CP-VIOLATION IN THE NEUTRALINO SYSTEM

JAN KALINOWSKI

Institute of Theoretical Physics, University of Warsaw Hoża 69, 00-681 Warszawa, Poland

(Received April 24, 2003)

Dedicated to Jan Kwieciński in honour of his 65th birthday

Supersymmetric extensions of the Standard Model provide new sources of CP-violation. Here the CP properties of neutralinos are described and possible experimental signatures of CP-violation in the neutralino production processes at e^+e^- linear colliders are discussed.

PACS numbers: 11.30.Er, 12.60.Jv, 13.10.+q

1. Introduction

The electroweak sector of the Standard Model (SM) contains only one CP-violating phase which arises in the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix. Adding right-handed neutrinos to account for nonzero neutrino masses and their mixing opens up a possibility of new CPviolating phases in the Maki-Nakagawa-Sakata (MNS) lepton mixing matrix. In both cases unitarity imposes constraints on mixing matrices, which can be represented graphically as unitarity triangles. After many years of experimentation with the K and B mesons, the CKM unitarity triangle $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ is essentially reconstructed and the required amount of CP-violation can be accommodated within the SM. (Reconstructing MNS triangles will be far more difficult.) The CP-violation in the SM also induces electric dipole moments (EDM) of elementary particles. However, in the lepton sector¹ they are induced at a multiloop level and, as a result, EDM's generated by the CKM phase are extremely small [1] and beyond current and foreseeable future experiments. Therefore the observation of a lepton EDM would be a clear signal of new physics beyond the Standard Model.

¹ An additional CP-violation due to strong interactions of the form $\frac{\alpha_s}{8\pi}\theta G\tilde{G}$ could generate a very large neutron EDM, unless θ is tuned to be very small.

While the CP properties of the K and B systems appear to be consistent with the SM, another (indirect) piece of evidence of CP-violation, the baryon asymmetry in the universe, requires a new source of CP-violation beyond what is in the SM [2]. Thus new CP-violating phases must exist in nature.

Supersymmetric extensions of the SM based on soft supersymmetry breaking mechanism introduce a plethora of CP-phases. This poses a SUSY CP problem (assuming even that the strong CP is solved), since if the phases are large $\mathcal{O}(1)$, SUSY contributions to the lepton EDM can be too large to satisfy current experimental constraints [3]. Many models have been proposed [4] to overcome this problem: fine tune phases to be small, push sparticle spectra above a few TeV to suppress effects of large phases on the EDM, constrain phases present in the first two generations to be small, arrange for internal cancellations *etc*.

In the absence of any reliable theory that forces in a natural way the phases to be vanishing or small, it is mandatory to consider scenarios with some of the phases large and arranged consistent with experimental EDM data. In such scenarios many phenomena will be affected: sparticle masses, their decay rates and production cross sections, SUSY contributions to SM processes *etc.* It is one of the main physics goals of future collider experiments to find SUSY and verify its CP properties [5]. Detailed analyses of the neutralino sector can prove particularly useful in this respect. In this note we will discuss the CP properties of neutralinos strengthening an argument of Ref. [6] that measurements of the neutralino production cross sections can provide a qualitative, unambiguous evidence for non-trivial CP-phases.

2. Neutralino sector of the MSSM

In the minimal supersymmetric extension of the Standard Model (MSSM), the mass matrix of the spin-1/2 partners of the neutral gauge bosons, \tilde{B} and \tilde{W}^3 , and of the neutral Higgs bosons, \tilde{H}_1^0 and \tilde{H}_2^0 , takes the form

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix} .$$
(1)

Here M_1 and M_2 are the fundamental supersymmetry breaking parameters: the U(1) and SU(2) gaugino masses, and μ is the higgsino mass parameter. As a result of the electroweak symmetry breaking by the vacuum expectation values of the two neutral Higgs fields v_1 and v_2 ($s_\beta = \sin\beta$, $c_\beta = \cos\beta$ where $\tan\beta = v_2/v_1$), non-diagonal terms with $s_W = \sin\theta_W$ and $c_W =$ $\cos \theta_W$ appear and gauginos and higgsinos mix to form the neutralino mass eigenstates χ_i^0 (i=1,2,3,4).

In general the mass parameters M_1 , M_2 and μ in the mass matrix (1) can be complex. By reparameterization of the fields, M_2 can be taken real and positive; the two remaining non-trivial phases, which are therefore reparameterization-invariant, may be attributed to M_1 and μ :

$$M_1 = |M_1| e^{i\Phi_1}$$
 and $\mu = |\mu| e^{i\Phi_{\mu}} \quad (0 \le \Phi_1, \Phi_{\mu} < 2\pi).$ (2)

Since the existence of CP-violating phases in supersymmetric theories in general induces electric dipole moments (EDM), current experimental bounds can be exploited to derive indirect limits on the parameter space [3]. In fact the experimental limits on EDM's of the electron, neutron and mercury atom have been used to (partly) justify the assumption of real SUSY parameters, and most phenomenological studies on supersymmetric particle searches have been performed within the CP-conserving MSSM. However, the EDM constraints can be avoided assuming masses of the first and second generation sfermions large (above the TeV scale), or arranging cancellations between the different SUSY contributions to the EDMs. As a result, the complex phase of the higgsino mass parameter μ is much less restricted than previously assumed, while the complex phase of M_1 is practically unconstrained. The possibility of non-zero CP-phases should therefore be included in phenomenological analyses.

The neutralino mass eigenvalues $m_i \equiv m_{\tilde{\chi}_i^0}$ (i = 1, 2, 3, 4) can be chosen positive by a suitable definition of the unitary mixing matrix N. In general this matrix involves 6 angles and 10 phases, and can be written as [6,7]

$$N = \operatorname{diag} \left\{ e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3}, e^{i\alpha_4} \right\} R_{34} R_{24} R_{14} R_{23} R_{13} R_{12} , \qquad (3)$$

where \mathbf{R}_{jk} are rotations in the [jk] plane characterized by a mixing angle θ_{jk} and a (Dirac) phase β_{jk} . For example,

$$\mathbf{R}_{12} = \begin{pmatrix} c_{12} & s_{12}^* & 0 & 0\\ -s_{12} & c_{12} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}$$
(4)

with $c_{jk} \equiv \cos \theta_{jk}$, $s_{jk} \equiv \sin \theta_{jk} e^{i\beta_{jk}}$. One of (Majorana) phases α_i is nonphysical and, for example, α_1 may be chosen to vanish. None of the remaining 9 phases can be removed by rotating the fields since neutralinos are Majorana fermions.

Neutralino sector is CP conserving if μ and M_1 are real, which is equivalent to $\beta_{ij} = 0 \pmod{\pi}$ and $\alpha_i = 0 \pmod{\pi/2}$. Majorana phases $\alpha_i = \pm \pi/2$ indicate only different CP-parities of the neutralino states [8]. The matrix elements of N define the couplings of the mass eigenstates $\tilde{\chi}_i^0$ to other particles. Like in the quark sector, it is useful [6,9] to represent the unitarity constraints on the elements N_{ij}

$$M_{ij} = N_{i1}N_{j1}^* + N_{i2}N_{j2}^* + N_{i3}N_{j3}^* + N_{i4}N_{j4}^* = \delta_{ij}, \qquad (5)$$

$$D_{ij} = N_{1i}N_{1j}^* + N_{2i}N_{2j}^* + N_{3i}N_{3j}^* + N_{4i}N_{4j}^* = \delta_{ij}$$
(6)

in terms of unitarity quadrangles. For $i \neq j$, the above equations define the M- and D-type quadrangles in the complex plane. The M-type quadrangles are formed by the sides $N_{ik}N_{jk}^*$ connecting two rows i and j, and the D-type by $N_{ki}N_{kj}^*$ connecting two columns i and j of the mixing matrix. By a proper ordering of sides the quadrangles are assumed to be convex with areas given by

$$\operatorname{area}[M_{ij}] = \frac{1}{4} (|J_{ij}^{12}| + |J_{ij}^{23}| + |J_{ij}^{34}| + |J_{ij}^{41}|), \qquad (7)$$

$$\operatorname{area}[D_{ij}] = \frac{1}{4}(|J_{12}^{ij}| + |J_{23}^{ij}| + |J_{34}^{ij}| + |J_{41}^{ij}|), \qquad (8)$$

where J_{ij}^{kl} are the Jarlskog-type CP-odd "plaquettes" [10]

$$J_{ij}^{kl} = \Im m N_{ik} N_{jl} N_{jk}^* N_{il}^* \,. \tag{9}$$

Note that plaquettes, and therefore the areas of unitarity quadrangles, are not sensitive to the Majorana phases α_i .

Unlike in the quark or lepton sector, the orientation of all quadrangles is physically meaningful, and determined by the CP-phases of the neutralino mass matrix. In terms of quadrangles, CP is conserved if and only if all quadrangles have null area (collapse to lines or points) and are oriented along either real or imaginary axis.

3. Experimental signatures of CP-violation

In principle, the imaginary parts of the complex parameters involved could most directly and unambiguously be determined by measuring suitable CP-violating observables.

Neutralinos can copiously be produced at prospective e^+e^- linear colliders [5] via the *s*-channel *Z* exchange and *t*- and *u*-channel selectron exchange. The polarized differential cross section for the $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ production is given by [6]

$$\frac{\mathrm{d}\sigma^{\{ij\}}}{\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \frac{\alpha^2}{16\,s}\,\lambda^{1/2} \left[\left(1 - P_\mathrm{L}\bar{P}_\mathrm{L}\right)\Sigma_U + \left(P_\mathrm{L} - \bar{P}_\mathrm{L}\right)\Sigma_\mathrm{L} + P_\mathrm{T}\bar{P}_\mathrm{T}\cos(2\phi - \eta)\,\Sigma_\mathrm{T} + P_\mathrm{T}\bar{P}_\mathrm{T}\sin(2\phi - \eta)\,\Sigma_N \right], \quad (10)$$

where $P = (P_{\rm T}, 0, P_{\rm L})$ $[\bar{P} = (\bar{P}_{\rm T} \cos \eta, \bar{P}_{\rm T} \sin \eta, -\bar{P}_{\rm L})]$ is the electron [positron] polarization vector; the electron-momentum direction defines the z-axis and the electron transverse polarization-vector the x-axis; $\lambda = [1 - (\mu_i + \mu_j)^2][1 - (\mu_i - \mu_j)^2]$ with $\mu_i = m_i/\sqrt{s}$. The coefficients Σ_U , $\Sigma_{\rm L}$, $\Sigma_{\rm T}$ and Σ_N depend only on the polar angle θ and their explicit form is given in [6].

An interesting feature of neutralino production is encoded in the term Σ_N of Eq. (10). Unlike Σ_U , Σ_L and Σ_T , the Σ_N is a function of plaquettes

$$\Sigma_N = 4\lambda \sin^2 \theta \left[A_1 (J_{ij}^{31} - J_{ij}^{41}) - A_2 (\tilde{J}_{ij}^{32} - \tilde{J}_{ij}^{42}) + A_3 \tilde{J}_{ij}^{21} \right], \quad (11)$$

where tilde means that in calculating J the N_{i2} should be replaced by $N'_{i2} = s_W N_{i1} + c_W N_{i2}$. The combinations of propagator factors A_i are

$$A_{1} = \frac{1}{4c_{W}^{4}} D_{Z} (D_{tL} - D_{uL}),$$

$$A_{2} = \frac{s_{W}^{2} - 1/2}{16s_{W}^{4}c_{W}^{4}} D_{Z} (D_{tR} - D_{uR}),$$

$$A_{3} = \frac{1}{8s_{W}^{2}c_{W}^{4}} (D_{tL}D_{uR} - D_{tR}D_{uL}),$$
(12)

with $D_{tL,R} = s/(t - m_{\tilde{e}_{L,R}}^2)$, $D_{uL,R} = s/(u - m_{\tilde{e}_{L,R}}^2)$ and the Z-boson propagator $D_Z = s/(s - m_Z^2)$ is taken real by neglecting the Z width in the limit of high energies. Therefore Σ_N is nonvanishing only in CP-noninvariant theories and already the detailed measurement of the angular distribution of produced neutralino pairs in collisions of transverse polarized beams could indicate the presence of CP-phases.

However, since Σ_N depends on plaquettes, nonvanishing Σ_N requires specific form of CP-violation: the area of some unitarity quadrangles has to be non-zero, *i.e.* at least one of Dirac phases $\beta_{kl} \neq 0$. Moreover, the effect might be quite small due to cancellations between \tilde{H}_1^0 and \tilde{H}_2^0 higgsino components of $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$ in Eq. (11), and between *t*- and *u*-channel selectron exchanges in Eq. (12).

If the initial beams are not polarized, the CP-phases could be inferred from the \mathcal{P}_N component of the polarization of the $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ pairs produced in e^+e^- annihilation [6, 11]. The polarization vector $\vec{\mathcal{P}} = (\mathcal{P}_L, \mathcal{P}_T, \mathcal{P}_N)$ is defined in the rest frame of the particle $\tilde{\chi}_i^0$, with components parallel to the $\tilde{\chi}_i^0$ flight direction in the c.m. frame, in the production plane, and normal to the production plane, respectively. The normal component \mathcal{P}_N can only be generated by complex production amplitudes in the non-diagonal $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ pair production process with $i \neq j$. For example, the contribution to \mathcal{P}_N from the \tilde{e}_R exchange reads

$$\mathcal{P}_N = \frac{8\lambda^{1/2}\mu_j \sin\theta}{c_W^2 \Sigma_U} D_{u\mathrm{R}} D_{t\mathrm{R}} \Im \mathrm{m} \left[\left(N_{i1} N_{j1}^* \right)^2 \right] \,. \tag{13}$$

The normal polarization can be non-zero even if all the β -type CP-phases vanish, *i.e.* it could signal the existence of non-trivial α -type CP-phases.

The non-zero values of CP-odd characteristics Σ_N or \mathcal{P}_N would unambiguously indicate CP-violation in the neutralino sector. However, their experimental measurements will be very difficult. On the other hand, one can also try to identify the presence of CP-phases by studying their impact on the CP-even quantities, like neutralino masses, branching ratios *etc.* Since these quantities are non-zero in CP conserving case, the detection of CP-odd phases will require a careful quantitative analysis of a number of physical observables. In particular, for numerically small CP-odd phases, their deviations from CP-even values will also be small. As an example, in Fig. 1 the unitarity quadrangles for a particular point in the parameter space (consistent with all experimental constraints) are shown. The phase of μ is set to zero, and $\Phi_1 = \pi/5$. In this case the quadrangles are almost degenerate to lines parallel to either real or imaginary axis, and revealing the phase of M_1 will be quite difficult.



Fig. 1. The *D*-type (left panel) and *M*-type (right panel) quadrangles in the complex plane, illustrated for $\tan \beta = 10$, $|M_1| = 100.5$ GeV, $\Phi_1 = \pi/5$, $M_2 = 190.8$ GeV, $|\mu| = 365.1$ GeV and $\Phi_{\mu} = 0$; *ij* as indicated in the figure.

In this respect, as pointed out in Ref. [6], a clear indication of non-zero CP-violating phases can be provided by studying the energy behavior of the cross sections for non-diagonal neutralino pair production near thresholds.

In CP-invariant theories, the CP-parity of a pair of Majorana fermions $\tilde{\chi}_i^0 \tilde{\chi}_i^0$ is given by

$$\eta = \eta^i \eta^j (-1)^L \,, \tag{14}$$

where η^i is the CP-parity of $\tilde{\chi}_i^0$ and L is the angular momentum [12]. Therefore neutralinos with the same CP-parities (for example for i = j) can be excited only in the *P*-wave via the *s*-channel γ and Z production processes. The excitation in the S-wave, with the characteristic steep rise $\sim \lambda^{1/2}$ of the cross section near threshold, can occur only for non-diagonal pairs with opposite CP-parities of the produced neutralinos [8].

The power of the selection rule (14) can clearly be seen by inspecting the expressions for the total cross section $\sigma^{\{ij\}}$ $(i \neq j)$ near threshold

$$\sigma^{\{ij\}} \approx \frac{\pi \alpha^2 \,\lambda^{1/2}}{(m_i + m_j)^2} \left\{ \frac{4m_i m_j}{(m_i + m_j)^2} \left(|\Im m \, G_{\rm R}^{(0)}|^2 + |\Im m \, G_{\rm L}^{(0)}|^2 \right) + \mathcal{O}(\lambda) \right\},\tag{15}$$

where

$$\begin{aligned} G_{\rm R}^{(0)} &= \frac{1}{2c_W^2} D_0 (N_{i3} N_{j3}^* - N_{i4} N_{j4}^*) - \frac{1}{c_W^2} F_{\rm R} N_{i1} N_{j1}^*, \\ G_{\rm L}^{(0)} &= \frac{(s_W^2 - 1/2)}{2c_W^2 s_W^2} D_0 (N_{i3} N_{j3}^* - N_{i4} N_{j4}^*) + \frac{1}{4s_W^2 c_W^2} F_{\rm L} N_{i2}' N_{j2}'^*, \end{aligned}$$

and the kinematic functions

$$D_0 = (m_i + m_j)^2 / ((m_i + m_j)^2 - m_Z^2),$$

$$F_{\rm L,R} = (m_i + m_j)^2 / (m_{\tilde{e}_{\rm L,R}}^2 + m_i m_j).$$

In the CP-invariant theory, the imaginary parts of N_{ij} can only be generated by Majorana phases $\alpha_i = 0$ and $\alpha_j = \pi/2$ or vice versa, i.e. the S-wave excitation is possible when the CP-parities of the produced neutralinos are opposite, as dictated by the Eq. (14). This immediately implies that if the $\{ij\}$ and $\{ik\}$ pairs are excited in the S-wave, the pair $\{jk\}$ must be excited in the P-wave characterized by the slow rise $\sim \lambda^{3/2}$ of the cross section, Fig. 2, left panel.

If, however, CP is violated the angular momentum of the produced neutralino pair is no longer restricted by the Eq. (14) and all non-diagonal pairs can be excited in the S-wave. This is illustrated in Fig. 2, where the threshold behavior of the neutralino pairs $\{12\}$, $\{13\}$ and $\{23\}$ for the CP-conserving (left panel) case is contrasted to the CP-violating case (right panel).

Even for relatively small CP-phase $\Phi_1 = \pi/5$, implying small impact on CP-even quantities, the change in the energy dependence near threshold can be quite dramatic. Thus, observing the $\{ij\}$, $\{ik\}$ and $\{jk\}$ pairs to be excited *all* in *S*-wave states would therefore signal CP-violation.



Fig. 2. The threshold behavior of the neutralino production cross-sections $\sigma^{\{ij\}}$ for the CP-conserving (left panel) and the CP-violating (right panel) cases. Other parameters as in Fig. 1.

4. Conclusions

The supersymmetric extension of the Standard Model can come with new sources of CP-violation. In the absence of natural suppression of the SUSY CP-phases, their non-zero values have to be considered in phenomenological studies. In this paper we have discussed the CP properties of neutralinos, which are quite peculiar due to their Majorana nature.

The CP-violation in the neutralino sector can reveal itself in many different ways. The most ambitious analysis would require the experimental reconstruction of the unitarity quadrangles. Since all phases of the mixing matrix N (at least at the tree level) are ultimately determined by the phases of the fundamental parameters μ and M_1 , the reconstruction of the quadrangles would provide many consistency checks of the underlying theory.

On the other hand, the first qualitative indication of the CP-violation can be provided by the energy dependence of the neutralino production cross sections. The steep rise of cross sections for the production of at least three different non-diagonal neutralino pairs can be interpreted as a first direct signature of the presence of CP-violation in the neutralino sector.

Work supported in part by the Polish State Committee for Scientific Research (KBN) Grant 2 P03B 040 24 (2003-2005).

REFERENCES

- F. Hoogeveen, Nucl. Phys. B341, 322 (1990); I.B. Khriplovich, M. Pospelov, Sov. J. Nucl. Phys. 53, 638 (1991).
- [2] A.G. Cohen, D.B. Kaplan, A.E. Nelson, Ann. Rev. Nucl. Part. Sci. 43, 27 (1993); B.M. Gavela, P. Hernandez, J. Orloff, O. Pène, C. Quimbay, Nucl. Phys. B430, 382 (1994); V.A. Rubakov, M.E. Shaposhnikov, Usp. Fiz. Nauk 166, 493 (1996).
- [3] S. Pokorski, J. Rosiek, C.A. Savoy, Nucl. Phys. B570, 81 (2000); V. Barger,
 T. Falk, T. Han, J. Jiang, T. Li, T. Plehn, Phys. Rev. D64, 056007 (2001).
- [4] Y. Kizukuri, N. Oshimo, *Phys. Lett.* B249, 449 (1990); T. Ibrahim, P. Nath, *Phys. Rev.* D58, 111301 (1998); [Erratum-ibid. D60, 099902b (1998)].
 T. Ibrahim, P. Nath, *Phys. Rev.* D61, 093004 (2000); M. Brhlik, G.J. Good, G.L. Kane, *Phys. Rev.* D59, 115004 (1999); R. Arnowitt, B. Dutta, Y. Santoso, *Phys. Rev.* D64, 113010 (2001); T. Ibrahim, P. Nath, arXiv:hepph/0210251.
- [5] TESLA Technical Design Report, Part: III Physics at an e⁺e⁻ Linear Collider, eds. R.-D. Heuer, D. Miller, F. Richard and P. Zerwas, DESY 2001-011, arXiv:hep-ph/0106315; T. Abe et al. [American Linear Collider Working Group Collaboration], "Linear collider physics resource book for Snowmass 2001. 2: Higgs and supersymmetry studies," in Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) ed. N. Graf, arXiv:hep-ex/0106056; K. Abe et al., JLC Roadmap Report, presented at the ACFA LC Symposium, Tsukuba, Japan 2003, http://lcdev.kek.jp/RMdraft/.
- [6] S.Y. Choi, J. Kalinowski, G. Moortgat-Pick, P.M. Zerwas, Eur. Phys. J. C22, 563 (2001), Eur. Phys. J. C23, 769 (2002).
- [7] L. Chau, W. Keung, *Phys. Rev. Lett.* 53, 1802 (1984); M. Gronau, J. Schechter, *Phys. Rev. Lett.* 54, 385 (1985); [Erratum-ibid. 54, 1209 (1985)]; H. Fritzsch, J. Plankl, *Phys. Rev.* D35, 1732 (1987).
- [8] J. Ellis, J.M. Frère, J.S. Hagelin, G.L. Kane, S.T. Petcov, Phys. Lett. B132, 436 (1983).
- [9] F. del Aguila, J.A. Aguilar-Saavedra, *Phys. Lett.* B386, 241 (1996);
 J.A. Aguilar-Saavedra, G.C. Branco, *Phys. Rev.* D62, 096009 (2000).
- [10] C. Jarlskog, *Phys. Rev. Lett.* **55**, 1039 (1985).
- [11] G. Moortgat-Pick, H. Fraas, *Phys. Rev.* D59, 015016 (1999); G. Moortgat-Pick, H. Fraas, A. Bartl, W. Majerotto, *Eur. Phys. J.* C9, 521 (1999);
 [Erratum-ibid. C9, 549 (1999)]; G. Moortgat-Pick, A. Bartl, H. Fraas, W. Majerotto, *Eur. Phys. J.* C18, 379 (2000); S.Y. Choi, H.S. Song, W.Y. Song, *Phys. Rev.* D61, 075004 (2000).
- [12] G. Moortgat-Pick, H. Fraas, Eur. Phys. J. C25, 189 (2002).