## $\begin{array}{c} \sigma\text{-MESON AND } \omega\text{-}\rho \text{ MIXING EFFECTS} \\ \text{IN } \omega \rightarrow \pi^+\pi^-\gamma \text{ DECAY} \end{array}$

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We calculate the branching ratio of  $\omega \to \pi^+\pi^-\gamma$  decay in a phenomenological framework in which the contributions of VMD, chiral loops,  $\sigma$ -meson intermediate state amplitudes and the effects of  $\omega-\rho$  mixing are considered. We conclude that the  $\sigma$ -meson intermediate state amplitude and  $\omega-\rho$  mixing make substantial contribution to the branching ratio.

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The radiative decays of neutral vector mesons into a single photon and a pair of neutral pseudoscalar mesons have been a subject of continuous interest both theoretically and experimentally since their study provides tests for the theoretical ideas about the interesting mechanisms and new physics feature involved in these decays. The value  $B(\omega \to \pi^0 \pi^0 \gamma) =$  $(6.6^{+1.4}_{-0.8} \pm 0.6) \times 10^{-5}$  for the branching ratio of the  $\omega \to \pi^0 \pi^0 \gamma$  decay obtained in the recent experimental study of  $\rho \to \pi^0 \pi^0 \gamma$  and  $\omega \to \pi^0 \pi^0 \gamma$ decays by SND Collaboration [1] was larger than the theoretical estimates of this branching ratio to date which, therefore, requires reexamination of the mechanism of this decay.

The theoretical study of  $\omega \to \pi \pi \gamma$  decays was initiated by Singer [2] who postulated the sequential vector meson decay mechanism  $\omega \to (\rho)\pi \to \pi \pi \gamma$  involving the dominance of the intermediate vector meson contribution (VMD). Bramon *et al.*, [3] also considered the contribution of intermediate vector mesons to the vector meson decays into two pseudoscalars and a single photon  $V \to PP'\gamma$  using standard Lagrangians obeying the SU(3) symmetry. Bramon *et al.*, [4] later studied various such decays within the framework of chiral effective Lagrangians using chiral perturbation theory

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and they showed that there is no tree-level contribution to the amplitudes of such decay processes and the one-loop contributions are finite. Guetta and Singer [5] in a recent work combined all the improvements on the simple Born term of VMD mechanism. They considered  $\omega - \rho$  mixing, momentum dependence of intermediate state  $\rho$ -meson width, the inclusion of the chiral loop amplitude as given by Bramon *et al.*, [4], and using the resulting full amplitude which includes all these effects they obtained the theoretical result  $B(\omega \to \pi^0 \pi^0 \gamma) = (4.6 \pm 1.1) \times 10^{-5}$  for the branching ratio of the  $\omega \to \pi^0 \pi^0 \gamma$  decay which is considerably less than the latest experimental result [1].

Another interesting contribution to the decay mechanism of this decay may involve  $\sigma$ -meson as an intermediate state. The existence of  $\sigma$ -meson has long been controversial, however, recently a large number of analyses point to its existence [6]. The recent Fermilab E791 experiment found strong direct experimental evidence for  $\sigma$ -meson in the measured  $D^+ \to \sigma \pi^+ \to \pi \pi \pi$ decay [7]. Therefore, this important meson must be included in the analyses of hadronic processes and its contribution to the mechanisms of such processes should be examined. However, the nature and the quark substructure of  $\sigma$ -meson have not been established yet, whether it is a conventional  $q\bar{q}$ state or a  $\pi\pi$  resonance has been a subject of debate. Therefore, analysis of the role of  $\sigma$ -meson in  $V \to PP\gamma$  decays may also provide information about the controversial nature of  $\sigma$ -meson.

The three of the present authors in an attempt to explain the latest experimental result of the branching ratio of  $\omega \to \pi^0 \pi^0 \gamma$  decay, reconsidered the decay mechanism of this decay in a phenomenological framework in which the contributions of VMD and chiral loop amplitudes, the effects of the  $\omega - \rho$  mixing, and the contribution of  $\sigma$ -meson intermediate state amplitude are included, and as the result of their analysis they concluded that  $\sigma$ -meson intermediate state should be included in the decay mechanism of  $\omega \to \pi^0 \pi^0 \gamma$  decay in order to explain the latest experimental result, and utilizing the experimental value of the branching ratio of  $\omega \to \pi^0 \pi^0 \gamma$  decay. which resulted in a quadric equation for  $g_{\omega\sigma\gamma}$ , estimated the coupling constant  $g_{\omega\sigma\gamma}$  as  $g_{\omega\sigma\gamma} = 0.11$  and  $g_{\omega\sigma\gamma} = -0.21$  [8]. An essential assumption of that work was that there is no SU(3) vector meson-sigma-gamma vertex, therefore,  $\omega\sigma\gamma$ -vertex cannot be related to the  $\rho\sigma\gamma$ -vertex. The effects of  $\sigma$ -meson in the mechanism of radiative  $\rho^0$ -meson decays is included by assuming that  $\sigma$ -meson couples to  $\rho^0$ -meson through the pion-loop [8,9]. However, in the mechanism of  $\omega \to \pi^0 \pi^0 \gamma$  decay a  $\omega \sigma \gamma$ -vertex was assumed which may be considered as representing the effective final state interactions in  $\pi\pi$ -channel [8]. The small value of the coupling constant  $g_{\omega\sigma\gamma}$  obtained resulted in a change in the Born amplitude of  $\omega \to \pi^0 \pi^0 \gamma$  decay which was of the same order of magnitude as it is typical of final state interactions. Indeed, Levy and Singer [10] in a study of the  $\omega \to \pi^0 \pi^0 \gamma$  decay using dispersion-theoretical approach showed that final state interactions resulting in a decay rate of the same order of magnitude as the one calculated from the Born term can be parametrized with the effective-pole approximation.

In order to investigate the proposed role of  $\sigma$ -meson in radiative  $\rho^0$ - and  $\omega$ -meson decays further and to obtain more results that can be tested by experiment, we study in this work  $\omega \to \pi^+ \pi^- \gamma$  decay in a phenomenological approach by considering the vector meson dominance, chiral loops, and  $\sigma$ -meson intermediate state amplitudes as well as the effects of  $\omega - \rho$  mixing. We calculate the branching ratio of this decay and we obtain the photon spectra for the branching ratio of  $\omega \to \pi^+ \pi^- \gamma$  decay which can be tested experimentally. If only Born term VMD amplitude is used one has the relation  $\Gamma(\omega \to \pi^0 \pi^0 \gamma) = 1/2 \Gamma(\omega \to \pi^+ \pi^- \gamma)$  which follows from charge conjugation invariance to order  $\alpha$  which imposes pion pairs of even angular momentum as shown by Singer [2]. Since the factor 1/2 holds to the first order in  $\alpha$ , the present calculation is also of interest since the amplitude resulting from the assumed decay mechanism for  $\omega$ - and  $\rho$ -meson decays in the present work contains terms of order  $e^3$ .



Fig. 1. Feynman diagrams for the decay  $\omega \to \pi^+ \pi^- \gamma$ .

Our calculation is based on the Feynman diagrams shown in Fig. 1 for  $\omega \to \pi^+ \pi^- \gamma$  decay and in Fig. 2 for  $\rho^0 \to \pi^+ \pi^- \gamma$  decay. The direct terms shown in the diagrams in Fig. 2(a), (b), (c) are required to establish the gauge invariance. The  $\omega \rho \pi$ -vertex is described by the Wess–Zumino anomaly term of the chiral Lagrangian [11]

$$\mathcal{L}_{\omega\rho\pi}^{\text{eff}} = g_{\omega\rho\pi} \varepsilon^{\mu\nu\alpha\beta} \partial_{\mu} \omega_{\nu} \partial_{\alpha} \vec{\rho}_{\beta} \cdot \vec{\pi} \,. \tag{1}$$

This effective Lagrangian also defines the coupling constant  $g_{\omega\rho\pi}$  which was determined by Achasov *et al.*, [12] through an experimental analysis as  $g_{\omega\rho\pi} = (14.4 \pm 0.2) \text{ GeV}^{-1}$ . Similarly we describe the  $\rho\pi\gamma$ -vertex with the effective Lagrangian

$$\mathcal{L}_{\rho\pi\gamma}^{\text{eff}} = g_{\rho\pi\gamma} \varepsilon^{\mu\nu\alpha\beta} \partial_{\mu} \vec{\rho}_{\nu} \cdot \vec{\pi} \partial_{\alpha} A_{\beta} , \qquad (2)$$



Fig. 2. Feynman diagrams for the decay  $\rho^0 \to \pi^+ \pi^- \gamma$ .

and we use the experimental partial width of the radiative  $\rho \to \pi \gamma$  decay [13] to deduce the coupling constant  $g_{\omega\pi\gamma}$  as  $g_{\rho\pi\gamma} = (0.274 \pm 0.035) \,\text{GeV}^{-1}$ . The  $\omega\sigma\gamma$ -vertex is described by the effective Lagrangian

$$\mathcal{L}_{\omega\sigma\gamma}^{\text{eff}} = \frac{e}{M_{\omega}} g_{\omega\sigma\gamma} \partial^{\alpha} \omega^{\beta} (\partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha}) \sigma \,. \tag{3}$$

The coupling constant  $g_{\omega\sigma\gamma}$  was estimate as  $g_{\omega\sigma\gamma} = 0.11$  and  $g_{\omega\sigma\gamma} = -0.21$ by three of the present authors [8] in their analysis of  $\omega \to \pi^0 \pi^0 \gamma$  decay in a similar phenomenological framework. We describe the  $\sigma\pi\pi$ -vertex by the effective Lagrangian

$$\mathcal{L}_{\sigma\pi\pi}^{\text{eff}} = \frac{1}{2} g_{\sigma\pi\pi} M_{\sigma} \vec{\pi} \cdot \vec{\pi} \sigma \,, \tag{4}$$

and using the experimental values for  $M_{\sigma}$  and  $\Gamma_{\sigma}$  as  $M_{\sigma} = (483 \pm 31)$  MeV and  $\Gamma_{\sigma} = (338 \pm 48)$  MeV [7] where statistical and systematic errors are added in quadrature we determine the strong coupling constant  $g_{\sigma\pi\pi}$  as  $g_{\sigma\pi\pi} = (5.3 \pm 0.55)$ . The  $\rho\pi\pi$ -vertex is described by the effective Lagrangian

$$\mathcal{L}_{\rho\pi\pi}^{\text{eff}} = g_{\rho\pi\pi}\vec{\rho}_{\mu} \cdot \left(\partial^{\mu}\vec{\pi} \times \vec{\pi}\right),\tag{5}$$

and the experimental decay width of the decay  $\rho \to \pi \pi$  [13] yields the value  $g_{\rho\pi\pi} = (6.03 \pm 0.02)$  for the coupling constant  $g_{\rho\pi\pi}$ . The effective Lagrangians  $\mathcal{L}_{\sigma\pi\pi}^{\text{eff}}$  and  $\mathcal{L}_{\rho\pi\pi}^{\text{eff}}$  result from an extension of the  $\sigma$  model where

the isovector  $\rho$  is included through a Yang–Mills local gauge theory based on isospin with the vector meson mass generated through the Higgs mechanism [14].

Meson-meson interactions were studied by Oller and Oset [15] using the standard chiral Lagrangian in lowest order of chiral perturbation theory. We use their result for the isospin  $I = 0 \pi^+ \pi^- \rightarrow \pi^+ \pi^-$  amplitude that we need in the loop diagrams in Fig. 2(b), thus we neglect the small I = 2 amplitude. We note that as shown by Oller [16] due to gauge invariance the off-shell parts of the amplitudes, that should be kept inside the loop integration, do not contribute, and consequently the amplitude  $\mathcal{M}_{\chi}(\pi^+\pi^- \rightarrow \pi^+\pi^-)$  factorizes in the expression for the loop diagrams.

In our calculation of the invariant amplitude, we make the replacement  $q^2 - M^2 \rightarrow q^2 - M^2 + iM\Gamma$  in  $\rho$ -meson and  $\sigma$ -meson propagators. We use for  $\sigma$ -meson the momentum dependent width that follows from Eq. (4)

$$\Gamma_{\sigma} \left(q^{2}\right) = \Gamma_{\sigma} \frac{M_{\sigma}^{2}}{q^{2}} \sqrt{\frac{q^{2} - 4M_{\pi}^{2}}{M_{\sigma}^{2} - 4M_{\pi}^{2}}} \theta \left(q^{2} - 4M_{\pi}^{2}\right) , \qquad (6)$$

and for  $\rho$ -meson we use the following momentum dependent width as conventionally adopted [17]

$$\Gamma_{\rho}\left(q^{2}\right) = \Gamma_{\rho}\frac{M_{\rho}}{\sqrt{q^{2}}} \left(\frac{q^{2} - 4M_{\pi}^{2}}{M_{\rho}^{2} - 4M_{\pi}^{2}}\right)^{3/2} \theta\left(q^{2} - 4M_{\pi}^{2}\right) \,. \tag{7}$$

Loop integrals similar to the ones appearing in Fig. 1 and Fig. 2 were evaluated by Lucio and Pestiau [18] using dimensional regularization. We use their results and, for example, we express the contribution of the pion-loop amplitude corresponding to  $\rho^0 \rightarrow (\pi^+\pi^-)\gamma \rightarrow \pi^+\pi^-\gamma$  reaction in Fig. 2(b) as

$$\mathcal{M}_{\pi} = -\frac{e g_{\rho \pi \pi} \mathcal{M}_{\chi}(\pi^{+} \pi^{-} \to \pi^{+} \pi^{-})}{2\pi^{2} M_{\pi}^{2}} I(a, b) \left[ (p \cdot k) (\varepsilon \cdot u) - (p \cdot \varepsilon) (k \cdot u) \right],$$
(8)

where  $a = M_{\rho}^2/M_{\pi}^2$ ,  $b = (p-k)^2/M_{\pi}^2$ ,  $\mathcal{M}_{\chi} = -(2/f_{\pi}^2)(s+M_{\pi}^2/6)$ ,  $s = (p-k)^2$ ,  $f_{\pi} = 92.4$  MeV, p(u) and  $k(\varepsilon)$  being the momentum (polarization vector) of  $\rho$ -meson and photon, respectively. A similar amplitude corresponding to  $\rho^0 \to (\pi^+\pi^-)\gamma\sigma \to \pi^+\pi^-\gamma$  reaction can also be written. The function I(a, b) is given as

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2} \left[ f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right) \right] + \frac{a}{(a-b)^2} \left[ g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right) \right],$$
(9)

where

$$f(x) = \begin{cases} -\left[\arcsin\left(\frac{1}{2\sqrt{x}}\right)\right]^2, & x > \frac{1}{4} \\ \frac{1}{4}\left[\ln\left(\frac{\eta_+}{\eta_-}\right) - i\pi\right]^2, & x < \frac{1}{4} \end{cases}$$
$$g(x) = \begin{cases} (4x-1)^{\frac{1}{2}}\arcsin\left(\frac{1}{2\sqrt{x}}\right), & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{\frac{1}{2}}\left[\ln\left(\frac{\eta_+}{\eta_-}\right) - i\pi\right], & x < \frac{1}{4} \end{cases}$$
$$\eta_{\pm} = \frac{1}{2x}\left[1 \pm (1-4x)^{\frac{1}{2}}\right]. \tag{10}$$

We describe the  $\omega - \rho$  mixing by an effective Lagrangian of the form

$$\mathcal{L}_{\rho-\omega}^{\text{eff}} = \Pi_{\rho\omega}\omega_{\mu}\rho^{\mu}\,,\tag{11}$$

where  $\omega_{\mu}$  and  $\rho_{\mu}$  denote pure isospin field combinations. The mixing allows the transition  $\omega \to \rho$  in the process  $\omega \to \pi^+ \pi^- \gamma$ , thus the amplitude of the decay can be written as  $\mathcal{M} = \mathcal{M}_0 + \varepsilon \mathcal{M}'$  where  $\mathcal{M}_0$  includes the contributions coming from the diagrams shown in Fig. 1 for  $\omega \to \pi^+ \pi^- \gamma$  and  $\mathcal{M}'$ represents the contributions of the diagrams in Fig. 2 for  $\rho^0 \to \pi^+ \pi^- \gamma$ . The mixing parameter  $\varepsilon$  is given as [17]

$$\varepsilon = \frac{\Pi_{\rho\omega}}{M_{\omega}^2 - M_{\rho}^2 + iM_{\rho}\Gamma_{\rho} - iM_{\omega}\Gamma_{\omega}}.$$
(12)

O'Connell *et al.*, [17] determined  $\Pi_{\rho\omega}$  from fits to  $e^+e^- \rightarrow \pi^+\pi^-$  data as  $\Pi_{\rho\omega} = (-3800 \pm 370) \text{ MeV}^2$  from which the mixing parameter is obtained as  $\varepsilon = (-0.006 + i0.036)$ . Another effect of  $\omega - \rho$  mixing is that it modifies the  $\rho$ -propagator in diagrams in Fig. 1(a)

$$\frac{1}{D_{\rho}(s)} \to \frac{1}{D_{\rho}(s)} \left( 1 + \frac{g_{\omega\pi\gamma}}{g_{\rho\pi\gamma}} \frac{\Pi_{\rho\omega}}{D_{\omega}(s)} \right) , \qquad (13)$$

where  $D_{\rho}(s) = s - M_{\rho}^2 + iM_{\rho}\Gamma_{\rho}(s)$ . This effect is relevant since according to SU(3) relation  $g_{\omega\pi\gamma}/g_{\rho\pi\gamma} = 3$  it makes a sizable contribution.

We calculate the invariant amplitude  $\mathcal{M}(E_{\gamma}, E_1)$  this way for the decay  $\omega \to \pi^+\pi^-\gamma$  from the corresponding Feynman diagrams shown in Fig. 1 and Fig. 2 for the decays  $\omega \to \pi^+\pi^-\gamma$  and  $\rho^0 \to \pi^+\pi^-\gamma$ , respectively.

The differential decay probability for an unpolarized  $\omega$ -meson at rest is then given as

$$\frac{d\Gamma}{dE_{\gamma}dE_{1}} = \frac{1}{(2\pi)^{3}} \frac{1}{8M_{\omega}} |\mathcal{M}|^{2}, \qquad (14)$$

where  $E_{\gamma}$  and  $E_1$  are the photon and pion energies, respectively. We perform an average over the spin states of the vector meson and a sum over the polarization states of the photon. The decay width is then obtained by integration

$$\Gamma = \int_{E_{\gamma,\min}}^{E_{\gamma,\max}} dE_{\gamma} \int_{E_{1,\min}}^{E_{1,\max}} dE_1 \frac{d\Gamma}{dE_{\gamma}dE_1} \,. \tag{15}$$

Although the minimum photon energy is  $E_{\gamma,\min} = 0$ , in our calculations it is taken as  $E_{\gamma,\min} = 30$  MeV because of the presence of bremsstrahlung amplitude, and the maximum photon energy is given as  $E_{\gamma,\max} = (M_{\omega}^2 - 4M_{\pi}^2)/2M_{\omega} = 341$  MeV. The maximum and minimum values for pion energy  $E_1$ are given by

$$E_{1} = \frac{1}{2(2E_{\gamma}M_{\omega} - M_{\omega}^{2})} \times \left[ -2E_{\gamma}^{2}M_{\omega} + 3E_{\gamma}M_{\omega}^{2} - M_{\omega}^{3} \pm E_{\gamma}\sqrt{(-2E_{\gamma}M_{\omega} + M_{\omega}^{2})(-2E_{\gamma}M_{\omega} + M_{\omega}^{2} - 4M_{\pi}^{2})} \right].$$

As the result of our calculation, if we use the full amplitudes resulting from the Feynman diagrams in Fig. 1 and in Fig. 2, we obtain for the branching ratio of  $\omega \to \pi^+\pi^-\gamma$  decay the value  $B(\omega \to \pi^+\pi^-\gamma) = 0.43 \times 10^{-3}$  using the coupling constant  $g_{\omega\sigma\gamma} = 0.11$  and  $B(\omega \to \pi^+\pi^-\gamma) = 0.67 \times 10^{-3}$  if we use the coupling constant  $g_{\omega\sigma\gamma} = -0.21$ . These values are consistent with the present experimental upper limit  $B(\omega \to \pi^+\pi^-\gamma) < 3.6 \times 10^{-3}$  [13]. If we use only the VMD amplitude for  $\omega \to \pi^+\pi^-\gamma$  decay we obtain the branching ratio  $B(\omega \to \pi^+\pi^-\gamma) = 7.2 \times 10^{-5}$ . If we use the VMD amplitude for  $\omega \to \pi^+\pi^-\gamma$  and the bremsstrahlung amplitude for  $\rho^0 \to \pi^+\pi^-\gamma$ , as a result of  $\omega - \rho$  mixing we obtain the branching ratio  $B(\omega \to \pi^+\pi^-\gamma) = 0.46 \times 10^{-3}$ . On the other hand, if we use VMD and  $\sigma$ -meson intermediate state amplitudes for  $\omega \to \pi^+\pi^-\gamma$  decay and do not consider  $\omega - \rho$  mixing, the resulting branching ratio is  $B(\omega \to \pi^+\pi^-\gamma) = 0.13 \times 10^{-3}$  for  $g_{\omega\sigma\gamma} = 0.11$  and  $B(\omega \to \pi^+\pi^-\gamma) = 0.12 \times 10^{-3}$  for  $g_{\omega\sigma\gamma} = -0.21$ . The values of the branching ratio  $B(\omega \to \pi^+\pi^-\gamma)$  resulting from the different amplitudes for the

The values of the branching ratio  $B(\omega \to \pi^+ \pi^- \gamma)$  resulting from the VMD amplitude, VMD amplitude and  $(\omega - \rho)$  mixing with the bremsstrahlung amplitude,  $\sigma$ -meson intermediate state amplitude, VMD and  $\sigma$ -meson intermediate state amplitude, and full amplitude including VMD,  $\sigma$ -meson intermediate state and  $(\omega - \rho)$  mixing for the coupling constant  $g_{\omega\sigma\gamma} = 0.11$ .

Amplitude	VMD	VMD+	$\sigma$	${ m VMD}{+}\sigma$	${ m VMD}{+}\sigma$
		$(\omega -  ho)$			$+(\omega -  ho)$
$B(\omega \to \pi^+ \pi^- \gamma) \times 10^5$	7.2	46	2.5	13	43

coupling constant  $g_{\omega\sigma\gamma} = 0.11$  are shown in Table I for comparison. These values show the importance of not only  $\omega - \rho$  mixing but also of the  $\sigma$ -meson intermediate state amplitude in  $\omega \to \pi^+ \pi^- \gamma$  decay.



Fig. 3. The photon spectra for the branching ratio of  $\omega \to \pi^+ \pi^- \gamma$  decay for  $g_{\omega\sigma\gamma} = 0.11$ . The separate contributions resulting from the amplitudes of VMD; VMD and bremsstrahlung with  $\omega - \rho$  mixing; VMD and bremsstrahlung, chiral loop with  $\omega - \rho$  mixing; and from the full amplitude using the diagrams in Fig. 1 and in Fig. 2 including  $\sigma$ -meson intermediate state with  $\omega - \rho$  mixing.

The photon spectra for the branching ratio of the decay  $\omega \to \pi^+ \pi^- \gamma$ , which may be tested in future experiments, are plotted in Fig. 3 for  $g_{\omega\sigma\gamma} =$ 0.11 and in Fig. 4 for  $g_{\omega\sigma\gamma} = -0.21$  as a function of photon energy  $E_{\gamma}$ , and the contributions of different amplitudes are indicated. We note that the bremsstrahlung amplitude of  $\rho^0 \to \pi^+ \pi^- \gamma$  decay as the result of  $\omega - \rho$  mixing affects mostly the lower part of the photon spectra, changing it drastically, but becomes practically negligible toward the higher photon energy part of the spectrum. Therefore, the model and the ideas about the role of  $\omega - \rho$ mixing and the importance of the contribution of  $\sigma$ -meson intermediate state in  $\omega \to \pi^+ \pi^- \gamma$  decay presented in this work can be tested experimentally if few events are collected at the characteristic parts of the photon spectra.



Fig. 4. The photon spectra for the branching ratio of  $\omega \to \pi^+ \pi^- \gamma$  decay for  $g_{\omega\sigma\gamma} = -0.21$ . The separate contributions resulting from the amplitudes of VMD; VMD and bremsstrahlung with  $\omega - \rho$  mixing; VMD and bremsstrahlung, chiral loop with  $\omega - \rho$  mixing; and from the full amplitude using the diagrams in Fig. 1 and in Fig. 2 including  $\sigma$ -meson intermediate state with  $\omega - \rho$  mixing.

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