EXPERIMENTAL STUDY OF SUBDIFFUSION IN A MEMBRANE SYSTEM*

K. Dworecki, S. Wąsik and T. Kosztołowicz

Intitute of Physics, Świętokrzyska Academy
Świętokrzyska 15, 25-406 Kielce, Poland

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We present experimental results of the anomalous diffusion in a membrane system obtained by means of the laser interferometric technique. We show that in membrane system with glucose diffusing in a gel medium, the thickness of the concentration boundary layers manifest a subdiffusive character. Namely, the thickness of the concentration boundary layer δ scales as t^γ with $\gamma = 0.443 \pm 0.019$.

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1. Introduction

Diffusion is an important transport process of matter in various physical, chemical, and biological systems. A form of time evolution of so-called near-membrane layers (NMLs) or the concentration boundary layers (CBLs) characterizes the diffusion in a membrane system [1–3]. It can be also treated as a useful representation of the measured concentration profiles. Therefore, direct measurements of the CBLs properties, become essential for understanding of transport mechanisms present in the membrane system. When the thickness of NML grows in time as t^γ with $\gamma = 0.5$ we deal with normal or Gaussian diffusion, when $\gamma > 0.5$ with a superdiffusive behavior, and when $\gamma < 0.5$ with a subdiffusive behavior [4,5]. In the following, the attention will be focused on this latter case. To observe the time evolution of NMLs, we use the laser-interferometrical method: the interference fringes pattern provides quantitative measurement of the substance concentration [6,7].

2. Experiment

The membrane system under study is a cell with two glass cuvettes separated by a horizontally located membrane. Initially, we fill both cuvettes

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with a gel but in the upper one we add some aqueous solution of glucose. Then, the glucose diffuses to the lower cuvette. To measure the concentration profiles in the membrane system we employ the laser interferometric technique [6]. The experimental setup is described in [7]. At present we only recall that it consists of the measurement cell with the membrane, the Mach–Zehnder interferometer, including the He–Ne laser ($\lambda = 632,8$ nm), TV-CCD camera and computerized data acquisition system. The output beam of a 15 mW He–Ne laser is spatially filtered and split into two beams. The first beam goes through the membrane system parallel to the membrane surface, while the second reference beam goes directly to the light detecting system. The interferograms, which appear due to the interference of the two beams, are controlled by the refraction coefficient of the solute, which in turn depends on the substance concentration. The analysis of the interferograms allows one to reconstruct the time dependent concentration profiles of the substance transported across the membrane. The substance concentration at position x and at time t is determined by the deflection of fringes $d(x,t)$ from their straight line run. Since the relation between the concentration $C(x,t)$ and the refraction coefficient $n(x,t)$ is assumed to be linear, we have:

$$C(x,t) = C_0 + a\frac{\lambda d(x,t)}{hf}\,, \tag{1}$$

where C_0 is initial substance concentration, a is the proportionality factor between the concentration, and the refraction index, λ is the wavelength of the laser light, h denotes the distance between the fringes in the area where they run parallel, f is the thickness of the solution layer in the measurement cuvette. Having the profiles one can define the concentration boundary layer. When the substance diffuses across the membrane into pure solvent (water gel-agar, 1% by weight) the thickness δ of the CBL is defined as a length at which the concentration decreases k times, *i.e.*

$$kC(x=0,t) = C(x=\delta,t) \tag{2}$$

with $x = 0$ being the membrane surface. The arbitrary constant k is assumed to be $k = 0.08$ but one may take a different value (< 1) depending on a specific application.

3. Results

We have systematically collected several hundred interferograms showing various stages in the development of each combination of experimental variables. We restrict the presentation here to selected representative examples. Figure 1 depicts distinct cases. We show three, typical for this investigation interferograms. Changes in the interference fringes in these interferograms indicate that on both sides of membrane the concentration boundary layers are formed and they are hydrodynamically stabile. The results of the computer analysis of interference images which refer to interfaces solution/membrane, enable us to obtain concentration profiles $C(x,t)$ in the concentration boundary layers for different times and different initial concentration concentration solutions. The results of this investigations obtained for a given membrane system are presented in figure 2. The solute concentration profiles were used to calculate the thickness of CBLs. The experimental results $\delta(t)$ are presented in figure 3.

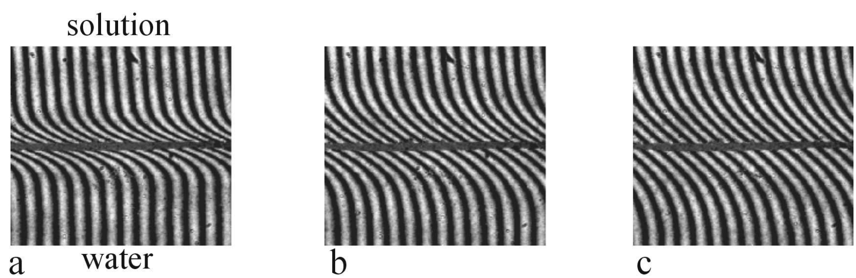

Fig. 1. The interferograms that are analyzed to obtain the concentration profiles. There is initially uniform glucose solution of the concentration 0.1 mol/l in the upper part of the measurement cuvette. The interferograms are taken at several values of time: (a) — 2 min, (b) — 10 min, and (c) — 20 min.

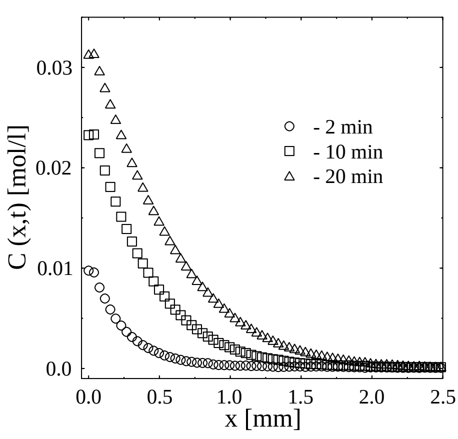

Fig. 2. The concentration profiles for C=0.1 mol/l glucose solution in a gel form taken at several values of time: 2 min, 10 min, and 20 min.

The solid line on this figure represents a fit $\log(\delta)$ *vs.* $\log(t)$. The results of the investigation of the dependence $\delta(t)$ indicate that it can be a power law $\delta \propto t^{0.443\pm0.019}$. Thus, the process under study is subdiffusive.

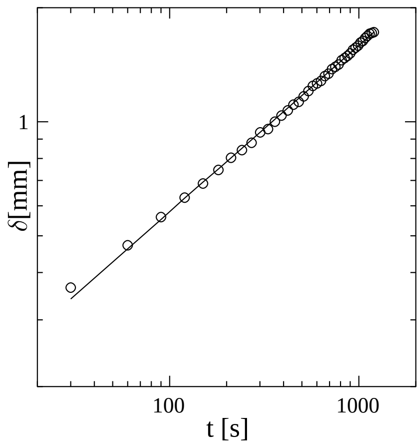

Fig. 3. The time evolution of the concentration boundary layer for $k = 0.08$ obtained for gelled solution of glucose. The initial concentration is 0.1 mol/l. The circles show the experimental values, the solid line is a fit to power law which yields an exponent of 0.443 ± 0.019.

4. Conclusions

It was demonstrated that the interferometric technique combined with digital image analysis allows one to determine the temporal structure of the concentration boundary layers in a membrane system. The experimental results of the investigation of the evolution of the CBLs show that their thickness grows in time as t^{γ}, with $\gamma < 0.5$. This demonstrate a subdiffusive mechanism of solute transport in the membrane system.

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