Vol. 34 (2003) ACTA PHYSICA POLONICA B No 8

# FREE MASSLESS SPIN-5/2 GAUGE FIELDS FROM THE LAGRANGIAN $Sp(3)$ BRST METHOD

S.O. Saliu

Faculty of Physics, University of Craiova
13 A. I. Cuza Str., Craiova RO-1100, Romania
e-mail: osaliu@central.ucv.ro

(Received March 17, 2003)

An irreducible version of free massless spin-5/2 gauge fields is analyzed from the point of view of the Lagrangian $Sp(3)$ BRST method. The irreducible formulation is obtained by means of introducing one purely gauge supplementary Majorana spinor. An appropriate gauge-fixing procedure is developed, such as to benefit from a direct link with the standard antifield-BRST method. The comparison with related results from the literature is discussed.

PACS numbers: 11.10.Ef

## 1. Introduction

The key point in the development of the BRST method is represented by the understanding of the BRST symmetry on a cohomological basis [1–7]. Although they do not play such an important role like the BRST symmetry itself, the extended versions of this symmetry helped at explaining some less known aspects. In this light, the $Sp(2)$ version [8–11] is important to many issues, like the study of renormalizability, the analysis of anomalies, and the understanding of the structure of non-minimal sectors involved with the standard antifield-BRST setting. The raised interest for constructing even more extended symmetries, like the antibracket-antifield $Sp(3)$ BRST symmetry [12], is due to many reasons, among which we mention: (a) the appearance of a larger spectrum of ghosts/antifields, which includes that present within the $Sp(2)$ version, and, moreover, ensures a more flexible choice and interpretation of non-minimal variables from the standard antibracket-antifield method; (b) the possibility to develop a gauge-fixing procedure based only on one fermionic functional, like in the standard method, that leaves an acceptable freedom within the class of eligible gauge-fixing conditions on the fields and ghosts, and thus a more transparent relationship with the gauge-fixed actions arising from the antibracket-antifield quantization; (c) the revealing of a beautiful and rich algebraic structure of differential tricomplex and homological triresolution; (d) the construction of a simple complex via a total degree, whose associated (co)homology remains isomorphic with those of the composing differentials.

In this paper we realize the construction of the Lagrangian $Sp(3)$ BRST symmetry for an irreducible formulation of free massless spin-5/2 fields. Free massless higher spin gauge fields [13–20] are important due to their connection with string theory, and, because of their remarkable gauge symmetries, they are promising candidates for building a unified physical theory. In the meantime, the existence of a large class of non-trivial interacting higher spin gauge theories [21], at least in four dimensions, reveals the necessity of investigating this type of models. Initially, we develop an irreducible formulation of free massless spin-5/2 fields by means of adding one purely gauge Majorana spinor. Next, we remain within the irreducible setting, and implement the following steps: *(i)* we triplicate the gauge transformations, and consequently derive the ghost and antifield spectra underlying the antibracket-antifield $Sp(3)$ tricomplex; *(ii)* we solve the fundamental equation of the Lagrangian $Sp(3)$ BRST formalism, called the extended classical master equation; *(iii)* we develop a gauge-fixing procedure that ensures a direct link with the standard antibracket-antifield approach, and consequently obtain the gauge-fixed action. The resulting gauge-fixed action presents all required physical characteristics, like spacetime locality, manifest Lorentz covariance and propagating behavior.

Our paper is structured in five sections. In Sec. 2 we derive an irreducible formulation of free massless spin-5/2 fields. Section 3 deals with the construction of the Lagrangian $Sp(3)$ BRST symmetry for this irreducible version. In Sec. 4 we apply a gauge-fixing procedure that enforces a direct relationship with the standard antifield-BRST approach, and infer the form of the gauge-fixed action. Finally, in Sec. 5 we discuss the link with other results from the literature.

## 2. Irreducible approach to free massless spin-5/2 gauge fields

We begin with the Lagrangian action for free massless spin-5/2 gauge fields [14, 19]

$$\begin{aligned} S_0^{\mathrm{L}}\left[\psi_{\mu\nu}\right] \;=\; \int d^4x \,\big( & -\tfrac{1}{2}\bar{\psi}_{\mu\nu}\partial\!\!\!/\psi_{\mu\nu} - \bar{\psi}_{\mu\nu}\gamma_\nu\partial\!\!\!/\gamma_\lambda\psi_{\lambda\mu} + 2\bar{\psi}_{\mu\nu}\gamma_\nu\partial_\lambda\psi_{\lambda\mu} \\ & +\tfrac{1}{4}\bar{\psi}_{\lambda\lambda}\partial\!\!\!/\psi_{\mu\mu} - \bar{\psi}_{\lambda\lambda}\partial_\mu\gamma_\nu\psi_{\mu\nu}\big)\,, \end{aligned} \tag{2.1}$$

where $\psi_{\mu\nu}$ is a symmetric tensorial Majorana spinor, the bar operation signifies spinor conjugation, and

$$\partial\!\!\!/ \equiv \gamma_\mu \partial_\mu \,. \tag{2.2}$$

In the sequel, we work with the Pauli metric ($\mu = 1, 2, 3, 4$) and Hermitian $\gamma$-matrices satisfying

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\delta_{\mu\nu} \,. \tag{2.3}$$

Action (2.1) is invariant under the Abelian gauge transformations

$$\delta_\epsilon \psi_{\mu\nu} = \left(\delta_{\nu\beta}\partial_\mu + \delta_{\mu\beta}\partial_\nu\right)\left(\delta_{\beta\alpha}\mathbf{1} - \tfrac{1}{4}\gamma_\beta\gamma_\alpha\right)\epsilon_\alpha \equiv Z_{\mu\nu\alpha}\epsilon_\alpha \tag{2.4}$$

with the gauge parameter $\epsilon_\alpha$ a fermionic Majorana vector spinor. Here and in the sequel the notation $\mathbf{1}$ signifies the unit matrix in the space of spinors. The transformations (2.4) are off-shell first-stage reducible

$$Z_{\mu\nu\alpha} Z_\alpha = 0 \tag{2.5}$$

with the reducibility functions

$$Z_\alpha = \gamma_\alpha \tag{2.6}$$

since if we perform the transformation $\epsilon_\alpha = \gamma_\alpha \epsilon$, with $\epsilon$ an arbitrary Majorana spinor, then we have that the gauge variation of the spin-5/2 fields identically vanishes, $\delta_\epsilon \psi_{\mu\nu} = 0$.

At this point we infer an irreducible formulation of the model under study, following the general line from Ref. [22]. Consider an arbitrary gauge theory, described by the Lagrangian action $S_0^{\mathrm{L}}[\Phi^{\alpha_0}]$, and invariant under the gauge transformations

$$\delta_\epsilon \Phi^{\alpha_0} = Z^{\alpha_0}_{\;\;\alpha_1} \epsilon^{\alpha_1}, \;\; \alpha_1 = 1, \cdots, M_1 \tag{2.7}$$

that are assumed to be first-stage reducible

$$Z^{\alpha_0}_{\;\;\alpha_1} Z^{\alpha_1}_{\;\;\alpha_2} \approx 0, \;\; \alpha_2 = 1, \cdots, M_2 \,, \tag{2.8}$$

where the weak equality leaves the possibility to deal with the general case where the reducibility holds on-shell. Acting like in Ref. [22], we pass to a new gauge theory, based on the irreducible gauge transformations (2.7) and

$$\delta_\epsilon \Phi^{\alpha_2} = A_{\alpha_1}^{\;\;\alpha_2} \epsilon^{\alpha_1} \,, \tag{2.9}$$

where $\Phi^{\alpha_2}$ are some purely gauge fields (that do not enter the Lagrangian action $S_0^{\mathrm{L}}[\Phi^{\alpha_0}]$), and $A_{\alpha_1}^{\;\;\alpha_2}$ are some functions that may involve at most the original fields $\Phi^{\alpha_0}$, taken to satisfy the condition

$$\mathrm{rank}\left(Z^{\alpha_1}_{\;\;\alpha_2} A_{\alpha_1}^{\;\;\beta_2}\right) = M_2 \,. \tag{2.10}$$

Under these circumstances, it has been shown in Ref. [22] that the standard antifield-BRST symmetry for the irreducible theory, based on the gauge transformations (2.7) and (2.9), truly exists. Moreover, the BRST symmetries $s$ and $s_{\mathrm{R}}$ associated with the irreducible, respectively, reducible gauge theories are essentially related through $s^2 = 0 = s_{\mathrm{R}}^2$ and $H^0(s) \simeq H^0(s_{\mathrm{R}})$. The last formulas indicate that the BRST symmetries $s$ and $s_{\mathrm{R}}$ are equivalent from the point of view of the fundamental equations of the antifield-BRST formalism, namely, the nilpotency of the BRST operator and the isomorphism between the zeroth-order cohomological space of the BRST differential and the algebra of physical observables. In view of this equivalence (since the physical content of the theory is not changed), one is allowed to replace the standard reducible antifield-BRST symmetry with the irreducible one. Then, according to the main observation from Ref. [12], that the antibracket-antifield $Sp(3)$ symmetry for a given gauge theories exists as long as the standard antifield-BRST symmetry can consistently be provided, we conclude that it is indeed legitimate to develop the Lagrangian $Sp(3)$ BRST method for the irreducible version instead of that for the initial reducible gauge theory.

In the case of free massless spin-5/2 gauge fields, we have that

$$\Phi^{\alpha_0} \to \psi_{\mu\nu},\ Z^{\alpha_0}_{\ \ \alpha_1} \to Z_{\mu\nu\alpha},\ Z^{\alpha_1}_{\ \ \alpha_2} \to \gamma_\alpha\,. \tag{2.11}$$

If we take

$$A_{\alpha_1}^{\ \ \alpha_2} \to A_\alpha = \partial_\alpha \mathbf{1}\,, \tag{2.12}$$

the condition (2.10) is indeed fulfilled, since $Z^{\alpha_1}_{\ \ \alpha_2} A_{\alpha_1}^{\ \ \beta_2} \to \partial\!\!\!/$, which is clearly invertible (its inverse is $\partial\!\!\!//\Box$). In consequence, the gauge transformations (2.9) take the concrete form

$$\delta_\epsilon \Phi = A_\alpha \epsilon_\alpha \equiv \partial_\alpha \epsilon_\alpha\,, \tag{2.13}$$

where the role of the purely gauge fields $\Phi^{\alpha_2}$ is played here by one fermionic Majorana spinor $\Phi$. It is easy to see that the new gauge transformations, (2.4) and (2.13), form a complete set of gauge transformations (generating set) for the action (2.1), that is irreducible and determines an Abelian gauge algebra, just like the original first-stage reducible generating set (2.4).

Based on the above discussion, in the sequel we work with the irreducible formulation of free massless spin-5/2 gauge fields, characterized by the action (2.1) and the gauge symmetries (2.4) and (2.13), to whom we apply the antibracket-antifield $Sp(3)$ BRST method [12].

## 3. "Irreducible" Lagrangian $Sp(3)$ symmetry

### *3.1. Sp(3) BRST tricomplex*

The main idea underlying the development of the Lagrangian $Sp(3)$ BRST symmetry related to the irreducible model constructed in the previous section is to triplicate both the gauge generators and gauge parameters, and thus to replace (2.4) and (2.13) with the modified transformations

$$\delta_{\bar{\epsilon}}\Phi^i = R^i_{\bar{\alpha}}\epsilon^{\bar{\alpha}}, \tag{3.1}$$

where

$$\Phi^i = \begin{pmatrix} \psi_{\mu\nu} \\ \Phi \end{pmatrix},\ \epsilon^{\bar{\alpha}} = \begin{pmatrix} \epsilon_{1\alpha} \\ \epsilon_{2\alpha} \\ \epsilon_{3\alpha} \end{pmatrix}, \tag{3.2}$$

$$R^i_{\bar{\alpha}} = \begin{pmatrix} Z_{\mu\nu\alpha} & Z_{\mu\nu\alpha} & Z_{\mu\nu\alpha} \\ A_\alpha & A_\alpha & A_\alpha \end{pmatrix}, \tag{3.3}$$

where $Z_{\mu\nu\alpha}$ and $A_\alpha$ are given in (2.4) and (2.12). Accordingly, the relations (3.1) take the concrete form

$$\delta_\epsilon\psi_{\mu\nu} = (\delta_{\nu\beta}\partial_\mu + \delta_{\mu\beta}\partial_\nu)\left(\delta_{\beta\alpha}\mathbf{1} - \tfrac{1}{4}\gamma_\beta\gamma_\alpha\right)(\epsilon_{1\alpha} + \epsilon_{2\alpha} + \epsilon_{3\alpha}), \tag{3.4}$$

$$\delta_\epsilon\Phi = \partial_\alpha(\epsilon_{1\alpha} + \epsilon_{2\alpha} + \epsilon_{3\alpha}), \tag{3.5}$$

and they are off-shell second-stage reducible, with the first-, respectively, second-stage reducibility functions

$$Z^{\bar{\alpha}}_{\bar{\beta}} = \begin{pmatrix} \mathbf{0} & \delta_{\alpha\beta}\mathbf{1} & -\delta_{\alpha\beta}\mathbf{1} \\ -\delta_{\alpha\beta}\mathbf{1} & \mathbf{0} & \delta_{\alpha\beta}\mathbf{1} \\ \delta_{\alpha\beta}\mathbf{1} & -\delta_{\alpha\beta}\mathbf{1} & \mathbf{0} \end{pmatrix}, \tag{3.6}$$

$$Z^{\bar{\beta}}_{\gamma} = \begin{pmatrix} -\delta_{\beta\gamma}\mathbf{1} \\ -\delta_{\beta\gamma}\mathbf{1} \\ -\delta_{\beta\gamma}\mathbf{1} \end{pmatrix}, \tag{3.7}$$

where the first- and second-stage reducibility relations are

$$R^i_{\bar{\alpha}}Z^{\bar{\alpha}}_{\bar{\beta}} = 0,\ Z^{\bar{\alpha}}_{\bar{\beta}}Z^{\bar{\beta}}_{\gamma} = 0\,. \tag{3.8}$$

We cannot stress enough that the reducibility relations (3.8) are completely due to the triplication of the gauge transformations in the $Sp(3)$ setting, and are not related in any point to the reducible formulation of free massless spin 5/2-fields.

As the main result from Ref. [12] states, it is then possible to consistently construct the Lagrangian $Sp(3)$ BRST symmetry for the irreducible model under study. This reduces to the construction of a BRST tricomplex, generated by three anticommuting differentials $(s_m)_{m=1,2,3}$

$$s_m s_n + s_n s_m = 0\,, m, n = 1, 2, 3\,, \tag{3.9}$$

each of them decomposing like

$$s_m = \delta_m + D_m + \cdots\,. \tag{3.10}$$

The operators $(\delta_m)_{m=1,2,3}$ are the three differentials from the Koszul–Tate tricomplex, which is required to furnish a triresolution of smooth functions defined on the stationary surface of field equations, while $(D_m)_{m=1,2,3}$ represent the exterior longitudinal derivatives associated with the new second-stage redundant description of the gauge orbits due to the triplication of the gauge transformations like in (3.4),(3.5). The graduation of the $Sp(3)$ BRST algebra is expressed in terms of the ghost tridegree

$$\mathrm{trigh} = (\mathrm{gh}_1, \mathrm{gh}_2, \mathrm{gh}_3)\ , \tag{3.11}$$

where we set

$$\mathrm{trigh}\,(s_1) = (1,0,0)\ ,\ \mathrm{trigh}\,(s_2) = (0,1,0)\ ,\ \mathrm{trigh}\,(s_3) = (0,0,1)\ . \tag{3.12}$$

On account of the triplication of the gauge transformations and of the induced redundancy structure, we find that the ghost spectrum from the exterior longitudinal tricomplex is organized like

$$\overset{(1,0,0)}{\eta}{}_{1\alpha},\ \overset{(0,1,0)}{\eta}{}_{2\alpha}\,,\ \overset{(0,0,1)}{\eta}{}_{3\alpha},\ \overset{(0,1,1)}{\pi}{}_{1\alpha}\,,\ \overset{(1,0,1)}{\pi}{}_{2\alpha},\ \overset{(1,1,0)}{\pi}{}_{3\alpha}\,,\ \overset{(1,1,1)}{\lambda}{}_{\alpha}\,, \tag{3.13}$$

where $\overset{(i,j,k)}{F}$ denotes an element with the ghost tridegree equal to $(i,j,k)$. It is understood that

$$\mathrm{trigh}\,(\psi_{\mu\nu}) = \mathrm{trigh}\,(\varPhi) = (0,0,0)\ . \tag{3.14}$$

All ghosts are Majorana vector spinors, with the Grassmann parities

$$\varepsilon\,(\eta_{m\alpha}) = \varepsilon\,(\lambda_\alpha) = 0,\ \varepsilon\,(\pi_{m\alpha}) = 1,\ m = 1,2,3\,. \tag{3.15}$$

An essential feature of the Lagrangian $Sp(3)$ formalism is represented by the presence of three antibrackets [12], denoted by $(,)_m$, $m = 1, 2, 3$, which yields that we need three antifields (star variables) conjugated to each field/ghost, one for each antibracket, like below

$$\overset{(-1,0,0)*(1)}{\bar{\psi}}{}_{\mu\nu}\,,\quad \overset{(0,-1,0)*(2)}{\bar{\psi}}{}_{\mu\nu}\,,\quad \overset{(0,0,-1)*(3)}{\bar{\psi}}{}_{\mu\nu}\,,\quad \overset{(-1,0,0)*(1)}{\bar{\varPhi}}\,,\quad \overset{(0,-1,0)*(2)}{\bar{\varPhi}}\,,\quad \overset{(0,0,-1)*(3)}{\bar{\varPhi}}\,, \tag{3.16}$$

$$\overset{(-2,0,0)*(1)}{\bar{\eta}}{}_{1\alpha}\,,\quad \overset{(-1,-1,0)*(2)}{\bar{\eta}}{}_{1\alpha}\,,\quad \overset{(-1,0,-1)*(3)}{\bar{\eta}}{}_{1\alpha}\,,\quad \overset{(-1,-1,0)*(1)}{\bar{\eta}}{}_{2\alpha}\,,\quad \overset{(0,-2,0)*(2)}{\bar{\eta}}{}_{2\alpha}\,, \tag{3.17}$$

$$\overset{(0,-1,-1)*(3)}{\bar{\eta}}{}_{2\alpha}\,,\quad \overset{(-1,0,-1)*(1)}{\bar{\eta}}{}_{3\alpha}\,,\quad \overset{(0,-1,-1)*(2)}{\bar{\eta}}{}_{3\alpha}\,,\quad \overset{(0,0,-2)*(3)}{\bar{\eta}}{}_{3\alpha}\,,\quad \overset{(-1,-1,-1)*(1)}{\bar{\pi}}{}_{1\alpha}\,, \tag{3.18}$$

$$\overset{(0,-2,-1)*(2)}{\bar{\pi}}{}_{1\alpha}\,,\quad \overset{(0,-1,-2)*(3)}{\bar{\pi}}{}_{1\alpha}\,,\quad \overset{(-2,0,-1)*(1)}{\bar{\pi}}{}_{2\alpha}\,,\quad \overset{(-1,-1,-1)*(2)}{\bar{\pi}}{}_{2\alpha}\,,\quad \overset{(-1,0,-2)*(3)}{\bar{\pi}}{}_{2\alpha}\,, \tag{3.19}$$

$$\overset{(-2,-1,0)*(1)}{\bar{\pi}}{}_{3\alpha}\,,\quad \overset{(-1,-2,0)*(2)}{\bar{\pi}}{}_{3\alpha}\,,\quad \overset{(-1,-1,-1)*(3)}{\bar{\pi}}{}_{3\alpha}\,, \tag{3.20}$$

$$\overset{(-2,-1,-1)*(1)}{\bar{\lambda}}{}_{\alpha}\,,\quad \overset{(-1,-2,-1)*(2)}{\bar{\lambda}}{}_{\alpha}\,,\quad \overset{(-1,-1,-2)*(3)}{\bar{\lambda}}{}_{\alpha}\,. \tag{3.21}$$

Still, this antifield spectrum cannot guarantee the basic properties of nilpotency and acyclicity of the Koszul–Tate differentials. In order to surpass this inconvenience, we need to enlarge it with the bar (B) and tilde (T) variables [12]

$$\overset{(0,-1,-1)^{(1)}}{\bar{\psi}^{\mathrm{B}}}{}_{\mu\nu},\quad \overset{(-1,0,-1)^{(2)}}{\bar{\psi}^{\mathrm{B}}}{}_{\mu\nu},\quad \overset{(-1,-1,0)^{(3)}}{\bar{\psi}^{\mathrm{B}}}{}_{\mu\nu},\quad \overset{(0,-1,-1)^{(1)}}{\bar{\varPhi}^{\mathrm{B}}},\quad \overset{(-1,0,-1)^{(2)}}{\bar{\varPhi}^{\mathrm{B}}},\quad \overset{(-1,-1,0)^{(3)}}{\bar{\varPhi}^{\mathrm{B}}}, \tag{3.22}$$

$$\overset{(-1,-1,-1)^{(1)}}{\bar{\eta}^{\mathrm{B}}}{}_{1\alpha},\quad \overset{(-2,0,-1)^{(2)}}{\bar{\eta}^{\mathrm{B}}}{}_{1\alpha},\quad \overset{(-2,-1,0)^{(3)}}{\bar{\eta}^{\mathrm{B}}}{}_{1\alpha},\quad \overset{(0,-2,-1)^{(1)}}{\bar{\eta}^{\mathrm{B}}}{}_{2\alpha},\quad \overset{(-1,-1,-1)^{(2)}}{\bar{\eta}^{\mathrm{B}}}{}_{2\alpha}, \tag{3.23}$$

$$\overset{(-1,-2,0)^{(3)}}{\bar{\eta}^{\mathrm{B}}}{}_{2\alpha},\quad \overset{(0,-1,-2)^{(1)}}{\bar{\eta}^{\mathrm{B}}}{}_{3\alpha},\quad \overset{(-1,0,-2)^{(2)}}{\bar{\eta}^{\mathrm{B}}}{}_{3\alpha},\quad \overset{(-1,-1,-1)^{(3)}}{\bar{\eta}^{\mathrm{B}}}{}_{3\alpha},\quad \overset{(0,-2,-2)^{(1)}}{\bar{\pi}^{\mathrm{B}}}{}_{1\alpha}, \tag{3.24}$$

$$\overset{(-1,-1,-2)^{(2)}}{\bar{\pi}^{\mathrm{B}}}{}_{1\alpha},\quad \overset{(-1,-2,-1)^{(3)}}{\bar{\pi}^{\mathrm{B}}}{}_{1\alpha},\quad \overset{(-1,-1,-2)^{(1)}}{\bar{\pi}^{\mathrm{B}}}{}_{2\alpha},\quad \overset{(-2,0,-2)^{(2)}}{\bar{\pi}^{\mathrm{B}}}{}_{2\alpha},\quad \overset{(-2,-1,-1)^{(3)}}{\bar{\pi}^{\mathrm{B}}}{}_{2\alpha}, \tag{3.25}$$

$$\overset{(-1,-2,-1)^{(1)}}{\bar{\pi}^{\mathrm{B}}}{}_{3\alpha},\quad \overset{(-2,-1,-1)^{(2)}}{\bar{\pi}^{\mathrm{B}}}{}_{3\alpha},\quad \overset{(-2,-2,0)^{(3)}}{\bar{\pi}^{\mathrm{B}}}{}_{3\alpha}, \tag{3.26}$$

$$\overset{(-1,-2,-2)^{(1)}}{\bar{\lambda}^{\mathrm{B}}}{}_{\alpha}\,,\quad \overset{(-2,-1,-2)^{(2)}}{\bar{\lambda}^{\mathrm{B}}}{}_{\alpha}\,,\quad \overset{(-2,-2,-1)^{(3)}}{\bar{\lambda}^{\mathrm{B}}}{}_{\alpha}\,, \tag{3.27}$$

$$\overset{(-1,-1,-1)}{\bar{\psi}^{\mathrm{T}}}{}_{\mu\nu},\quad \overset{(-1,-1,-1)}{\bar{\varPhi}^{\mathrm{T}}}{}_{\alpha},\quad \overset{(-2,-1,-1)}{\bar{\eta}^{\mathrm{T}}}{}_{1\alpha},\quad \overset{(-1,-2,-1)}{\bar{\eta}^{\mathrm{T}}}{}_{2\alpha},\quad \overset{(-1,-1,-2)}{\bar{\eta}^{\mathrm{T}}}{}_{3\alpha}, \tag{3.28}$$

$$\overset{(-1,-2,-2)}{\bar{\pi}^{\mathrm{T}}}{}_{1\alpha},\quad \overset{(-2,-1,-2)}{\bar{\pi}^{\mathrm{T}}}{}_{2\alpha},\quad \overset{(-2,-2,-1)}{\bar{\pi}^{\mathrm{T}}}{}_{3\alpha},\quad \overset{(-2,-2,-2)}{\bar{\lambda}^{\mathrm{T}}}{}_{\alpha}\,. \tag{3.29}$$

All antifields (star, bar and tilde) must be viewed like conjugated Majorana spinors, with the Grassmann parities of the star and tilde ones (generically denoted by $\bar{\Phi}_A^{*(m)}$ and $\bar{\Phi}_A^{\mathrm{T}}$) opposite to those of the corresponding fields or ghosts, while the Grassmann parities of the bar variables (collectively denoted by $\bar{\Phi}_A^{\mathrm{B}(m)}$) coincide with those of the associated fields/ghosts. The main characteristics of the antifields are appropriately synthesized with the help of the formulas

$$\varepsilon\left(\bar{\Phi}_A^{*(m)}\right) = (\varepsilon_A+1) \bmod 2 = \varepsilon\left(\bar{\Phi}_A^{\mathrm{T}}\right),\ \varepsilon\left(\bar{\Phi}_A^{\mathrm{B}(m)}\right) = \varepsilon_A\,, \tag{3.30}$$

$$\mathrm{trigh}\left(\bar{\Phi}_A^{*(1)}\right) = \left(-\mathrm{gh}_1\left(\Phi_A\right)-1, -\mathrm{gh}_2\left(\Phi_A\right), -\mathrm{gh}_3\left(\Phi_A\right)\right)\,, \tag{3.31}$$

$$\mathrm{trigh}\left(\bar{\Phi}_A^{*(2)}\right) = \left(-\mathrm{gh}_1\left(\Phi_A\right), -\mathrm{gh}_2\left(\Phi_A\right)-1, -\mathrm{gh}_3\left(\Phi_A\right)\right)\,, \tag{3.32}$$

$$\mathrm{trigh}\left(\bar{\Phi}_A^{*(3)}\right) = \left(-\mathrm{gh}_1\left(\Phi_A\right), -\mathrm{gh}_2\left(\Phi_A\right), -\mathrm{gh}_3\left(\Phi_A\right)-1\right)\,, \tag{3.33}$$

$$\mathrm{trigh}\left(\bar{\Phi}_A^{\mathrm{B}(1)}\right) = \left(-\mathrm{gh}_1\left(\Phi_A\right), -\mathrm{gh}_2\left(\Phi_A\right)-1, -\mathrm{gh}_3\left(\Phi_A\right)-1\right)\,, \tag{3.34}$$

$$\mathrm{trigh}\left(\bar{\Phi}_A^{\mathrm{B}(2)}\right) = \left(-\mathrm{gh}_1\left(\Phi_A\right)-1, -\mathrm{gh}_2\left(\Phi_A\right), -\mathrm{gh}_3\left(\Phi_A\right)-1\right)\,, \tag{3.35}$$

$$\mathrm{trigh}\left(\bar{\Phi}_A^{\mathrm{B}(3)}\right) = \left(-\mathrm{gh}_1\left(\Phi_A\right)-1, -\mathrm{gh}_2\left(\Phi_A\right)-1, -\mathrm{gh}_3\left(\Phi_A\right)\right)\,, \tag{3.36}$$

$$\mathrm{trigh}\left(\bar{\Phi}_A^{\mathrm{T}}\right) = \left(-\mathrm{gh}_1\left(\Phi_A\right)-1, -\mathrm{gh}_2\left(\Phi_A\right)-1, -\mathrm{gh}_3\left(\Phi_A\right)-1\right)\,. \tag{3.37}$$

where $\Phi_A$ stands for all fields and ghosts

$$\Phi_A = \left(\psi_{\mu\nu}, \Phi, \eta_{m\alpha}, \pi_{m\alpha}, \lambda_\alpha\right)\,, \tag{3.38}$$

and $\varepsilon_A$ signifies the Grassmann parity of a given $\Phi_A$. The supplementary superscript between parentheses carried by the antifields (3.16)–(3.21) emphasizes in which antibracket are they (anti)canonically conjugated to the corresponding field/ghost

$$\left(\Phi_A, \bar{\Phi}_B^{*(m)}\right)_n = \delta_{mn}\delta_{AB}. \tag{3.39}$$

The antifield sector is also graded by the resolution tridegree, defined as $\mathrm{trires} = (\mathrm{res}_1, \mathrm{res}_2, \mathrm{res}_3) = -\mathrm{trigh}$, while the induced simple grading, named total resolution degree, is $\mathrm{res} = \mathrm{res}_1 + \mathrm{res}_2 + \mathrm{res}_3$, and is found useful at solving the fundamental equation of the $Sp(3)$ formalism, namely, the extended classical master equation.

### 3.2. *Extended classical master equation*

Instead of constructing the $Sp(3)$ algebra (3.9), and hence three differentials, it is easier to determine one functional only, namely, the anticanonical generator $S$ of this symmetry

$$sF = (F,S) + VF \Leftrightarrow s_m F = (F,S)_m + V_m F,\ m = 1,2,3\,, \tag{3.40}$$

which is bosonic and constrained to satisfy the extended classical master equation

$$\tfrac{1}{2}(S,S) + VS = 0 \Leftrightarrow \frac{1}{2}(S,S)_m + V_m S = 0,\ m = 1,2,3 \tag{3.41}$$

as well as the property $\mathrm{trigh}\,(S) = (0,0,0)$. The symbol $(,)$ denotes the total antibracket

$$(,) = (,)_1 + (,)_2 + (,)_3\ , \tag{3.42}$$

and $V$ represents the non-canonical part of the total Koszul–Tate differential

$$\delta = \delta_{\mathrm{can}} + V = \sum_{m=1}^{3} \delta_m,\ \delta_m = \delta_{\mathrm{can}m} + V_m\,. \tag{3.43}$$

Their features are

$$\varepsilon\left((,)_m\right) = \varepsilon\left(V_m\right) = 1\,, \tag{3.44}$$

$$\mathrm{trigh}\left((,)_m\right) = \mathrm{trigh}\left(s_m\right),\ \mathrm{trigh}\left(V_m\right) = \mathrm{trigh}\left(s_m\right) = -\mathrm{trires}\left(V_m\right)\ . \tag{3.45}$$

It is understood that the individual antibrackets, as well as the total one, satisfy the usual properties of the antibracket from the antifield-BRST formalism, while the operators $V_m$ and $V$ behave like derivations with respect to the antibrackets. The operators $V_m$ act only on the (B) and (T) conjugated spinors through

$$V_m \bar{\Phi}_A^{\mathrm{B}(n)} = (-)^{\varepsilon_A} \varepsilon_{mnp} \bar{\Phi}_A^{*(p)},\ V_m \bar{\Phi}_A^{\mathrm{T}} = (-)^{\varepsilon_A + 1} \bar{\Phi}_A^{\mathrm{B}(m)}\,, \tag{3.46}$$

where $\varepsilon_{mnp}$ is completely antisymmetric, with $\varepsilon_{123} = +1$. Based on the concrete realizations of both the exterior and Koszul–Tate tricomplexes, we deduce the following boundary conditions on the solution to the extended classical master equation

$$\overset{[0]}{S} = S_0^{\mathrm{L}}\left[\psi_{\mu\nu}\right], \tag{3.47}$$

$$\overset{[1]}{S} = \int d^4x \left(\bar{\Phi}^{*(m)}\partial_\alpha + \bar{\psi}_{\mu\nu}^{*(m)}\left(\delta_{\nu\beta}\partial_\mu + \delta_{\mu\beta}\partial_\nu\right) \times \left(\delta_{\beta\alpha} - \tfrac{1}{4}\gamma_\beta\gamma_\alpha\right)\right)\eta_{m\alpha}\,, \tag{3.48}$$

$$\begin{aligned}\overset{[2]}{S} &= \int d^4x \Big(\varepsilon_{mnp}\bar{\eta}^{*(m)}_{n\alpha} + \bar{\Phi}^{\mathrm{B}(p)}\partial_\alpha \\ &\quad +\bar{\psi}^{\mathrm{B}(p)}_{\mu\nu}\left(\delta_{\nu\beta}\partial_\mu + \delta_{\mu\beta}\partial_\nu\right)\left(\delta_{\beta\alpha} - \tfrac{1}{4}\gamma_\beta\gamma_\alpha\right)\Big)\,\pi_{p\alpha}\,, \qquad (3.49)\\ \overset{[3]}{S} &= \int d^4x \left(-\bar{\pi}^{*(m)}_{m\alpha} - \bar{\eta}^{\mathrm{B}(m)}_{m\alpha} + \bar{\Phi}^{\mathrm{T}}\partial_\alpha \right.\\ &\quad \left.+\bar{\psi}^{\mathrm{T}}_{\mu\nu}\left(\delta_{\nu\beta}\partial_\mu + \delta_{\mu\beta}\partial_\nu\right)\left(\delta_{\beta\alpha} - \tfrac{1}{4}\gamma_\beta\gamma_\alpha\right)\right)\lambda_\alpha\,, \qquad (3.50)\end{aligned}$$

where the supplementary superscript between brackets in $\overset{[0]}{S}$, $\overset{[1]}{S}$, *etc.*, refers to a decomposition of the solution to the extended master equation via the total resolution degree.

The equation (3.41), subject to the boundary conditions (3.47)–(3.50), is solved by means of expanding $S$ in terms of the total resolution degree,

$$S = \sum_{k\geq 0} \overset{[k]}{S},\ \mathrm{res}\left(\overset{[k]}{S}\right) = k,\ \mathrm{trigh}\left(\overset{[k]}{S}\right) = (0,0,0)\,. \qquad (3.51)$$

In the case of the irreducible version of free massless spin-5/2 fields, since the generating set is Abelian and irreducible, we find that the solution to (3.41) simply reduces to the sum of the boundary terms (3.47)–(3.50)

$$\begin{aligned} S &= \int d^4x \Big(\bar{\Phi}^{*(m)}\partial_\alpha\eta_{m\alpha} + \bar{\Phi}^{\mathrm{B}(m)}\partial_\alpha\pi_{m\alpha} + \bar{\Phi}^{\mathrm{T}}\partial_\alpha\lambda_\alpha \\ &\quad +\bar{\psi}^{*(m)}_{\mu\nu}\left(\delta_{\nu\beta}\partial_\mu + \delta_{\mu\beta}\partial_\nu\right)\left(\delta_{\beta\alpha} - \tfrac{1}{4}\gamma_\beta\gamma_\alpha\right)\eta_{m\alpha} \\ &\quad +\bar{\psi}^{\mathrm{B}(m)}_{\mu\nu}\left(\delta_{\nu\beta}\partial_\mu + \delta_{\mu\beta}\partial_\nu\right)\left(\delta_{\beta\alpha} - \tfrac{1}{4}\gamma_\beta\gamma_\alpha\right)\pi_{m\alpha} \\ &\quad +\bar{\psi}^{\mathrm{T}}_{\mu\nu}\left(\delta_{\nu\beta}\partial_\mu + \delta_{\mu\beta}\partial_\nu\right)\left(\delta_{\beta\alpha} - \tfrac{1}{4}\gamma_\beta\gamma_\alpha\right)\lambda_\alpha \\ &\quad +\varepsilon_{mnp}\bar{\eta}^{*(m)}_{n\alpha}\pi_{p\alpha} - \left(\bar{\pi}^{*(m)}_{m\alpha} + \bar{\eta}^{\mathrm{B}(m)}_{m\alpha}\right)\lambda_\alpha\Big) + S_0^{\mathrm{L}}\left[\psi_{\mu\nu}\right]\,. \qquad (3.52)\end{aligned}$$

The main inconvenience presented by the above solution is that it is not yet invariant under any of the differentials from the $Sp(3)$ BRST tricomplex, while still preserves the original gauge symmetries of the fields. To surpass these difficulties, we need such a gauge-fixing procedure that on the one hand "kills" the gauge symmetries of (3.52), and, on the other hand, outputs a gauge-fixed action that is simultaneously invariant under the differentials that stay at the core of the $Sp(3)$ BRST tricomplex.

## 4. Gauge-fixing process

It is useful [12] to add further variables in order to realize a proper connection with the gauge-fixing procedure from the standard antibracket-antifield formalism. The main role of these supplementary fields is to impose some irreducible gauge-fixing conditions. For a proper relation with the results from the literature, we choose to implement some irreducible gauge-fixing conditions, based on the Majorana spinors

$$G_\alpha \equiv \gamma_\nu \psi_{\alpha\nu} - \tfrac{1}{4}\gamma_\alpha \psi_{\nu\nu} + \gamma_\alpha \Phi \tag{4.1}$$

that we want to induce a Gaussian term in the gauge-fixed path integral, of the type

$$\int \cdots Dl_\alpha \exp\left(-i\int d^4x\, \bar{l}_\alpha \partial\!\!\!/ \left(\gamma_\nu \psi_{\alpha\nu} - \tfrac{1}{4}\gamma_\alpha \psi_{\nu\nu} + \gamma_\alpha \Phi - \tfrac{1}{2} l_\alpha\right)\right). \tag{4.2}$$

In view of the above gauge-fixing conditions, we introduce a supplementary bosonic (already conjugated) Majorana vector spinor

$$\overset{(0,0,0)}{\bar{\varphi}}{}_{\alpha} \tag{4.3}$$

that is regarded as a purely gauge field (does not enter the original action), endowed with the gauge invariance

$$\delta_\xi \bar{\varphi}_\alpha = \bar{\xi}_\alpha\,, \tag{4.4}$$

where obviously the gauge parameters $\xi_\alpha$ are independent of $\epsilon_\alpha$ (they have opposite statistics, anyway). This means that we remain with the Lagrangian action (2.1), but regarded as being subject to the irreducible and Abelian gauge transformations (2.4), (2.13) and (4.4). Then, according to the discussion from the previous section, by triplicating all the above mentioned gauge transformations, we find that the ghost spectrum from the $Sp(3)$ BRST tricomplex for the overall gauge theory includes the ghosts (3.13), as well as

$$\overset{(1,0,0)}{\bar{C}}{}_{1\alpha},\ \overset{(0,1,0)}{\bar{C}}{}_{2\alpha},\ \overset{(0,0,1)}{\bar{C}}{}_{3\alpha},\ \overset{(0,1,1)}{\bar{p}}{}_{1\alpha},\ \overset{(1,0,1)}{\bar{p}}{}_{2\alpha},\ \overset{(1,1,0)}{\bar{p}}{}_{3\alpha},\ \overset{(1,1,1)}{\bar{l}}{}_{\alpha}\,, \tag{4.5}$$

displaying the Grassmann parities

$$\varepsilon\left(\bar{C}_{m\alpha}\right) = \varepsilon\left(\bar{l}_\alpha\right) = 1,\ \varepsilon\left(\bar{p}_{m\alpha}\right) = 0,\ m = 1,2,3\,. \tag{4.6}$$

We will see that $\bar{l}_\alpha$ from (4.5) is precisely the vector spinor that yields the Gaussian term (4.2) in the gauge-fixed path integral. For notational simplicity, we make the collective notation

$$\bar{\varphi}_I = \left(\bar{\varphi}_\alpha, \bar{C}_{m\alpha}, \bar{p}_{m\alpha}, \bar{l}_\alpha\right),\ m = 1,2,3\,. \tag{4.7}$$

The antifield spectrum for the larger irreducible gauge theory will contain the variables (3.16)–(3.29), together with the additional ones

$$\left(\varphi_I^{*(m)}, \varphi_I^{\mathrm{B}(m)}, \varphi_I^{\mathrm{T}}\right), \quad m = 1,2,3 \tag{4.8}$$

whose properties are correctly described by the formulas (3.30)–(3.37), where we replace $\Phi_A$ by $\bar{\varphi}_I$, and also remove the spinor conjugation. Along the line exposed in the above section, we find that the solution to the classical master equation of the $Sp(3)$ BRST formalism associated with the richer gauge theory, $(1/2)\,(S', S') + VS' = 0$, will be

$$\begin{aligned} S' \;=\; & S + \int d^4x \left(-\bar{C}_{m\alpha}\varphi_\alpha^{*(m)} + \bar{p}_{p\alpha}\left(\varepsilon_{mnp}C_{n\alpha}^{*(m)} + \varphi_\alpha^{\mathrm{B}(p)}\right)\right. \\ & \left. -\bar{l}_\alpha\left(-p_{m\alpha}^{*(m)} - C_{m\alpha}^{\mathrm{B}(m)} + \varphi_\alpha^{\mathrm{T}}\right)\right), \end{aligned} \tag{4.9}$$

where $S$ is given by (3.52), the non-vanishing fundamental antibrackets are defined by (3.39), together with

$$\left(\bar{\varphi}_I, \varphi_J^{*(m)}\right)_n = \delta_{mn}\delta_{IJ}\,, \tag{4.10}$$

and the components of the operator $V$ act only on the bar and tilde spinors via (3.46) and

$$V_m\varphi_I^{\mathrm{B}(n)} = (-)^{\varepsilon_I}\,\varepsilon_{mnp}\varphi_I^{*(p)},\; V_m\varphi_I^{\mathrm{T}} = (-)^{\varepsilon_I+1}\,\varphi_I^{\mathrm{B}(m)}\,. \tag{4.11}$$

We are now prepared to develop a consistent gauge-fixing procedure at the level of the antibracket-antifield $Sp(3)$ formalism. Initially, we begin by restoring an anticanonical structure for all the variables (including the bar and tilde ones) in order to bring the classical master equation of the $Sp(3)$ BRST formalism to a more familiar form. We pick up, for instance, the first antibracket, and forget about the other two. As only $(\Phi_A, \bar{\varphi}_I)$ (field/ghost spectra (3.38) and (4.7)) and $\left(\bar{\Phi}_A^{*(1)}, \varphi_I^{*(1)}\right)$ form (anti)canonical pairs in the first antibracket, we need to extend the algebra of the $Sp(3)$ BRST tricomplex [12] by adding the variables

$$\rho_{2A}, \rho_{3A}, \kappa_{1A}, \mu_{2A}, \mu_{3A}, \nu_{1A}, \bar{r}_{2I}, \bar{r}_{3I}, \bar{k}_{1I}, \bar{m}_{2I}, \bar{m}_{3I}, \bar{n}_{1I}\,, \tag{4.12}$$

respectively (anti)canonically conjugated in the first antibracket to

$$\begin{aligned} & \left(\bar{\Phi}_A^{*(3)}, \bar{\Phi}_A^{*(2)}, \bar{\Phi}_A^{\mathrm{B}(1)}, \bar{\Phi}_A^{\mathrm{B}(3)}, \bar{\Phi}_A^{\mathrm{B}(2)}, \bar{\Phi}_A^{\mathrm{T}},\right. \\ & \left.\varphi_I^{*(3)}, \varphi_I^{*(2)}, \varphi_I^{\mathrm{B}(1)}, \varphi_I^{\mathrm{B}(3)}, \varphi_I^{\mathrm{B}(2)}, \varphi_I^{\mathrm{T}}\right)\,. \end{aligned} \tag{4.13}$$

The Grassmann parity and ghost tridegree of the new variables follow from those of the first antibracket

$$\varepsilon\left((,)_1\right)=1,\ trigh\left((,)_1\right)=(1,0,0)\ . \tag{4.14}$$

We make the convention that the variables

$$\begin{aligned}&\left(\Phi_A,\bar{\varphi}_I,\bar{\Phi}_A^{*(3)},\varphi_I^{*(3)},\rho_{3A},\bar{r}_{3I},\bar{\Phi}_A^{\mathrm{B}(1)},\right.\\&\left.\varphi_I^{\mathrm{B}(1)},\mu_{3A},\bar{m}_{3I},\bar{\Phi}_A^{\mathrm{B}(3)},\varphi_I^{\mathrm{B}(3)},\nu_{1A},\bar{n}_{1I}\right)\end{aligned} \tag{4.15}$$

are regarded as 'fields', while

$$\begin{aligned}&\left(\bar{\Phi}_A^{*(1)},\varphi_I^{*(1)},\rho_{2A},\bar{r}_{2I},\bar{\Phi}_A^{*(2)},\varphi_I^{*(2)},\kappa_{1A},\right.\\&\left.\bar{k}_{1I},\bar{\Phi}_A^{\mathrm{B}(2)},\varphi_I^{\mathrm{B}(2)},\mu_{2A},\bar{m}_{2I},\bar{\Phi}_A^{\mathrm{T}},\varphi_I^{\mathrm{T}}\right)\end{aligned} \tag{4.16}$$

are viewed like their respectively (anti)canonically conjugated 'antifields'. Accordingly, we obtain that the functional

$$\begin{aligned}S_1' \ =\ & S'+\int d^4x\left(\bar{\Phi}_A^{*(2)}\mu_{2A}+\bar{\Phi}_A^{*(3)}\mu_{3A}+\bar{\Phi}_A^{\mathrm{B}(1)}\nu_{1A}\right.\\&\left.+(-)^{\varepsilon_I+1}\left(\bar{m}_{2I}\varphi_I^{*(2)}+\bar{m}_{3I}\varphi_I^{*(3)}\right)+(-)^{\varepsilon_I}\,\bar{n}_{1I}\varphi_I^{\mathrm{B}(1)}\right)\end{aligned} \tag{4.17}$$

satisfies the equation

$$\left(S_1',S_1'\right)_1=0, \tag{4.18}$$

which is precisely the familiar form of the classical master equation from the standard BRST method in the first antibracket.

This suggests that we can employ the gauge-fixing procedure from the standard BRST formalism, which requires the choice of a fermionic functional $\psi_1$, with the help of which we eliminate half of the variables in favor of the other half. For definiteness, we eliminate the variables

$$\begin{aligned}&\left(\bar{\Phi}_A^{*(1)},\rho_{2A},\rho_{3A},\kappa_{1A},\bar{\Phi}_A^{\mathrm{B}(2)},\bar{\Phi}_A^{\mathrm{B}(3)},\bar{\Phi}_A^{\mathrm{T}},\right.\\&\left.\varphi_I^{*(1)},\bar{r}_{2I},\bar{r}_{3I},\bar{k}_{1I},\varphi_I^{\mathrm{B}(2)},\varphi_I^{\mathrm{B}(3)},\varphi_I^{\mathrm{T}}\right),\end{aligned} \tag{4.19}$$

and, in the meantime, enforce the gauge-fixing conditions

$$\rho_{2A}=\rho_{3A}=\kappa_{1A}=0,\ \bar{r}_{2I}=\bar{r}_{3I}=\bar{k}_{1I}=0, \tag{4.20}$$

which can be realized by taking

$$\psi_1=\psi_1\left[\Phi_A,\mu_{2A},\mu_{3A},\nu_{1A},\bar{\varphi}_I,\bar{m}_{2I},\bar{m}_{3I},\bar{n}_{1I}\right]. \tag{4.21}$$

A variable is eliminated by one of the formulas

$$\text{antifield} = \frac{\delta^L \psi_1}{\delta\,(\text{field})},\ \text{field} = -\frac{\delta^L \psi_1}{\delta\,(\text{antifield})}, \tag{4.22}$$

depending if it is a 'field' or an 'antifield' (see (4.15),(4.16)). The gauge-fixing fermion that implements the irreducible gauge conditions inferred from (4.1) and (4.20) reads as

$$\begin{aligned}\psi_1 \ = \ & a\int d^4x \left(\bar{n}^{(\varphi)}_{1\alpha}\partial\!\!\!/ \left(\gamma_\nu\psi_{\alpha\nu} - \frac{1}{4}\gamma_\alpha\psi_{\nu\nu} + \gamma_\alpha\Phi - \frac{1}{2}l_\alpha\right)\right. \\ & +\bar{\varphi}_\alpha\partial\!\!\!/ \left(\gamma_\nu\nu^{(\psi)}_{1\alpha\nu} - \frac{1}{4}\gamma_\alpha\nu^{(\psi)}_{1\nu\nu} + \gamma_\alpha\nu^{(\Phi)}_1 - \frac{1}{2}n^{(l)}_{1\alpha}\right) \\ & +\bar{m}^{(\varphi)}_{3\alpha}\partial\!\!\!/ \left(\gamma_\nu\mu^{(\psi)}_{2\alpha\nu} - \frac{1}{4}\gamma_\alpha\mu^{(\psi)}_{2\nu\nu} + \gamma_\alpha\mu^{(\Phi)}_2 - \frac{1}{2}m^{(l)}_{2\alpha}\right) \\ & \left.-\bar{m}^{(\varphi)}_{2\alpha}\partial\!\!\!/ \left(\gamma_\nu\mu^{(\psi)}_{3\alpha\nu} - \frac{1}{4}\gamma_\alpha\mu^{(\psi)}_{3\nu\nu} + \gamma_\alpha\mu^{(\Phi)}_3 - \frac{1}{2}m^{(l)}_{3\alpha}\right)\right), \end{aligned} \tag{4.23}$$

where we put an extra superscript between parentheses ($(\Phi_A)$ or $(\varphi_I)$) where necessary, in order to distinguish the variables that carry the same indices, and $a$ is a non-vanishing constant. Eliminating the variables (4.19) with the help of (4.23) from (4.17), and also some auxiliary variables from the resulting functional, after some computation we infer the gauge-fixed action

$$\begin{aligned}S'_{\rm gf} \ = \ & S_0^{\rm L}\left[\psi_{\mu\nu}\right] + a\int d^4x \left(-\bar{p}_{m\lambda}\partial\!\!\!/ M_{\lambda\alpha}\eta_{m\alpha} + \bar{C}_{m\lambda}\partial\!\!\!/ M_{\lambda\alpha}\pi_{m\alpha}\right. \\ & \left.+\bar{\varphi}_\lambda\partial\!\!\!/ M_{\lambda\alpha}\lambda_\alpha - \bar{l}_\alpha\partial\!\!\!/ \left(\gamma_\nu\psi_{\alpha\nu} - \frac{1}{4}\gamma_\alpha\psi_{\nu\nu} + \gamma_\alpha\Phi - \frac{1}{2}l_\alpha\right)\right), \end{aligned} \tag{4.24}$$

where we used the notation

$$\begin{aligned}M_{\lambda\alpha} \ \equiv \ & \gamma_\lambda\partial_\alpha + \frac{1}{2}\left(\delta_{\lambda\mu}\gamma_\nu + \delta_{\lambda\nu}\gamma_\mu - \frac{1}{2}\delta_{\mu\nu}\gamma_\lambda\right) \\ & \times\left(\delta_{\nu\beta}\partial_\mu + \delta_{\mu\beta}\partial_\nu\right)\left(\delta_{\beta\alpha}\mathbf{1} - \frac{1}{4}\gamma_\beta\gamma_\alpha\right). \end{aligned} \tag{4.25}$$

Eliminating the auxiliary variables $l_\alpha$ from (4.24), we infer the gauge-fixed action

$$\begin{aligned}S''_{\rm gf} \ = \ & S_0^{\rm L}\left[\psi_{\mu\nu}\right] + a\int d^4x \left(-\tfrac{1}{2}\bar{G}_\alpha\partial\!\!\!/ G_\alpha - \bar{p}_{m\alpha}\left(\delta_{\alpha\beta}\Box + \partial_\alpha\partial_\beta\right)\eta_{m\beta}\right. \\ & +\tfrac{3}{8}\bar{p\!\!\!/}_m\Box\eta\!\!\!/_m - \tfrac{1}{4}\bar{p}_{m\alpha}\partial_\alpha\partial\!\!\!/\eta\!\!\!/_m + \tfrac{1}{2}\bar{p\!\!\!/}_m\partial\!\!\!/\partial_\alpha\eta_{m\alpha} \end{aligned}$$

$$
\begin{aligned}
&+\bar{C}_{m\alpha}\left(\delta_{\alpha\beta}\Box+\partial_\alpha\partial_\beta\right)\pi_{m\beta}-\tfrac{3}{8}\bar{\not{C}}_m\Box\not{\pi}_m \\
&+\tfrac{1}{4}\bar{C}_{m\alpha}\partial_\alpha\not{\partial}\not{\pi}_m-\tfrac{1}{2}\bar{\not{C}}_m\not{\partial}\partial_\alpha\pi_{m\alpha}+\bar{\varphi}_\alpha\left(\delta_{\alpha\beta}\Box+\partial_\alpha\partial_\beta\right)\lambda_\beta \\
&-\tfrac{3}{8}\bar{\not{\varphi}}\Box\not{\lambda}+\tfrac{1}{4}\bar{\varphi}_\alpha\partial_\alpha\not{\partial}\not{\lambda}-\frac{1}{2}\bar{\not{\varphi}}\not{\partial}\partial_\alpha\lambda_\alpha\Big)
\end{aligned}
\tag{4.26}
$$

with $G_\alpha$ given in (4.1). In the above we used the generic notations $\not{g}=\gamma_\alpha g_\alpha$ and $\bar{\not{g}}=\bar{g}_\alpha\gamma_\alpha$.

We can equally infer the expression (4.26) in the context of the other two antibrackets. Indeed, if we focus on the second antibracket, then only $\Phi_A$ and $\bar{\varphi}_I$ admit the antifields $\bar{\Phi}_A^{*(2)}$ and $\varphi_I^{*(2)}$ (anti)canonically conjugated to them, such that we need the supplementary variables

$$
\rho_{3A},\rho_{1A},\kappa_{2A},\mu_{3A},\mu_{1A},\nu_{2A},\bar{r}_{3I},\bar{r}_{1I},\bar{k}_{2I},\bar{m}_{3I},\bar{m}_{1I},\bar{n}_{2I},
\tag{4.27}
$$

respectively (anti)canonically conjugated in the second antibracket to

$$
\begin{aligned}
&\Big(\bar{\Phi}_A^{*(1)},\bar{\Phi}_A^{*(3)},\bar{\Phi}_A^{\mathrm{B}(2)},\bar{\Phi}_A^{\mathrm{B}(1)},\bar{\Phi}_A^{\mathrm{B}(3)},\bar{\Phi}_A^{\mathrm{T}}, \\
&\varphi_I^{*(1)},\varphi_I^{*(3)},\varphi_I^{\mathrm{B}(2)},\varphi_I^{\mathrm{B}(1)},\varphi_I^{\mathrm{B}(3)},\varphi_I^{\mathrm{T}}\Big).
\end{aligned}
\tag{4.28}
$$

Consequently, we find that the functional

$$
\begin{aligned}
S_2' \;=\; S'+\int d^4x\,\Big(&\bar{\Phi}_A^{*(1)}\mu_{1A}+\bar{\Phi}_A^{*(3)}\mu_{3A}+\bar{\Phi}_A^{\mathrm{B}(2)}\nu_{2A} \\
&+(-)^{\varepsilon_I+1}\left(\bar{m}_{1I}\varphi_I^{*(1)}+\bar{m}_{3I}\varphi_I^{*(3)}\right)+(-)^{\varepsilon_I}\,\bar{n}_{2I}\varphi_I^{\mathrm{B}(2)}\Big)
\end{aligned}
\tag{4.29}
$$

fulfills the standard master equation in this antibracket

$$
\left(S_2',S_2'\right)_2=0.
\tag{4.30}
$$

In this case, we have to choose a fermionic functional $\psi_2$, with the help of which we eliminate half of the variables in favor of the other half. If we eliminate the variables

$$
\begin{aligned}
&\Big(\bar{\Phi}_A^{*(2)},\rho_{1A},\rho_{3A},\kappa_{2A},\bar{\Phi}_A^{\mathrm{B}(1)},\bar{\Phi}_A^{\mathrm{B}(3)},\bar{\Phi}_A^{\mathrm{T}}, \\
&\varphi_I^{*(2)},\bar{r}_{1I},\bar{r}_{3I},\bar{k}_{2I},\varphi_I^{\mathrm{B}(1)},\varphi_I^{\mathrm{B}(3)},\varphi_I^{\mathrm{T}}\Big)\,,
\end{aligned}
\tag{4.31}
$$

and implement the gauge-fixing conditions

$$
\rho_{1A}=\rho_{3A}=\kappa_{2A}=0,\;\bar{r}_{1I}=\bar{r}_{3I}=\bar{k}_{2I}=0\,,
\tag{4.32}
$$

then we have to set

$$\psi_2 = \psi_2 \left[\Phi_A, \mu_{1A}, \mu_{3A}, \nu_{2A}, \bar{\varphi}_I, \bar{m}_{1I}, \bar{m}_{3I}, \bar{n}_{2I}\right]. \tag{4.33}$$

Now, the 'fields' are precisely

$$\Big(\Phi_A, \bar{\varphi}_I, \bar{\Phi}_A^{*(1)}, \varphi_I^{*(1)}, \rho_{1A}, \bar{r}_{1I}, \bar{\Phi}_A^{\mathrm{B}(2)}, \tag{4.34}$$

$$\varphi_I^{\mathrm{B}(2)}, \mu_{1A}, \bar{m}_{1I}, \bar{\Phi}_A^{\mathrm{B}(1)}, \varphi_I^{\mathrm{B}(1)}, \nu_{2A}, \bar{n}_{2I}\ , \tag{4.35}$$

and, naturally,

$$\Big(\bar{\Phi}_A^{*(2)}, \varphi_I^{*(2)}, \rho_{3A}, \bar{r}_{3I}, \bar{\Phi}_A^{*(3)}, \varphi_I^{*(3)}, \kappa_{2A}, \tag{4.36}$$

$$\bar{k}_{2I}, \bar{\Phi}_A^{\mathrm{B}(3)}, \varphi_I^{\mathrm{B}(3)}, \mu_{3A}, \bar{m}_{3I}, \bar{\Phi}_A^{\mathrm{T}}, \varphi_I^{\mathrm{T}}\Big) \tag{4.37}$$

are interpreted like their 'antifields'. The fermionic functional that takes into account the irreducible gauge conditions furnished by (4.1), as well as by (4.32), is simply given by

$$\begin{aligned}
\psi_2 \;=\; & a\int d^4x \left(\bar{n}_{2\alpha}^{(\varphi)} \partial\!\!\!/ \left(\gamma_\nu \psi_{\alpha\nu} - \frac{1}{4}\gamma_\alpha \psi_{\nu\nu} + \gamma_\alpha \Phi - \frac{1}{2} l_\alpha\right)\right.\\
& + \bar{\varphi}_\alpha \partial\!\!\!/ \left(\gamma_\nu \nu_{2\alpha\nu}^{(\psi)} - \frac{1}{4}\gamma_\alpha \nu_{2\nu\nu}^{(\psi)} + \gamma_\alpha \nu_2^{(\Phi)} - \frac{1}{2} n_{2\alpha}^{(l)}\right)\\
& + \bar{m}_{1\alpha}^{(\varphi)} \partial\!\!\!/ \left(\gamma_\nu \mu_{3\alpha\nu}^{(\psi)} - \frac{1}{4}\gamma_\alpha \mu_{3\nu\nu}^{(\psi)} + \gamma_\alpha \mu_3^{(\Phi)} - \frac{1}{2} m_{3\alpha}^{(l)}\right)\\
& \left. - \bar{m}_{3\alpha}^{(\varphi)} \partial\!\!\!/ \left(\gamma_\nu \mu_{1\alpha\nu}^{(\psi)} - \frac{1}{4}\gamma_\alpha \mu_{1\nu\nu}^{(\psi)} + \gamma_\alpha \mu_1^{(\Phi)} - \frac{1}{2} m_{1\alpha}^{(l)}\right)\right),
\end{aligned} \tag{4.38}$$

and, after some computation, we get that it also leads to the gauge-fixed action (4.26). A similar reasoning is valid with respect to the third antibracket, in which case (4.26) is again recovered.

## 5. Comments and conclusions

We observe that the resulting gauge-fixed action has all the desired features, namely, it is local, manifestly covariant, and exhibits no residual gauge symmetries. Meanwhile, it has the same form, no matter what antibracket we start with. We are now able to make the comparison with the standard results from the literature [4, 19]. In both previously mentioned Refs. the authors work in the framework of the reducible formulation of free massless spin-5/2 fields, and propose some gauge-fixing conditions expressed via the vector Majorana spinor $F_\alpha \equiv \gamma_\nu \psi_{\alpha\nu} - \frac{1}{4}\gamma_\alpha \psi_{\nu\nu}$, that are reducible since $\gamma_\alpha F_\alpha = 0$. Our approach has, among others, the advantage that allows the gauge-fixing conditions to rely on the vector Majorana spinor (4.1), such that they are irreducible. Let us consider the terms of total resolution degree equal to one from the gauge-fixed action (4.24), namely,

$$-a \int d^4x\ \bar{p}_{m\lambda} \partial\!\!\!/ M_{\lambda\alpha} \eta_{m\alpha} \equiv -a \int d^4x\ \left( \bar{p}_{m\lambda} \partial\!\!\!/ \gamma_\lambda \partial_\alpha \eta_{m\alpha} \right.$$
$$\left. + \tfrac{1}{2} \bar{p}_{m\lambda} \partial\!\!\!/ \left( \delta_{\lambda\mu}\gamma_\nu + \delta_{\lambda\nu}\gamma_\mu - \tfrac{1}{2}\delta_{\mu\nu}\gamma_\lambda \right) \left( \partial_{(\mu}\eta_{m\nu)} - \tfrac{1}{4}\gamma_{(\mu}\partial_{\nu)} \eta\!\!\!/_m \right) \right), \quad (5.1)$$

and the similar pieces (of antighost number one) in $\mathcal{S}_{\mathrm{ghost}}$ from [4]. The notation $(\mu\nu)$ signifies symmetrization (without the factor 1/2) with respect to the indices between parentheses. The latter terms from the right hand-side of (5.1) are invariant under the residual gauge transformations

$$\eta_{m\alpha} \to \eta_{m\alpha} + \gamma_\alpha \eta_m \quad (5.2)$$

with $\eta_m$ three arbitrary bosonic Majorana spinors, just like the similar component in $\mathcal{S}_{\mathrm{ghost}}$ from [4] (up to the difference that in [4] there appears a single ghost). In our setting, this residual gauge symmetry is frozen by the former pieces in the right hand-side of (5.1) (proportional with $\partial_\alpha \eta_{m\alpha}$), while in the reducible version there is necessary to introduce a convenient non-minimal sector in order to fix it. We remark that the "Nielsen–Kallosh ghost" for spin-5/2 gauge fields from [4] is absent in our procedure, while the role of the extraghost $C'_1$ is played here by the purely gauge Majorana spinor $\Phi$, involved with the irreducible formulation of spin-5/2 gauge fields (see Sec. 2).

The reducible approach is thus deeply focused on finding an appropriate non-minimal sector to compensate for the redundancy of both gauge transformations and gauge-fixing conditions. In turn, our $Sp(3)$ treatment applied to the irreducible formulation of spin 5/2-gauge fields exhibits the advantage of avoiding these issues. It equally offers some correct irreducible gauge-fixing conditions, and the opportunity to enforce them within the gauge-fixed action without the fear that some undetected residual gauge symmetries might occur.

This work has been supported by a research contract with the Romanian Ministry of Education and Scientific Research (M.E.C.).

## REFERENCES


[1] C. Becchi, A. Rouet, R. Stora, *Phys. Lett.* **B52**, 344 (1974); *Commun. Math. Phys.* **42**, 127 (1975).

[2] I.V. Tyutin, preprint Lebedev Inst. no. 39 (1975); E.S. Fradkin, G.A. Vilkovisky, *Phys. Lett.* **B55**, 224 (1975); E.S. Fradkin, T.E. Fradkina, *Phys. Lett.* **B72**, 343 (1978); I.A. Batalin, E.S. Fradkin, *Phys. Lett.* **B122**, 157 (1983).

[3] I.A. Batalin, G.A. Vilkovisky, *Phys. Lett.* **B69**, 309 (1977); **B102**, 27 (1981); **B120**, 166 (1983).

[4] I.A. Batalin, G.A. Vilkovisky, *Phys. Rev.* **D28**, 2567 (1983).

[5] I.A. Batalin, G.A. Vilkovisky, *J. Math. Phys.* **26**, 172 (1985).

[6] M. Henneaux, C. Teitelboim, *Quantization of Gauge Systems*, Princeton University Press, Princeton 1992.

[7] M. Henneaux, *Phys. Rep.* **126**, 1 (1985); *Nucl. Phys.* **B** *(Proc. Suppl.)* **18A**, 47 (1990); J. Gomis, J. Paris, S. Samuel, *Phys. Rep.* **259**, 1 (1995).

[8] I.A. Batalin, P.M. Lavrov, I.V. Tyutin, *J. Math. Phys.* **31**, 6 (1990); **31**, 1487 (1990); **31**, 2708 (1990); **32**, 532 (1991).

[9] Ph. Grégoire, M. Henneaux, *Phys. Lett.* **B277**, 459 (1992); *Commun. Math. Phys.* **157**, 279 (1993); *J. Phys. A: Math. Gen.* **26**, 6073 (1993); M. Henneaux, *Phys. Lett.* **B282**, 372 (1992); Ph. Grégoire, Ph.D. thesis, Université Libre de Bruxelles, Bruxelles 1993.

[10] J. Hoyos, M. Quiros, J.R. Mittelbrunn, F.J. de Urries, *Nucl. Phys.* **B218**, 159 (1983); V.P. Spiridonov, *Nucl. Phys.* **B308**, 257 (1988); J. Gomis, J. Roca, *Nucl. Phys.* **B342**, 152 (1990); E. Dur, S.J. Gates Jr., *Nucl. Phys.* **B343**, 622 (1990); C.M. Hull, *Mod. Phys. Lett.* **A5**, 1871 (1990); C.M. Hull, B. Spence, J.L. Vasquez-Bello, *Nucl. Phys.* **B348**, 108 (1991).

[11] L. Tatar, R. Tatar, *Phys. Lett.* **B326**, 238 (1994); L. Tataru, *J. Phys. A: Math. Gen.* **28**, 4175 (1995); L. Tataru, R. Tatar, *Int. J. Mod. Phys.* **A13**, 1981 (1998).

[12] C. Bizdadea, S.O. Saliu, *Int. J. Mod. Phys.* **A16**, 2975 (2001); C. Bizdadea, S.O. Saliu, *Mod. Phys. Lett.* **A17**, 269 (2002).

[13] C. Fronsdal, *Phys. Rev.* **D18**, 3624 (1978).

[14] J. Fang, C. Fronsdal, *Phys. Rev.* **D18**, 3630 (1978).

[15] B. de Wit, D.Z. Freedman, *Phys. Rev.* **D21**, 358 (1980).

[16] C. Fronsdal, *Some Open Problems with Higher Spins, in "Supergravity"*, Eds. P. van Nieuwenhuizen, D. Z. Freedman, North Holland Publishing Company, Amsterdam 1979.

[17] C. Fronsdal, *Phys. Rev.* **D20**, 848 (1979); J. Fang, C. Fronsdal, *Phys. Rev.* **D22**, 1361 (1980).

[18] J. Schwinger, *Particles, Sources, Fields*, Addison Wesley, Reading (MA) 1970.

[19] F.A. Berends, J.W. van Holten, P. van Nieuwenhuizen, B. de Wit, *Phys. Lett.* **B83**, 188 (1979); *Nucl. Phys.* **B154**, 261 (1979).

[20] T. Curtright, *Phys. Lett.* **B85**, 219 (1979); M.A. Vasiliev, *Sov. J. Nucl. Phys.* **32**, 855 (1980) (p. 439 in English translation); C. Aragone, S. Deser, *Nucl. Phys.* **B170**, 329 (1980); M. A. Vasiliev, *Fortschr. Phys.* **35**, 741 (1987).

[21] M.A. Vasiliev, *Consistent Higher Spin Gauge Theories in 3+1 Dimensions, in "Supermembranes, Physics in 2+1 Dimensions"*, Eds. M.J. Duff, C.N. Pope, E. Sezgin, World Scientific, Singapore 1990.

[22] C. Bizdadea, S.O. Saliu, *J. Phys. A Math. Gen.* **32**, 3919 (1999).