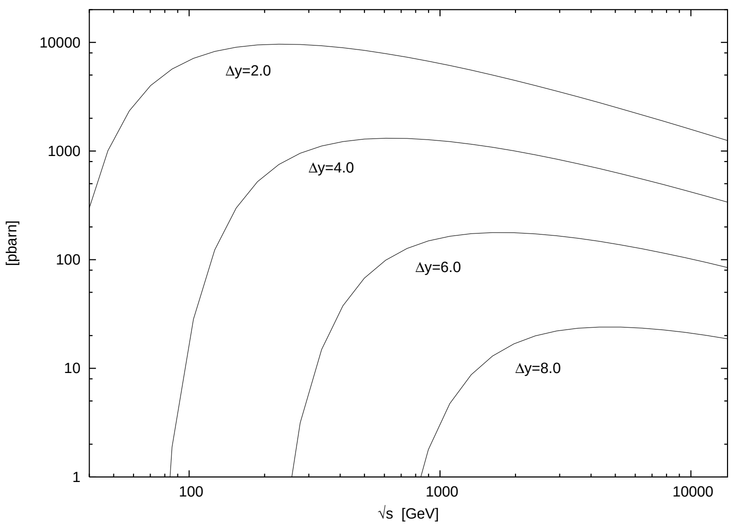

Fig. 7. Differential cross section $\frac{d}{d\Delta y}[\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})]$ incorporating LO DGLAP + $\ln^2 x\ q_{\rm val}(x, Q^2)$ for the bottom quark production at different large Δy.

TABLE I

Differential cross section $\mathrm{F} \equiv x_1 x_2 \left[\frac{d\sigma(P\bar{P}\to h\bar{h})}{dx_1 dx_2} - \frac{d\sigma(PP\to h\bar{h})}{dx_1 dx_2}\right]$ for a charm quark at large $\Delta y \geq 4.0$ and different $\sqrt{s}$.

$x_1 = x_2$	$\sqrt(s)$ [GeV]	Δy	F [pb]
0.54	39	4.0	3776
0.034	630	4.0	3811
0.091	630	6.0	1090
0.25	630	8.0	236
0.012	1800	4.0	1628
0.032	1800	6.0	496
0.087	1800	8.0	143
0.0015	14000	4.0	277
0.0041	14000	6.0	90
0.011	14000	8.0	29

where $\hat{s}$ is defined in (2.19). When the energy becomes larger ($\sqrt{s} > 50$ GeV for charm and $\sqrt{s} > 150$ GeV for bottom production) because of a suppression factor s^{-1} in the cross section formula (2.18), $\sigma(P\bar{P}) - \sigma(PP)$ decreases with the further increasing of energy $\sqrt{s}$. More interesting is analysis of the differential cross section $\frac{d\sigma(P\bar{P}\to h\bar{h})}{d\Delta y} - \frac{d\sigma(PP\to h\bar{h})}{d\Delta y}$. When the rapidity gap Δy between a quark and an antiquark defined in (2.20) is large, according to (2.23) high order corrections $\sim (\alpha_{\rm s} \ln^2 x)^n$ are important. It is shown in Fig. 5 where we compare the LO DGLAP + $\ln^2 x$ predictions for the cross section (2.24) with pure LO DGLAP ones as a function of $\sqrt{s}$ at large rapidity gap $\Delta y = 4.0$. In the high energy region $\sqrt{s} \in$ (1800 GeV; 14TeV) LO DGLAP + $\ln^2 x$ $q_{\rm val}$ give larger value of the cross section $\frac{d}{d\Delta y}[\sigma(P\bar{P}) - \sigma(PP)]$ than the pure LO DGLAP $q_{\rm val}$ approach. This results simply from larger values of $q_{\rm val}$ distribution functions in LO DGLAP + $\ln^2 x$ approach at small x ($x \leq 10^{-2}$) in comparison to those obtained via LO DGLAP (or even NLO) evolution method. However double logarithmic terms $\ln^2 x$ play a significant role not only in the very small x region. Indeed, for large Δy one deals with not too small x in the integrals (2.24) or (2.27) because the lower limit for x is

$$x_{\rm low} = \frac{4m_h^2}{s} {\rm e}^{\Delta y} \,. \tag{4.4}$$

In a case of a very large rapidity gap *e.g.* $\Delta y = 8.0$ at Tevatron energy $\sqrt{s} = 1800$ GeV and for $m_h = m_c$ one should know parton distributions at $x \geq 10^{-2}$ instead of $x \geq 3 \times 10^{-6}$ ($\Delta y = 0$). It could be very convenient because the very small x region is still experimentally unattainable. Large rapidity gap Δy implies however rapid decrease of suitable cross sections.

So one should minimize $\hat{s}$ which is a suppression factor ($\hat{s}^{-1}$) keeping still large enough $\Delta y \sim \ln \hat{s}$. Our plots in Figs. 6, 7 exhibit that for larger Δy the differential cross section $\frac{d}{d\Delta y}[\sigma(P\bar{P}) - \sigma(PP)]$ can change by almost 2 (3) orders of magnitude from 81 nb (8 nb) at $\sqrt{s} = 630$ GeV and $\Delta y = 2.0$ to 1.2 nb (10 pb) at $\Delta y = 7.0$ and the same energy in a charm (bottom) production. For one value of $\Delta y = 4.0$ we can find $\frac{d}{d\Delta y}[\sigma(P\bar{P}) - \sigma(PP)]$ for a charm (bottom) case in the range from 22 nb (28 pb) at $\sqrt{s} = 100$ GeV to 2.8 nb (340 pb) at $\sqrt{s} = 14$TeV. Table I shows values of double differential cross section $x_1 x_2 \left[\frac{d\sigma(P\bar{P}\to h\bar{h})}{dx_1 dx_2} - \frac{d\sigma(PP\to h\bar{h})}{dx_1 dx_2}\right]$ for a charm quark at large $\Delta y \geq 4.0$ and different $\sqrt{s}$. These DGLAP + $\ln^2 x$ predictions which are equal from 29 pb ($\Delta y = 8.0$, $\sqrt{s} = 14$TeV) to 3.8 nb ($\Delta y = 4.0$, $\sqrt{s} = 39$ GeV) as not too small should probably be measurable.

5. Summary

Difference of two cross sections $[\sigma(P\bar{P}) - \sigma(PP)]$ for heavy quark production is in the Regge theory represented by pure mesonic Reggeon exchange process. In this way only valence quarks and antiquarks are taken into account in this hadron–hadron collisions. Distribution functions of valence quarks $q_{\rm val}(x, Q^2)$ which via factorization theorem enter to a suitable formula for quark–antiquark elastic scattering in cross sections can be obtained within PQCD approach. In our paper we have used a perturbative method which incorporates LO DGLAP evolution and resummation of double logarithmic terms $\ln^2 x$ as well. Using the dynamical GRV input parametrization at $k_0^2 = 1$ GeV^2 we have found numerically in our LO DGLAP + $\ln^2 x$ approach $q_{\rm val}(x, Q^2)$ at higher scale $Q^2 \sim 4m_h^2$ and hence the cross section $\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})(s)$ and the differential cross section $\frac{d}{d\Delta y}[\sigma(P\bar{P} \to h\bar{h}) - \sigma(PP \to h\bar{h})]$ for the charm and bottom production. Very interesting is an analysis where the rapidity gap Δy between the quark and the antiquark is large. The large rapidity gap ($\Delta y > 2.0$) causes that double logarithmic terms $\ln^2 x \sim \Delta y^2$ become important. Processes with large rapidity gaps in high energy PP and $P\bar{P}$ collisions, observed at the Tevatron [12] have been theoretically investigated *e.g.* in [13–16]. The main reason that the large rapidity gap processes are so intensively studied is the opportunity to search for new particles *e.g.* Higgs bosons. Besides, the hard partonic processes with a rapidity gap between two jets in a final state enable examination of BFKL Pomeron behaviour [17, 18]. Even though the pure mesonic exchange process gives a small contribution to cross sections in high energy hadron–hadron collisions, it could be interesting because it enable the PQCD test in double logarithmic $\ln^2 x$ approximation. In our paper we have modified the mesonic Regge picture by PQCD LO DGLAP method combined with $\ln^2 x$ terms resummation. We have found that for the large rapidity gap Δy between a quark and an antiquark double logarithmic terms $\ln^2 x$ become significant. It must be however emphasized that in an unpolarized case it is very difficult to observe a dominant role of the $\ln^2 x$ contribution to the flavour nonsinglet part of heavy quark production cross sections. There are two reasons of this fact. First: $\ln^2 x$ terms in $q_{\rm val}(x, Q^2)$, which by definition are significant in the small x region ($x \leq 10^{-2}$) are partially hidden behind leading Regge $x^{\alpha_2(0)}$ behaviour of $q_{\rm val}$. Second: the quark–antiquark scattering cross section $d\hat{\sigma}/d\hat{t}$ is suppressed by a factor of $\hat{s}^{-2}$, so one should minimize $\hat{s}$ with a reasonably large rapidity gap $\Delta y = \ln(\hat{s}/4m^2)$. Larger Δy causes rapid decrease of the suitable cross sections, which however can be measured at Tevatron and LHC.

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REFERENCES

[1] J.C. Collins, D.E. Soper, G. Sterman, *Nucl. Phys.* **B308**, 833 (1988).

[2] F. Halzen, A.D. Martin, *Quarks and Leptons*, Wiley, New York 1984.

[3] W.-K. Tang, SLAC-PUB-6099, 1993.

[4] P.D.B. Collins, *An Introduction to Regge Theory and High Energy Physics*, Cambridge University Press, Cambridge 1977.

[5] A. Donnachie, P.V. Landshoff, *Phys. Lett.* **296B**, 257 (1992).

[6] J. Bartels, B.I. Ermolaev, M.G. Ryskin, *Z. Phys.* **C70**, 273 (1996); J. Bartels, B.I. Ermolaev, M.G. Ryskin, *Z. Phys.* **C72**, 627 (1996).

[7] J. Kwieciński, *Acta Phys. Pol. B* **27**, 893 (1996).

[8] B. Badełek, J. Kwieciński, *Phys. Lett.* **B418**, 229 (1998).

[9] J. Kwieciński, *Acta Phys. Pol. B* **29**, 1201 (1998).

[10] D. Kotlorz, *Acta Phys. Pol. B* **31**, 1721 (2000).

[11] M. Glück, E. Reya, A. Vogt, *Z. Phys.* **C67**, 433 (1995).

[12] D0 Collab., *Phys. Rev. Lett.* **72**, 2332 (1994), *Phys. Rev. Lett.* **76**, 734 (1996), *Phys. Lett.* **B440**, 189 (1998); CDF Collab., *Phys. Rev. Lett.* **74**, 855 (1995), *Phys. Rev. Lett.* **80**, 1156 (1998), *Phys. Rev. Lett.* **81**, 5278 (1998).

[13] J.D. Bjorken, *Int. J. Mod. Phys.* **A7**, 4189 (1992), *Phys. Rev.* **D45**, 4077 (1992), *Phys. Rev.* **D47**, 101 (1993).

[14] J. Pumplin, *Phys. Rev.* **D52**, 1477 (1995).

[15] A. Berera, J.C. Collins, *Nucl. Phys.* **B474**, 183 (1996).

[16] V. Del Duca, W.-K. Tang, *Phys. Lett.* **B312**, 225 (1993).

[17] A.H. Mueller, H. Navelet, *Nucl. Phys.* **B282**, 727 (1987).

[18] A.H. Mueller, W.-K. Tang, *Phys. Lett.* **B284**, 123 (1992).