PHOTONS PRODUCED INSIDE A CAVITY WITH A MOVING WALL

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The production of particles inside one-dimensional cavity with a moving wall is discussed. Cavities with periodically driven wall motions are analyzed numerically for long times. We formulate the conditions under which the particle production is being efficiently run. The conditions are independent of a specific type of cavity motions.

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1. Introduction

The relativistic theory of quantized fields to be defined in the presence of boundaries or interfaces is usually a troublesome issue. Especially, handling with a non-stationary case of moving boundaries theorists are entangled in embarrassing underlying problems. For the most part, remarkable phenomena arising in applications of the quantum field theory in spaces with moving boundaries are widely discussed in literature under the name of dynamical Casimir effect (or non-stationary Casimir effect). One consequence of confining quantum fields to live in some bounded space is that the boundaries experience attractive or repulsive forces. Unlike a static case, Casimir vacuum forces can change in time. However, it is assumed in usual settings that the motion of boundaries is firmly fixed by some external environment. Therefore, no attention is paid to Casimir forces. A prominent feature of the dynamical Casimir effect is the phenomenon of quantum radiation attributable to vacuum fluctuations. Most of extensive studies of the subject are mainly contributions to the knowledge of this purely quantum mechanism of particle production (called motion-induced radiation as well). In spite of the significant number of researches, even the simplest systems of idealized cavities with moving boundaries are not fully understood (see a recent review of the subject and a long list of references in [1]). One can believe that such systems are truly fundamental with respect to motion-induced radiation and that their investigation will clarify conceptual issues raised by more complicated models. This subject is also important in its own right, especially concerning prospects for experimental tests to measure an emission of vacuum photons out of a real cavity with vibrating walls.

In this paper, we continue our investigation of one-dimensional vibrating cavities with perfectly reflecting walls carried out in [2]. For such systems, it is clearly recognized that a significant amount of photons may be produced inside the cavity only under some special circumstances. The crucial point is to maintain a constructive interference of quantum waves from the perturbed field vacuum. It is provided for a cavity on condition of some parametric "opto-mechanical" resonance. A frequency of cavity oscillations should be a multiple of some static cavity eigenfrequency [3-16]. Only few papers analyzed the process of radiation generated under a disturbed resonance condition. Dodonov *et al.*, [1,9] thoroughly studied harmonic motions considering approximate solutions for small amplitudes of oscillations. In our previous paper [2], we analyzed energy densities and total energies for off resonant oscillations. But for open dynamical systems it happens that there is no direct correspondence between the total accumulated energy and the number of observed particles. The aim of this paper is to extend our previous analysis. In what follows, we will analyze the number of radiated photons and conditions in order to observe the enhanced photon production in one-dimensional cavity.

From practical point of view, the one-dimensional cavity model with perfectly reflecting point-like walls seems to be a crude oversimplification. Let us remind the arguments why the investigation of this toy model can be really a first step to gain insight into a realistic lifelike situation. First, the analysis of three-dimensional cavities shows that each transverse photon mode can be truly described by one-dimensional model. Therefore, the quantum electrodynamics inside one-dimensional cavity is really a single polarization approximation. Next, from all papers we know that realistic features of cavity walls (like finite size, imperfect shape, frequency-dependent conductivity) are included in fact as suitable modifications of the theoretical model with perfect cavity walls. They do not change the basic mechanism (resonance enhancement of the photon production, the qualitative laws for the growth of the energy and the number of photons, the formation of traveling peaks in the energy density (pulse shaping) *etc.*), but diminish the cavity finesse. On the other hand, the detector involved to count photons will have its influence on the photon production process as well. But the most important factor is provided in case we take into consideration the influence of non-zero temperature upon the production process. The finite temperature modifications lead to the opposite effect. A cavity moving in thermal fluctuations produces more photons. They are generated rather from a thermal field than from a vacuum field. However, all qualitative features of motion-induced radiation known from the zero temperature case are observed here. The experimental detection of radiation may be considerably simpler in this case [13]. All the above considerations of including important cavity features which should be taken into account in realistic experiments suggest that the modifications preserve the basic picture of the production process. They influence only the photon production rate.

The most important and interesting phenomenon seen in the oscillating cavity system is the opto-mechanical resonance. The production of photons inside the cavity can be greatly enhanced due to some constructive interference. We cannot describe all circumstances which directly affect it. It is fairly certain that the mechanical oscillation frequency must be a multiple of optical resonance frequencies. But other features of the cavity motion seem to be less important. The cases of cavities with one or two oscillating walls are similar [14, 15]. There are no important differences between the cases when the cavity oscillates as a whole (translational vibrations) and when the cavity length oscillates (breathing vibrations) [11]. There is no need for sinusoidal type of oscillations to have the opto-mechanical resonance [10,16]. It is not proved, but maybe there is no need for exact periodicity and we need only some "anchor" points of the cavity motion [8] to take profit of the resonance enhancement. The existence of an anchor point means that the cavity should always return to some fixed position with a resonant frequency. In fact, it is possible to find a set of different cavity motions or perturbations with the common parametric opto-mechanical resonance condition leading to the exponential growth of produced photons. We recognize the same parametric resonance condition for any periodic cavity motion. It gives a motivation to study thoroughly the parametric resonance condition. In the presented paper, we analyze two different motions of a cavity with perturbed resonance conditions. Our goal is to find the resonance width, where the exponential growth of photons still occurs.

The paper is organized as follows. In Sec. 2 we remind the setup of quantum electrodynamics inside one-dimensional cavity with perfectly reflecting walls. The quantization procedure is carefully described. We address the question of particle production induced by some cavity motion which is assumed to be bounded in time. In Sec. 3 we describe how to explore more general cavity motions than harmonically oscillating walls. Then, we discuss examples of harmonic oscillations and some non-harmonic oscillations. For both types of cavity oscillations, we recall main results well-known from studies of resonant harmonic motions. Finally, the off-resonant oscillations are discussed. We give two conditions of the frequency adjustment necessary to keep the exponential growth of produced particles. This adjustment relies on the inequalities between the frequency modulation depth, the relative amplitude of oscillations and the total number of oscillations (or the total time of the cavity motion). Some conclusions for prospected experiments are drawn.

2. Quantum electrodynamics inside one-dimensional cavity

We study the electromagnetic field inside one-dimensional oscillating cavity made of two perfectly reflecting walls. We mean in fact the quantum theory of linearly polarized light [17,18]. It is equivalent to the problem of the scalar field with the corresponding field equation and boundary conditions:

$$\begin{aligned} (-\partial_t^2 + \partial_x^2)A(x,t) &= 0\,, \\ A(x=0,t) = A(x=q(t),t) &= 0 \qquad \text{for all times.} \end{aligned} \tag{1}$$

The left wall is assumed to be fixed at position $x = 0$, while the right one is oscillating with some prescribed time-like trajectory $q(t)$. The physical system is just open, therefore, the total field energy and the number of field quanta can change their values over the period of wall motion. We restrict ourselves to some regular wall motions. In particular, a velocity of the wall is never close to the speed of light. The allowed wall trajectories are specified by the following set of requirements [2]:

$$\begin{aligned} (i) &\quad q(t) = L\,, \quad \text{for} \quad t \le 0 \\ (ii) &\quad |\dot{q}(t)| < 1\,, \\ (iii) &\quad q(t) > 0\,, \\ (iv) &\quad q(t) = L \quad \text{for} \quad t \ge T\,. \end{aligned} \tag{2}$$

The fourth requirement concerns the calculation of the number of "photons". In our one-dimensional physics, we mean massless scalar particles. Requiring that the wall motion lasts a finite period of time, we have guaranteed the equivalence between in and out photon states. In general case, Fock spaces related to both asymptotic states are not unitary equivalent. It complicates the definition of "particles". As usual the specification of particles corresponds to the choice of a suitable number of quanta operator. This choice should be made on physical grounds, so we cannot determine it from the formalism of the theory. Because of the lack of direct experimental verification of the idealized and simplified model, in literature it is commonly assumed that a particle detector responds to standing-wave field modes of a static cavity. These modes correspond to asymptotic solutions of (1):

$$A_k^{\text{in}}(x,t) = \frac{1}{\sqrt{\pi k}} \exp\left(-i\omega_k t\right)\, \sin\left(\omega_k x\right), \qquad \omega_k \equiv \frac{k\pi}{L}\,. \tag{3}$$

Note that each ω_k corresponds to an eigenfrequency of the static cavity. Due to the fourth requirement of (2), the quantized field for $t > T$ can be now represented in terms of creation $\hat{a}_k^\dagger$ and annihilation $\hat{a}_k$ operators associated with the static cavity system:

$$\hat{A}(x,t) = \sum_{k=1}^{\infty} \left[\hat{a}_k A_k^{\mathrm{in}}(x,t) + \hat{a}_k^\dagger A_k^{in*}(x,t) \right]. \tag{4}$$

The quantization of the electromagnetic field inside a static cavity is done here in a manner analogous to the canonical quantization of the electromagnetic field in the whole space. The choice of operators $\{a_k, a_k^\dagger\}$ allows us to set up the Fock representation. In other words, the notion of particles is then introduced. Before going further, we make some comment about the quantization. It has been recognized very early by von Neumann (1938) that there exist infinitely many unitary inequivalent representations in the quantum field theory. We should remember that the quantum system settled down in a space bounded by reflecting walls is in fact an effective theory. The boundary conditions for fields come from idealizations of the actual physics. If we think about the description of real physics, then we must conclude that in order to simplify the problem we have replaced complicated interactions of fields with boundary walls with simple boundary conditions for fields themselves. In any quantum theory, the price for such idealization is that we lost some information and even worse we have injected an infinite amount of information which we did not have. This manifests itself later in calculations of physical observables. The calculations may involve infinite numbers and need for physical cut-offs. Moreover, any effective theory always sets the stage for a discussion which choice of quantum representation is appropriate. In other words and referring directly to our context, the notion of particles in the presence of idealized (moving) boundaries is not judiciously defined. In the framework of an oversimplified effective theory, it may happen that we cannot justify rules for selecting operators for counting real particles. If we are not in a position to analyze a full theory, then the relevant selection can be justified only on physical grounds with reference to some experimental setup. Therefore, at the level of theoretical considerations based on the oversimplified effective theory we are not able to address this question. To avoid possible confusion we stress that the quantization procedure [17] seems to build a well defined quantum theory. Nevertheless, the quantization of effective theories always must be handled with care and confronted with a physical context.

Let us review briefly the procedure for calculating a number of produced particles inside a vibrating cavity. The complete set of mode functions $A_k(x,t)$ for any time may be chosen in such a way that left-moving and right-moving wave packets have the same shape [17]:

$$A_k(x,t) = N_k \left[\mathrm{e}^{ik\pi R(t+x)} - \mathrm{e}^{ik\pi R(t-x)} \right] , \qquad N_k \equiv \frac{i}{\sqrt{4\pi k}} , \tag{5}$$

where the phase function R is subject to the Moore's equation

$$R(t+q(t)) = R(t-q(t)) + 2 \, . \tag{6}$$

In fact, the phase function R contains the whole information about the physical system. For a static case, the basic solutions (5) match exactly the standing-wave solutions (3). Actually, this generalized fundamental set of solutions can be easily constructed from standing-wave solutions if we make use of the conformal symmetry that our two-dimensional theory possesses. One can immediately recognize this way if the solutions (5) are written down in the following alternative form:

$$A_k(x,t) = \frac{1}{\sqrt{\pi k}} \exp\left(-i\omega_k u_+\right) \sin\left(\omega_k u_-\right) , \tag{7}$$

where we have introduced new coordinates:

$$u_\pm = \frac{R(t+x) \pm R(t-x)}{2} \, L \, . \tag{8}$$

Since we are equipped with the basis (5), we can evaluate the sum over field modes referring to any point in time. The decomposition of the final state ($t > T$) of the field operator in the basis (5) yields:

$$\hat{A}(x,t) = \sum_{k=1}^{\infty} \left[\hat{b}_k A_k(x,t) + \hat{b}_k^\dagger A_k^*(x,t) \right] \, . \tag{9}$$

The particle content of the system can be now recognized in the following standard way [17]. We use here the Heisenberg picture. For $t > T$, the cavity system is being in the quantum state that corresponds to the state $|out\rangle$. The state $|out\rangle$ is uniquely defined as the state which is annihilated by all operators $\{b_k\}$. In this construction, the operators $\{b_k, b_k^\dagger\}$ do not annihilate and create any particles. This action is realized by the set of operators $\{a_k, a_k^\dagger\}$. The family of b-operators helps us to trace the evolution of particle states. After a time period of cavity motion T, the vacuum state $|0\rangle$ goes to the squeezed state $|out\rangle$, which is actually a formal Fock vacuum state with respect to the action of b-operators. To count for particles produced inside the cavity during the cavity motion, we should analyze the particle content of the final state $|out\rangle$. The number of quanta n_k produced in the k-th mode is just the expectation value of the appropriate number operator $\hat{a}_k^\dagger \hat{a}_k$ in the final state $|out\rangle$. The total number of created photons is, therefore, given by $N = \sum_{k=1}^{\infty} n_k$. Now, we can perform all suitable calculations.

A standard method is to calculate the Bogoliubov coefficients at first. The passage from one basis (3) to another one (5) can be represented by some linear transformation:

$$A_k(x,t) = \sum_{l=1}^{\infty} \left[\alpha_{kl} A_l^{\rm in}(x,t) + \beta_{kl} A_l^{in\star}(x,t) \right] . \tag{10}$$

With the help of the Bogoliubov coefficients α_{kl} and β_{kl}, one can also establish the following operator transformations

$$\begin{aligned} \hat{a}_k &= \sum_{l=1}^{\infty} \left(\alpha_{lk} \hat{b}_l + \beta_{lk}^\star \hat{b}_l^\dagger \right) , \qquad & \hat{b}_k &= \sum_{l=1}^{\infty} \left(\alpha_{kl}^\star \hat{a}_l - \beta_{kl}^\star \hat{a}_l^\dagger \right) , \\ \hat{a}_k^\dagger &= \sum_{l=1}^{\infty} \left(\beta_{lk} \hat{b}_l + \alpha_{lk}^\star \hat{b}_l^\dagger \right) , \qquad & \hat{b}_k^\dagger &= \sum_{l=1}^{\infty} \left(-\beta_{kl} \hat{a}_l + \alpha_{kl} \hat{a}_l^\dagger \right) . \end{aligned} \tag{11}$$

It is now straightforward to calculate the Bogoliubov coefficients starting from the knowledge of the phase function $R(z)$ [3,19] (note some minor sign error in [3] and other papers):

$$\alpha_{kl} = \frac{1}{2L} \sqrt{\frac{l}{k}} \int\limits_{t-L}^{t+L} dz \; \mathrm{e}^{-ik\pi R(z) + i\omega_l z} ,$$

$$\beta_{kl} = -\frac{1}{2L} \sqrt{\frac{l}{k}} \int\limits_{t-L}^{t+L} dz \; \mathrm{e}^{-ik\pi R(z) - i\omega_l z} . \tag{12}$$

The above formulas are meaningful provided that $t > T$. Obviously, the Bogoliubov coefficients are in fact independent of time. The number of photons in k-th mode and the total number of photons produced in the oscillating cavity are given by

$$n_k = \sum_{l=1}^{\infty} |\beta_{lk}|^2 \;, \qquad N = \sum_{k=1}^{\infty} n_k \,. \tag{13}$$

The knowledge of the phase function $R(z)$ comes from the solution of Moore's equation (6) for a given trajectory of a moving cavity wall. This solution usually can be obtained only numerically.

3. Photon production inside a cavity with a periodically oscillating wall

In this paper, we will consider the problem of a vibrating cavity under perturbed parametric resonance conditions. Its motion is assumed to be periodic but a corresponding frequency differs from the resonant one $\omega_n = n\pi/L$. Moreover, the cavity oscillations will die after some period of time T. Then, we will be looking for the number of created particles. As it was mentioned in the previous section, the key point of the corresponding calculation is to find the phase function $R(z)$. The method to solve Moore's equation (6) was presented in [2] (we have found recently that a similar method was described in [20] in the context of classical strings). We define an auxiliary function $f(z)$ as the unique solution to the equation:

$$f(t + q(t)) = t - q(t). \tag{14}$$

The function $f(z)$ itself represents a physically reasonable motion of the cavity wall (2) provided that the following conditions are satisfied:

$$\begin{aligned} (i) \quad & f(z) = z - 2L\,, \quad \text{for} \quad z \le L\,, && (15)\\ (ii) \quad & 0 < f'(z) < \infty\,, \\ (iii) \quad & f(z) < z\,, \\ (iv) \quad & f(z) = z - 2L\,, \quad \text{for} \quad z \ge L + T\,. && (16) \end{aligned}$$

Using this function, we can evaluate the corresponding solution of (6) as

$$R(z) = 2n + \frac{f^n(z)}{L} \qquad \text{for} \quad z \in [L_{n-1}, L_n]\,, \tag{17}$$

where we denote $L_0 \equiv L$ and $L_n = (f^{-1})^n(L)$. The symbol f^n is used here for something else than the power of the function, namely for the n-fold multicomposition $f \circ f \circ \ldots \circ f$. In the static region, for all $z \le L$ we have always $R(z) = z/L$. The relevant periodicity conditions for a motion of the cavity wall can be expressed referring either directly to the trajectory $q(t)$ or to the auxiliary function $f(z)$:

$$q(t + T_0) = q(t) \qquad \text{for} \quad 0 \le t \le T - T_0\,,$$

$$f(z + T_0) = f(z) + T_0 \qquad \text{for} \quad L \le z \le L + T - T_0\,. \tag{18}$$

The period of wall oscillations is denoted here by T_0. For our resonant motions, we have $T_0 = 2\pi/\omega_n = 2L/n$. If we are dealing with resonant periodic wall motions, then the phase function $R(z)$ always develops a perfect staircase-shape [2, 8]. In this case, the energy and the total number of photons usually grow with time. The vibrating cavity system being in a parametric resonance can continuously increase the amount of photons created from vacuum. This phenomenon is referred as constructive interference. Evidently it happens, but we need yet to ask whether this mechanism of particle production runs efficiently only when the system parameters are fine tuned for a resonance. The practical importance of this mechanism of particle production depends crucially on conditions needed in order to keep constructive interference inside a cavity system. Their investigation should cast light on the feasibility for setting up experimental tests.

To follow our discussion of nearly resonant behavior of the oscillating cavity system, we specify the motion of the cavity wall. We investigate two different wall motions with either $q(t)$ or $f(z)$ being a harmonic function:

$$q(t) = \begin{cases} L\,, & t < 0 \\ L\left\{1 + d \sin\left[\omega_n(1+\varepsilon)t\right]\right\}, & 0 < t < T \\ L\,, & t > T \end{cases}\,, \tag{19}$$

$$f(z) = \begin{cases} z - 2L\,, & z < L \\ z - 2L + d \sin\left[\omega_n(1+\varepsilon)(z-L)\right], & L < z < L+T \\ z - 2L\,, & z > L+T \end{cases}\,. \tag{20}$$

Note that the above examples (19) and (20) correspond to two different types of wall motions. The example (19) yields just harmonic oscillations, while the example (20) corresponds to a more sophisticated type of oscillations. In both cases, we can interpret ε as a perturbation of the resonant frequency (detuning of the resonance). For small amplitudes, an analysis of the off resonant systems (19) was carried out in [1]. Here, we will discuss such vibrating cavity systems for more general motions, arbitrary amplitudes and long-time limits. We will calculate numerically the number of particles created in different modes. The most interesting question is the long-time behavior of these quantities.

First, we review some results concerning a resonant motion with some finely adjusted frequency $\omega_n = n\pi/L$. Let us remind that we have restricted ourselves to motions of type (19) or (20). The parametric resonance means $\varepsilon = 0$ there. The corresponding phase function $R(z)$ develops a well known staircase shape. It is checked that the total average energy exponentially grows in time. If we look at the energy density, it is concentrated into narrow traveling wave packets. The energy density wave packets grow rapidly in time under the resonance conditions. Looking outside of the wave packets, the energy density is very small. Its average value is smaller n^2 times than the value of Casimir energy density inside a static cavity. It results that the photons are produced in sharp and intense pulses. In the long time limit, the total number of photons created from the vacuum increases in time. The exception is a resonant oscillation with the lowest frequency ω_1 ("semi-resonance"). There is no rapid proliferation of photons with time there. The picture of traveling narrow wave packets in the energy density does not appear in this case as well. In all resonant cases with the frequency ω_2, photons are created practically only in the odd modes (see Fig. 1), their amount in even modes is small (in the linear approximation [1] there is no photon production in even modes). The significant number of particles is created just in the lowest mode (see Fig. 1), after a long time the system looks like a Bose–Einstein condensate. The results described in this paragraph have been previously drawn for small-amplitude cavity oscillations. In this paper, we have checked them numerically to be true in general.

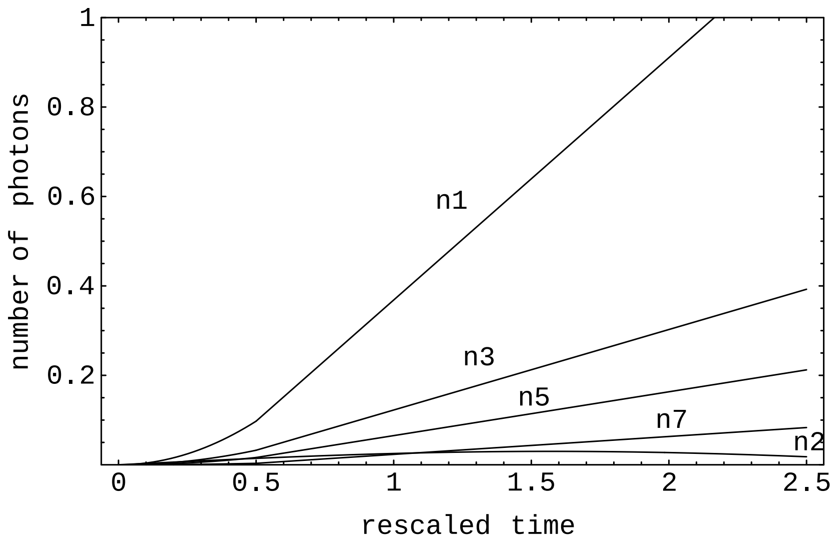

Fig. 1. Number of created photons in particular modes n_1,n_2,n_3,n_5 and n_7. Time is rescaled as $\pi dt/2L$ [1]. A cavity wall oscillates with resonant frequency ω_2. Even modes are suppressed. The lowest mode n_1 dominates for a long-time regime.

Let us now discuss cavity systems defined by (19) or (20) and assume that we have there $\varepsilon \neq 0$. It means that the parametric resonance is violated. Unlike the parametric resonance case, there are unknown exact solutions of Moore's equation (6) for off resonant motions. Some analytical information was given in [2]: the long-time pattern of phase function $R(z)$ is characterized by a "long period" MT_0, where M is some positive integer which can be defined by the following relation:

$$L_N = L + MT_0 \,. \tag{21}$$

For many examples of cavity motions the above relation can be exact, and in general we assume that it holds up to some desired accuracy. From the relation (21) follows immediately: $L_{N+n} = L_n + MT_0$. It results that the phase function $R(z)$ after the long period MT_0 is just reproduced but shifted. The field energy inside a cavity that oscillates with a perturbed resonant frequency grows exponentially only during the time $MT_0/2$. Later, the total energy rapidly decreases down to its minimal value. The minimal energy matches the value of Casimir energy for a static cavity. After reaching its minimal value, the energy becomes to grow again. Thus, the value of the total field energy inside the cavity oscillates with the long period MT_0. It allows us to define the first condition to keep our mechanism of particle production efficient: $MT_0/2 > T$. Let us now summarize our numerical results concerning the number of produced particles for off resonant cavity motions. We are interested to know whether the mechanism of constructive interference disappear and how the photon production rate is influenced. The relevant parameter to control that is $g = \varepsilon/d$. Figs. 2 and 3 show the behavior of the number of particles produced in the lowest mode for small and large g respectively. These data were taken from numerical calculations for the cavity motion (19) with frequency ω_2, but calculations for motions of type (20) and other resonant frequencies lead to similar results and plots. The particles are still produced mainly in "principal" modes [1]. The lowest mode n_1 is always preferred, however, if we detune the system the lowest mode is more suppressed than the higher ones (see Fig. 4). We notice that the numbers of produced particles in different modes grows in time linearly

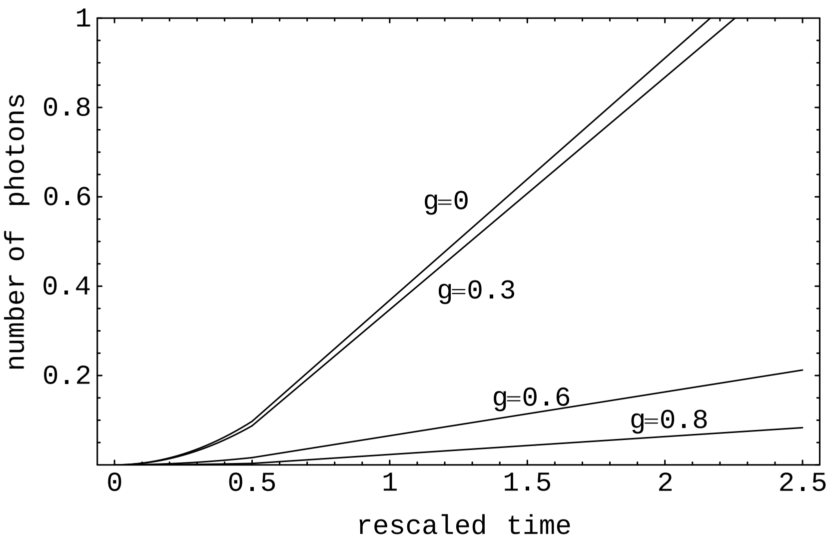

Fig. 2. The change of the number of particles produced in the dominating lowest mode n_1 during the detuning of the parametric resonance. The detuning process is described by dimensionless parameter $g = \varepsilon/d$. The unperturbed frequency is ω_2.

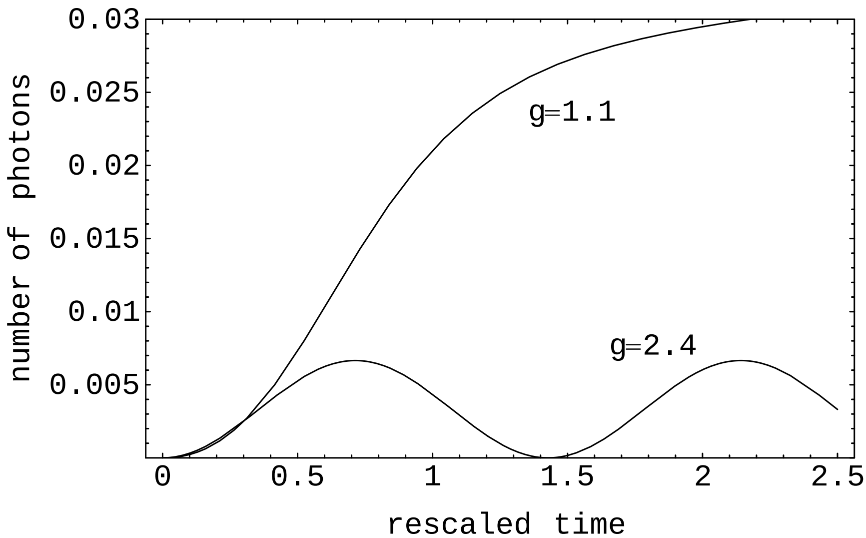

Fig. 3. The change of n_1 for a strong detuning $g > 1$. The production of particles is suppressed.

provided that $g < 1$. For a strong violence of parametric resonance conditions $g > 1$, we can observe negligible oscillations of particle numbers around null values. Let us then summarize that we observe the sharp transition of the photon production rate about $g = 1$. If the parameters of the vibrating cavity system are out of the resonant width defined by the inequality $g < 1$,

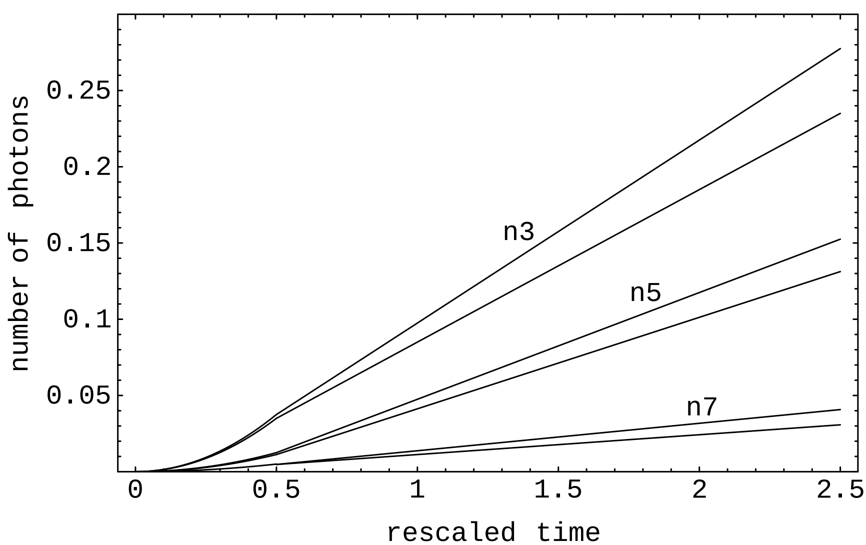

Fig. 4. Number of created photons in higher modes n_3,n_5 and n_7. Upper curves correspond to the parametric resonance $g = 0$, and lower ones are taken for $g = 0.6$.

then the exponential rate of the photon growth disappears. This feature is characteristic for resonant behavior and our parametric condition is common for cavity motions of both types (19) and (20).

4. Conclusions

Each of our examples of cavity motions we have investigated in this paper enables us to draw some common conclusions. The main results can be summarized by the following relations arising from analytical and numerical calculations. First, we proved that the measurement of produced particles must be set in the first half of the "long period", before the process of destruction of particles starts. The corresponding condition $MT_0/2 > T$ can be rewritten as:

$$\frac{\Delta\omega}{\omega} < \frac{T_0}{T}\,. \tag{22}$$

In the above condition, ω is the (resonant) frequency of cavity oscillations, $\Delta\omega$ denotes the accuracy of tuning this frequency, T_0 is a period of oscillations and T stands for the total time of cavity motion. It is the first condition to keep resonant enhancement of the production of photons inside a vibrating cavity. The second condition comes from the numerical calculations discussed in the previous section. They show that the total number of photons produced inside a cavity grows in time up to significant amounts provided that the following relation between parameters occurs (we rewrite here the requirement $g < 1$ for the constructive interference from the Sec. 3)

$$\frac{\Delta\omega}{\omega} < \frac{\Delta L}{L}\,. \tag{23}$$

The cavity length is denoted by L, and its maximal change is ΔL.

The observation of photons created by virtue of the cavity oscillations in realistic experiments is hoped in near future [21]. Instead of oscillations of a cavity wall as a whole they consider rather surface oscillations induced by strong acoustic waves. Then ΔL would correspond to the maximal possible displacement of a wall material. The condition (23) states simply that the modulation depth of the resonant frequency should be smaller than the relative depth of surface oscillations. This can be used as a crude estimation of the sufficient amplitude of the high-frequency surface vibrations that should be excited inside the wall. The second condition (22) says that the number of oscillations the cavity performs should be less that the inverse of this relative frequency modulation. This condition gives the estimated total time of the growing production of photons.

The production of photons inside a vibrating cavity due to the dynamical Casimir effect would be detected only if the parameters of our experimental setup are inside the width of the opto-mechanical resonance. Our simple and general conditions confronted with feasible parameters of mechanical waves which can be generated inside cavity walls suggest that an experimental evidence is now still out of reach.

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