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# THREE PAPERS ON MULTIPARTICLE PRODUCTION

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*Dedicated to Jan Kwieciński in honour of his 65th birthday*

In multiparticle production one has the sensation that problems are never solved. They come back again and again. Here I try to illustrate how it happens.

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## 1. Paper I (1987)

In 1987 Jan Kwiecinski, Mário Pimenta and myself wrote a paper on minijets and multiparticle distributions [1]. The idea was to use the occurrence of minijets to generate, via unitarity, significant changes in elastic scattering and inelastic production [2–7].

The model is a two component eikonal model where the (imaginary part of the) eikonal, $\Omega(b^2,s)$, $b$ being the impact parameter and $\sqrt{s}$ the centre of mass energy, is given by the sum of two terms,

$$\Omega(b^2,s)=\Omega_{\mathrm{S}}(b^2,s)+\Omega_{\mathrm{SH}}(b^2,s), \tag{1.1}$$

corresponding to the two driving interactions, the soft interaction and the semi-hard interaction. The inelastic cross section is then written as

$$\sigma_{\mathrm{in.}}(s)=\pi\int\{1-\exp[-2\Omega(b^2,s)]\}db^2. \tag{1.2}$$

The important piece (relatively new at the time) is $\Omega_{\mathrm{SH}}(b^2,s)$. Jan had, in fact, to explain to the two other authors of the paper what was it about: the semi-hard component is a fast rising with energy contribution (semi-hard Pomeron with a trajectory intercept $\alpha_0\simeq 1.35$ and vanishing slope, $\alpha'=0$), central in impact parameter. This means noticeable effects with increasing energy, such as large $|t|$ changes in the differential elastic cross-section [7] and additional large multiplicity contributions.

The main features of the model, regarding multiparticle production, are, essentially,

(1) A gamma function distribution at each impact parameter

$$\bar{n}(b^z, s)P(n, b^2, s) = k^k z^{k-1} \mathrm{e}^{-kz} / \Gamma(k)\,, \tag{1.3}$$

where $P(b^2, n, s)$ is the probability of emitting $n$ particles in a collision at energy $\sqrt{s}$ and impact parameter $b$, $\bar{n}(b^2, s)$ is the average multiplicity, and $z$ is the KNO variable, $z \equiv n/\bar{n}$. Note that the limits of the gamma function are, for $k \to \infty$, the $\delta$-function and, for $k \to 1$, the exponential. The gamma function parameter $k$ is treated as a function of $b^2$ and $s$: $k(b^2, s)$.

(2) The emission of particles is assumed to take place from independent sources. That requires the proportionality between $k$ and $\bar{n}$,

$$k(b^2, s) \sim \bar{n}(b^2, s)\,. \tag{1.4}$$

(3) The multiplicity $\bar{n}$ is approximately proportional to the average number of collisions,

$$\bar{n}(b^2, s) \sim \nu(b^2, s)\,, \tag{1.5}$$

with

$$\nu(b^2, s) \equiv \frac{2\Omega(b^2, s)}{1 - \exp[-2\Omega(b^2, s)]}\,. \tag{1.6}$$

(4) The mechanism of particle production is the same in soft and semi-hard processes, which implies, because of centrality, larger multiplicities in hard processes.

The model, adjusted to low energy data, $\sqrt{s} \simeq$ 20–900 GeV, gives predictions for the KNO moments $C_q$ of the multiparticle distribution,

$$C_q \equiv \frac{\langle n^q \rangle}{\langle n \rangle^q}\,, \tag{1.7}$$

and they are shown in the Table (see column `Paper 1`). The 1.8 TeV experimental $C_q$ moments — not existing in 1987 — do remarkably agree with the prediction. At LHC energies, $\sim$ 20 TeV, the model predicts very large values for the $C_q$ moments.

## 2. Paper II (1999)

In 1999 — more than 10 years after the paper with Jan and Mário — Roberto Ugoccioni and myself wrote a paper on the same subject: soft and semi-hard components in multiplicity distributions at TeV energies [8]. The idea was much simpler than in Paper I and was inspired in the work of [9]. Ref. [9] successfully describes the shoulder of the multiplicity distribution at 1.8 TeV and the oscillations of the $H_q$ moments.

In this model, the soft component and the semi-hard component are represented by two negative binomial distributions, $P_{\rm S}(n, \bar{n}_{\rm S}, k_{\rm S})$ and $P_{\rm SH}$ $(n, \bar{n}_{\rm SH}, \bar{k}_{\rm SH})$, respectively, where $\bar{n}$ and $k$ are the negative binomial parameters, and the multiplicity particle distribution is written as

$$P(n, \langle n\rangle, s) = \alpha P(n, \bar{n}_{\rm S}, k_{\rm S}) + (1 - \alpha_{\rm S}) P(n, \bar{n}_{\rm SH}, k_{\rm SH}) \,, \qquad (2.1)$$

with

$$\alpha_{\rm S} \equiv \frac{\sigma_{\rm S}}{\sigma_{\rm in.}} \,, \quad 1 - \alpha_{\rm S} \equiv \frac{\sigma_{\rm SH}}{\sigma_{\rm in.}} \,. \qquad (2.2)$$

The assumptions in this model are:

(1’) Just two components, one for the soft interaction and the other one for the hard interaction, with two weight factors $\alpha_{\rm S}$ and $(1 - \alpha_{\rm S})$ estimated from data. There is no attempt to unitarization.

(2’) The elementary collisions are independent, which implies

$$\frac{k_{\rm SH}}{\bar{n}_{\rm SH}} = \frac{k_{\rm S}}{\bar{n}_{\rm S}} \,. \qquad (2.3)$$

(3’) Particle production is of the same nature for the two components, and thus one expects the same kind of distribution: the negative binomial distribution.

(4’) As minijets are triggers for central collisions one naturally expects

$$\bar{n}_{\rm SH}(s) > \bar{n}_{\rm S}(s) \,. \qquad (2.4)$$

The $C_q$ moments obtained in this model are given in the Table $C_q$ (see column `Paper 2`):

If one compares (1’), (2’), (3’) and (4’) of Paper II to (1), (2), (3) and (4) of Paper I, the similarities appear quite clearly: emission from independent sources, same mechanism of particle production in soft and semi-hard interactions and effect of centrality. In Paper I the unitarization is explicit, in Paper II it is not so obvious.

$C_q$, KNO moments.

| $q$ | 1.8 TeV | | | 14 TeV | |
|---|---|---|---|---|---|
| | Paper 1 | Paper 2 | Data (E735) | Paper 1 | Paper 2 |
| 2 | 1.44 | 1.46 | $1.45 \pm 0.07$ | 2.68 | 1.34 |
| 3 | 2.68 | 2.72 | $2.70 \pm 0.18$ | 10.3 | 2.14 |
| 4 | 5.87 | 5.97 | $5.96 \pm 0.52$ | 45.0 | 3.92 |
| 5 | 14.2 | 14.7 | $14.9 \pm 1.6$ | 212.0 | 8.04 |

Let us come back to Paper I and write the probability of having an inelastic collision at a given impact parameter $b$:

$$P(b^2, s) = \frac{\pi}{\sigma_{\text{in.}}}\{1 - \exp[-2\Omega(b^2, s)]\}\,, \tag{2.5}$$

with $\Omega(b^2, s)$ given by (1.1). If one expands ( ) around the soft components one obtains:

$$\begin{aligned} P(b^2, s) \;=\; & \frac{\pi}{\sigma_{\text{in.}}}\{[1 - \exp[-2\Omega_{\text{S}}(b^2, s)]] + [\exp[-2\Omega_{\text{S}}(b^2, s)](2\Omega_{\text{SH}}(b^2, s))] \\ & -[\exp[-2\Omega_{\text{S}}(b^2, s)]\tfrac{1}{2}(2\Omega_{\text{S}}(b^2, s))^2] + \ldots\}\,. \end{aligned} \tag{2.6}$$

The first two (positive!) terms correspond precisely to the soft and the (absorbed) semi-hard contributions. While in Paper II as the energy increases one moves from the soft limit to the semi-hard one, in Paper I, as the energy increases, more and more terms of the expansion contribute.

From this difference it results that while in `Paper 1` the width of the KNO distribution, and in general the $C_q$ moments **increase** continuously with energy, in `Paper 2`, as the width of the soft distribution is larger that the width of the semi-hard one ($k_{\text{SH}} > k_{\text{S}}$), at some stage the width and, in general, the $C_q$ numbers start to **decrease** with energy.

In the Table we see that both models agree at Tevatron ($\sqrt{s} = 1.8$ TeV). However, at LHC ($\sqrt{s} \simeq 14$ TeV) the predictions are completely different!

## 3. Paper III (2003)

Who is right, who is wrong: Paper I or Paper II? In my opinion, they are both wrong! Paper II does not take into account the role of fluctuations in the number of collisions [10], or impact parameter fluctuations. Paper I does not take into account collective effects [11,12].

Multi-collision/impact parameter fluctuations is the reasonable way of explaining the growth with energy of the $C_q$ numbers (including the growth of the width of the KNO distribution), as done in Paper I. However the elementary collisions are treated as independent, without additional interactions. Or, in other words, saturation phenomena have to be taken into account — as Jan knows very well.

I shall now turn to the percolation approach to the problem and to Paper III, written in collaboration with Ugoccioni, Ferreiro and Pajares [13].

Multiparticle production is described as resulting from multiple collisions at the parton level and, in the case of nucleus–nucleus collisions, also at nucleon level, with formation of colour strings stretched between the projectile and the target, which decay into other strings that subsequently hadronize into the observed hadrons [14]. There are long strings in rapidity, valence strings, associated to valence quark (diquark) interactions, and short strings in rapidity, centrally produced (sea strings) associated to interactions of sea partons, mostly gluons. In a symmetrical AA collisions, with $N_{\rm A}$ participants from each nucleus, the number of valence strings equals the number of participants, as in the wounded nucleon model [15], while the number of sea strings, which is proportional to the number of collisions, behaves roughly as $N_{\rm s} \approx N_{\rm A}^{4/3}$ [16], increasing with the energy.

In [13] it was adopted as mechanism of particle production the Schwinger model mechanism as developed in [16,18]. In particular, the particle density and transverse momentum square will be considered proportional to the field (and the charge) carried by the string.

In multicollision models, many strings are produced, the number increasing with energy, atomic mass and centrality. If the strings are identical and independent, and approximately align with the collision axis, we have, for the rapidity particle density, $dn/dy$, and for the average of the square of the transverse momentum, $\langle p_{\rm T}^2 \rangle$,

$$\frac{dn}{dy} = N_{\rm s}\bar{n}_1\,, \tag{3.1}$$

$$\langle p_{\rm T}^2 \rangle = \overline{p_1^2}\,, \tag{3.2}$$

where $N_{\rm s}$ is the number of strings, $\bar{n}_1$ is the single string particle density and $\overline{p_1^2}$ the average transverse momentum squared of the single string. Eq. (3.1) is natural in Paper I.

If the strings fuse in a rope [17], the colour randomly grows as $\sqrt{N_{\rm s}}$ and we have

$$\frac{dn}{dy} = \frac{1}{\sqrt{N_{\rm s}}}N_{\rm s}\bar{n}_1\,, \tag{3.3}$$

$$\langle p_{\rm T}^2 \rangle = \overline{p_1^2}\,\sqrt{N_{\rm s}}\,. \tag{3.4}$$

In the situation of a hadron–hadron or nucleus–nucleus central collision, the strings overlap in the impact parameter plane and the problem becomes similar to a 2-dimensional continuum percolation problem [12]. If the strings are randomly distributed in the impact parameter plane then, in the thermodynamical approximation [19], the overlapping colour reducing factor is given by

$$F(\eta) = \sqrt{\frac{1-\mathrm{e}^{-\eta}}{\eta}}\,, \tag{3.5}$$

where $\eta$ is the transverse density percolation parameter,

$$\eta \equiv \left(\frac{r_{\mathrm{s}}}{R}\right)^2 N_{\mathrm{s}}\,, \tag{3.6}$$

where $\pi r_{\mathrm{s}}^2$ is the string transverse area and $\pi R^2$ the interaction transverse area. We thus have

$$\frac{dn}{dy} = F(\eta) N_{\mathrm{s}} \bar{n}_1\,, \tag{3.7}$$

$$\langle p_{\mathrm{T}}^2 \rangle = \frac{1}{F(\eta)} \overline{p_1^2}\,. \tag{3.8}$$

Equations similar to (3.7) and (3.8) were written in [19]. As with $\eta \to 0$ (low density limit) $F(\eta) \to 0$ and with $\eta \to \infty$ (high density limit) $F(\eta) \to 1/\sqrt{\eta}$, the behaviour of relations (3.1) and (3.2),and (3.3) and (3.4) is recovered from (3.7) and (3.8).

What are the consequences of (3.7) and (3.8)? Two straightforward results follow:

*(i) slow increase of particle density with energy and saturation of the normalised particle densities as $N_{\mathrm{s}}$ increases*

As the number of strings, $N_{\mathrm{s}}$, increases with energy, at large energy $\eta$ also increases and

$$F(\eta) \approx \frac{1}{\sqrt{\eta}}\,, \tag{3.9}$$

which means, (3.7),

$$\frac{dn}{dy} \approx \left(\frac{R}{r_{\mathrm{s}}}\right) N_{\mathrm{s}}^{1/2} \bar{n}_1\,. \tag{3.10}$$

Instead of growing with $N_{\mathrm{s}}$, as one should have naively expected with independent strings, (3.1), the density grows more slowly, as $N_{\mathrm{s}}^{1/2}$.

On the other hand, as

$$N_{\mathrm{s}} \approx N_{\mathrm{A}}^{4/3}, \qquad R \approx R_1 N_{\mathrm{A}}^{1/3}, \tag{3.11}$$

where $R_1$ is a quantity of the order of the nucleon radius,

$$\frac{1}{N_{\rm A}}\frac{dn}{dy} \approx \left(\frac{R_1}{r_{\rm s}}\right)\bar{n}_1 \tag{3.12}$$

tends to saturate as $N_{\rm A}$ increase. Both behaviours (3.10) and (3.12) were confirmed by data [20].

The saturation, in the framework of Paper III, is a consequence of string percolation. At the level of QCD it can be seen as resulting from low-$x$ parton saturation in the colliding nuclei [2].

*(ii) a universal relation between $dn/dy$ and $\langle p_{\rm T}\rangle$*

For large density, Eqs. (3.7) and (3.8) become

$$\frac{dn}{dy} = \left(\frac{R}{r_{\rm s}}\right) N_{\rm s}^{1/2}\bar{n}_1\,, \tag{3.13}$$

$$\langle p_{\rm T}^2\rangle = \left(\frac{r_{\rm s}}{R}\right) N_{\rm s}^{1/2}\overline{p_1^2}\,, \tag{3.14}$$

and, eliminating $N_{\rm s}^{1/2}$,

$$\sqrt{\langle p_{\rm T}^2\rangle} = c\sqrt{\frac{1}{N_{\rm A}^{2/3}}\frac{dn}{dy}}\,, \tag{3.15}$$

with

$$c \equiv \left(\frac{r_{\rm s}}{R_1}\right)\left(\frac{\overline{p_1^2}}{\bar{n}_1}\right)^{1/2}. \tag{3.16}$$

A relation of this type,

$$\sqrt{\langle p_{\rm T}^2\rangle} \approx \sqrt{\frac{1}{N_{\rm A}^{2/3}}\frac{dn}{dy}} \tag{3.17}$$

was obtained, in the framework of the Colour Glass Condensate (CGC) model [11], in [21]. Our formula (3.15) includes not only the functional dependence, but, as well, the proportionality factor $c$.

We can make an order of magnitude estimate of the proportionality factor $c$. In the dual string model $r_{\rm s} \approx 0.2$ fm [12,22], $R_1$ should be of the order of the proton radius ($\approx 1$ fm) and for the string charged particle production parameters one has $\bar{p}_1 \approx 0.3$ and $\bar{n}_1 \approx 0.7$, as observed from low energy data [23], and $(\overline{p_1^2}/\bar{n}_1)^{1/2} \approx 0.35$. The proportionality factor is then $\approx 0.07$ to be compared with 0.0348 for pions and 0.100 for kaons [21]. In the comparison with data we shall identify $\sqrt{\langle p_{\rm T}^2\rangle}$ with $\langle p_{\rm T}\rangle$ and $\sqrt{\overline{p_1^2}}$ with $\bar{p}_1$ (this overestimates the average values of $\langle p_{\rm T}\rangle$ and $\bar{p}_1$).

We have just considered the high $\eta$ limit. In the low density end, which means low energy and peripheral collisions, we have just valence strings and $\langle p_{\rm T}\rangle \to \bar{p}_1 \approx 0.3$ GeV. This is, in practice, the value of $\langle p_{\rm T}\rangle$ in $pp$ collisions at low ($\sqrt{s} \lesssim 10$ GeV) energies.

By putting these two limits together, we arrive at the formula obtained in [21], but now with all the parameters theoretically constrained:

$$\langle p_{\rm T}\rangle = \bar{p}_1 \left(1 + \frac{r_{\rm s}}{R}\frac{1}{\bar{n}_1^{1/2}}\sqrt{\frac{1}{N_{\rm A}^{2/3}}\frac{dn}{dy}}\right). \tag{3.18}$$

In Fig. 1 we compare Eq. (3.18) with data. The agreement is not perfect, but there is an indication that some truth exists in CGC and string percolation models.

In [13] an attempt is made to relate $p_{\rm T}$ distributions to multiplicity distributions.

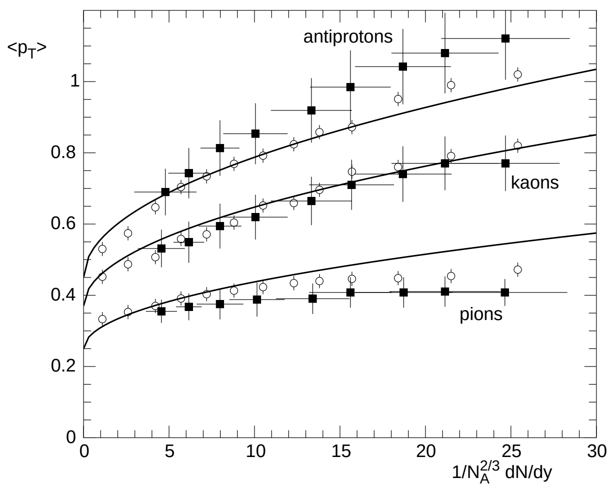


Fig. 1. $\langle p_{\rm T}\rangle$ *vs.* multiplicity density in p$\bar{\rm p}$ collisions (where $N_{\rm A} = 1$) at 1800 GeV [24] (open circles) and in central Au + Au collisions at 200 $A$GeV [25] (filled squares). Solid lines represent Eq. (3.18) with $\bar{p}_1$ adjusted separately to each species.

And what happens to the width of the KNO distribution, which was monotonically increasing with energy in `Paper 1` and decreasing at some stage in `Paper 2`? The parameter $k$ of the negative binomial distribution parametrization, increases with density (independent sources). In $pp$ collisions (see [26]), so far, $\eta$ is decreasing with the energy, as the increase of the proton radius compensates the slowest increase of the number of the strings. This brings back memories of the old "geometrical scaling" [27]. However we know, from SPS and Tevatron, that the parton density is increasing and cross-section seem to approach the Froissart limit. This requires that at some stage $\eta$ has to start increasing and, as a consequence, $k$ has to start increasing (as in `Paper 2`!). However, and contrary to `Paper 2`, this increase will not stop. Asymptotically, due to percolation, we shall end up with a single cluster fully covering the impact parameter plane.

This is an occasion to thank very, very much Jan for the kind help that for many years so generously he gave to me. Allow me also to thank all the other friends that collaborated in the three papers: Mário Pimenta, Roberto Ugoccioni, Elena Ferreiro and Carlos Pajares.

## REFERENCES

[1] J. Dias de Deus, J. Kwiecinski, M. Pimenta, *Phys. Lett.* **B202**, 397 (1988).

[2] L.V. Gribov, E.M. Levin, M.G. Ryskin, *Phys. Rep.* **100**, 1 (1983).

[3] T.V. Gaisser, F. Halzen, *Phys. Rev. Lett.* **54**, 1754 (1985); G. Pancheri, Y. Srivastava, *Phys. Lett.* **B159**, 1754 (1985).

[4] J. Dias de Deus, *Nucl. Phys.* **B252**, 369 (1985).

[5] L. Durand, Pi Hong, *Phys. Rev. Lett.* **58**, 303 (1985); A. Capella, J. Tran Thanh Van, J. Kwiecinski, *Phys. Rev. Lett.* **58**, 2015 (1987).

[6] W.R. Chen, R.C. Hwa, *Phys. Rev.* **D36**, 761 (1987).

[7] J. Dias de Deus, J. Kwiecinski, *Phys. Lett.* **B196**, 537 (1987).

[8] J. Dias de Deus, R. Ugoccioni, *Phys. Lett.* **B469**, 243 (1999).

[9] A. Giovannini, R. Ugoccioni, *Phys. Rev.* **D59**, 094020 (1999).

[10] J. Dias de Deus, C. Pajares, C.A. Salgado, *Phys. Lett.* **B407**, 299 (1997).

[11] L. McLerran, R. Venugopalan, *Phys. Rev.* **D49**, 2233 (1994); L. McLerran, R. Venugopalan, *Phys. Rev.* **D49**, 3352 (1994); Yu.V. Kovchegov, *Phys. Rev.* **D54**, 5463 (1996).

[12] N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares, *Phys. Rev. Lett.* **77**, 3736 (1996); M. Nardi, H. Satz, *Phys. Lett.* **B442**, 14 (1998).

[13] J. Dias de Deus, E.G. Ferreiro, C. Pajares, R. Ugoccioni, `hep-ph/0303220`.

[14] A. Capella, U.P. Sukhatme, C.I. Tan, J. Trân Thanh Vân, *Phys. Rep.* **236**, 225 (1994); N.S. Amelin, M.A. Braun, C. Pajares, *Phys. Lett.* **B306**, 312 (1993); N.S. Amelin, M.A. Braun, C. Pajares, *Z. Phys.* **C63**, 507 (1994).

[15] A. Białas, B. Bleszyński, W. Czyż, *Nucl. Phys.* **B111**, 461 (1976); C. Pajares, A. Ramallo, *Phys. Rev.* **D31**, 2800 (1985).

[16] N. Armesto, C. Pajares, *Int. J. Mod. Phys.* **A15**, 2019 (2000).

[17] J. Schwinger, *Phys. Rev.* **82**, 664 (1951); E. Brezin, C. Itzykson, *Phys. Rev.* **D2**, 1191 (1970).

[18] T.S. Biro, H.B. Nielsen, J. Knoll, *Nucl. Phys.* **B245**, 449 (1984); A. Bialas, W. Czyz, *Nucl. Phys.* **B267**, 242 (1986); A. Casher, H. Neuberger, S. Nussinov, *Phys. Rev.* **D20**, 179 (1979).

[19] M.A. Braun, F. Del Moral, C. Pajares, *Phys. Rev.* **C65**, 024907 (2002).

[20] J. Dias de Deus, R. Ugoccioni, *Phys. Lett.* **B491**, 253 (2000); J. Dias de Deus, R. Ugoccioni, *Phys. Lett.* **B494**, 53 (2000).

[21] L. McLerran, J. Schaffner-Bielich, *Phys. Lett.* **B514**, 29 (2001); J. Schaffner-Bielich, D. Kharzeev, L. McLerran, R. Venugopalan, *Nucl. Phys.* **A705**, 494 (2002).

[22] H. Satz, *Nucl. Phys.* **A642**, 130c (1998).

[23] G. Giacomelli, M. Jacob, *Phys. Rep.* **55**, 1 (1979).

[24] T. Alexopoulos *et al.*, (E735 Collaboration), *Phys. Rev.* **D48**, 984 (1993).

[25] G. Van Buren (STAR Collaboration), `nucl-ex/0211021`.

[26] J. Dias de Deus, E.G. Ferreiro, C. Pajares, R. Ugoccioni, Universality of the Transverse Momentum Distribution in the Framework of Percolation Strings, to be published.

[27] J. Dias de Deus, *Nucl. Phys.* **B59**, 231 (1973).