ANALYSIS OF THE FREEZE-OUT PARAMETERS FOR RHIC, SPS AND AGS BASED ON $\frac{dE_\mathrm{T}}{d\eta}/\frac{dN_\mathrm{ch}}{d\eta}$ RATIO MEASUREMENTS

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The ratio $\frac{dE_T}{d\eta}/\frac{dN_\mathrm{ch}}{d\eta}$ is analyzed in the framework of a single-freeze-out thermal hadron gas model. Decays of hadron resonances are taken into account in evaluations of this ratio. The predictions of the model at the freeze-out parameters, established previously from observed particle yields, agree very well with the ratio measured at RHIC, SPS and AGS.

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In this paper, a statistical model is tested in recovering values of the ratio $\frac{dE_\mathrm{T}}{d\eta}_{|\mathrm{mid}}/\frac{dN_\mathrm{ch}}{d\eta}_{|\mathrm{mid}}$ measured at RHIC [1], SPS [2] and AGS [3] (mid means the midrapidity region). For RHIC and SPS the ratio equals about 0.8 GeV, but for AGS it is 0.72 GeV. So far, the model has been applied successfully in explanation of particle ratios and distributions observed in heavy-ion collisions [4–11]. Since the transverse energy measurement is independent and easier (no particle identification is necessary), it gives an unique opportunity to verify the concept of the appearance of a thermal system during such collisions.

The statistical model with single freeze-out is used [8–11]. The model reproduces very well ratios and p_T spectra of particles observed at RHIC [8,10,11]. The main assumption of the model is the simultaneous occurrence of chemical and thermal freeze-outs. The new data on $K^*(892)^0$ production revealed by the STAR Collaboration [12] support strongly this assumption. Since $\frac{dE_\mathrm{T}}{d\eta}_{|\mathrm{mid}}/\frac{dN_\mathrm{ch}}{d\eta}_{|\mathrm{mid}}$ measurements have been done at midrapidity, the presented analysis is valid in the Central Rapidity Region (CRR) of considered collisions.

Therefore, it is assumed that a noninteracting gas of stable hadrons and resonances at chemical and thermal equilibrium is present at the CRR. For consistency with previous works [4–9], where freeze-out parameters were found out from particle ratio measurements for a static fireball, this is also the case considered here. Then the distributions of various species of primordial particles are given by the usual ideal-gas formulae. Only baryon number μ_B and strangeness μ_S chemical potentials are taken into account here. The isospin chemical potential μ_{I_3} has very low value in analyzed cases [6, 8] and therefore can be neglected. For given T and μ_B, μ_S is determined from the requirement that the overall strangeness of the gas equals zero. In this way, the temperature T and the baryon chemical potential μ_B are the only independent parameters of the model.

Theoretically, the transverse energy is defined as the sum of transverse masses of all L interacting and produced particles [13],

$$E_{\mathrm{T}}^{\mathrm{th}} = \sum_{i=1}^{L} \sqrt{m_i^2 + (p_{\mathrm{T}}^i)^2}\,, \tag{1}$$

where m_i and p_{T}^i are the mass and transverse momentum, respectively. But experimentally, the measured quantity is

$$E_{\mathrm{T}} = \sum_{i=1}^{L} E_i \, \sin\theta_i\,, \tag{2}$$

where θ_i is the polar angel and E_i denotes the total energy. Precisely, what is measured in a calorimeter is the kinetic energy for nucleons and the total energy for all other particles [1].

If one deals with the thermal system, definitions (1) and (2) should be generalized appropriately. The transverse energy density $\varepsilon_{\mathrm{T}}^i$ of the particles of species i can be defined at the temperature T and the baryon chemical potential μ_B as ($\hbar = c = 1$ always)

$$\varepsilon_{\mathrm{T}}^i = (2s_i + 1) \int d^3\vec{p}\, e_{\mathrm{T}}^i(\vec{p})\, f_i(p; T, \mu_B)\,, \tag{3}$$

where $e_{\mathrm{T}}^i(\vec{p})$ is the suitable expression for the transverse energy of the particle and m_i, s_i and $f_i(p; T, \mu_B)$ are its mass, spin and momentum distribution (which is isotropic here), respectively. Since the theoretical estimates should correspond to what is actually measured, e_{T}^i takes the form

$$e_{\mathrm{T}}^i(\vec{p}) = E_i \, \sin\theta = E_i \, \frac{p_{\mathrm{T}}}{p} \tag{4}$$

in the analyzed model.

The thermal system ceases at the freeze-out and there are only free escaping particles instead of the fireball. The measured $\frac{dE_{\mathrm{T}}}{d\eta}_{|\mathrm{mid}}$ is fed from two sources: (a) stable hadrons which survived until catching in a detector, (b) secondaries produced by decays and sequential decays of primordial resonances after the freeze-out. Therefore, if the contribution to the transverse energy from particles (a) is described, the distribution f_i in (3) is either a Bose–Einstein or a Fermi–Dirac distribution at the freeze-out. But if the contribution from particles (b) is considered, the distribution f_i is the spectrum of the finally detected secondaries and can be obtained from the elementary kinematics of a many-body decay or from the superposition of two or more such decays (for details, see [8, 11]; also [14, 15] can be very useful). In fact, if one considers detected species i, then f_i is the sum of final i's spectra resulting from a single decay or a cascade. The sum is taken over all such decays and cascades of resonances in which at least one of the final secondaries is of the i kind.

Since $\frac{dN_{\mathrm{ch}}}{d\eta}_{|\mathrm{mid}}$ has also its origin in the above-mentioned sources (a) and (b), to define properly the density of charged particles, decays should be also taken into account. Thus the density of the measured charged particles of species j reads

$$n^j = n^j_{\mathrm{primordial}} + \sum_i \alpha(j,i)\, n^i_{\mathrm{primordial}}\,, \tag{5}$$

where $n^i_{\mathrm{primordial}}$ is the density of the ith particle species at the freeze-out and $\alpha(j,i)$ is the final number of particles of species j which can be received from all possible simple or sequential decays of particle i. The density $n^i_{\mathrm{primordial}}$ is given by the usual integral of either a Bose–Einstein or a Fermi–Dirac distribution. Now, in the midrapidity region, the theoretical equivalent of $\frac{dE_{\mathrm{T}}}{d\eta}_{|\mathrm{mid}} / \frac{dN_{\mathrm{ch}}}{d\eta}_{|\mathrm{mid}}$ can be postulated as

$$\frac{\frac{dE_{\mathrm{T}}}{d\eta}_{|\mathrm{mid}}}{\frac{dN_{\mathrm{ch}}}{d\eta}_{|\mathrm{mid}}} \equiv \frac{\varepsilon_{\mathrm{T}}}{n_{\mathrm{ch}}}\,, \tag{6}$$

where the transverse energy density ε_{T} and the density of charged particles n_{ch} are given by the expressions

$$\varepsilon_{\mathrm{T}} = \sum_{i\in A} \varepsilon^i_{\mathrm{T}}\,, \tag{7}$$

$$n_{\mathrm{ch}} = \sum_{j\in B} n^j\,. \tag{8}$$

Note that there are two different sets of final particles, A and B ($B \subset A$). B denotes final charged particles and these are π^+, π^-, K^+, K^-, p and $\bar{p}$, whereas A also includes photons, $K^0_{\rm L}$, n and $\bar{n}$ [1].

To proceed further, some simplifications are necessary. This is because the complete treatment of resonance decays in $\varepsilon_{\rm T}$ is complex and therefore it consumes a lot of computer working time in numerical calculations. So the initial set of resonances should be as small as possible. The lifetime of at least 10 fm is chosen as the necessary condition, with the exception of $K^*(892)$ mesons, since one of them is measured at RHIC [12]. This reduces constituents of the hadron gas to 40 species, but note that all particles listed in [4–8] are included. Of course, this condition is arbitrary but it makes sense because most neglected resonances have the lifetime of the order of a few fm, so one may assume they decay already at the pre-equilibrium stage. Anyway, on the basis of the analysis with decays not included in $\varepsilon_{\rm T}$, it has been found that the ratio (6) changes slowly with the number of species of the hadron gas in the considered region of T and μ_B.

In the following, a point-like gas is assumed. It has been checked that for the excluded volume hadron gas model [16–18] the results are exactly the same. There are two reasons for that: the first, the volume corrections placed in denominators of expressions for various densities cancel with each other in a ratio; the second, the eigenvolume of a hadron and the pressure in the considered region of T and μ_B are so small that their product correction to the chemical potential is negligible.

As it has been already mentioned, the main difficulty in complete treatment of decays in numerical evaluations of $\varepsilon_{\rm T}$ is their complexity. Therefore some further simplifications should be done. First of all, some decays and cascades are neglected: *(i)* four-body decays, *(ii)* superpositions of two three-body decays, *(iii)* superpositions of two three-body and one two-body decays, *(iv)* superpositions of four two-body decays, *(v)* some decays of heavy resonances with very small branching ratios. Their maximal contribution to $\varepsilon_{\rm T}$ has been evaluated as $\leq 2\%$. Thus, the real $\varepsilon_{\rm T}$ can be at most 2% higher than its evaluation. It should be stressed that the above-mentioned simplifications are done only in calculations of $\varepsilon_{\rm T}$, whereas $n_{\rm ch}$ given by (5) and (8) includes all possible decays and cascades. This means that the presented values of the ratio (6) (and (15)) can be also at most 2% higher.

In application to heavy-ion collisions, it is assumed that the rest frame of the hadron gas is the c.m.s of two colliding ions. But RHIC is the opposite beam experiment, whereas SPS and AGS are the fixed target ones. So, the laboratory frame is the c.m.s only in the RHIC case. Since the measurement is done in the laboratory frame, to treat SPS and AGS cases properly, it is assumed that there is the overall uniform flow (of the gas) with constant velocity v equal to the velocity $v_{\rm c.m.s}$ of the c.m.s relative to the target. The $v_{\rm c.m.s}$ is calculated for $158 \cdot A$ GeV Pb–Pb collisions at SPS (this results in $v_{\rm c.m.s} = 0.994$) and for $11 \cdot A$ GeV Au–Au collisions at AGS ($v_{\rm c.m.s} = 0.918$).

Now one applies the general description founded in [19] and developed in [20] for the case with decays taken into account. The invariant distribution of the measured particles of species j has the form [20]

$$E_j \frac{dN_j}{d^3\vec{p}} = \int p^\mu d\sigma_\mu \, f_j(p \cdot u) \,, \tag{9}$$

where $d\sigma_\mu$ is the normal vector on a freeze-out hypersurface, $p \cdot u = p^\mu u_\mu$, u is the appropriate four-velocity and f_j is the final momentum distribution of the particle in question. The final means here that f_j is the sum of primordial and decay contributions to the particle distribution. For the static (homogeneous) fireball, the freeze-out hypersurface is simply a volume V^* at a freeze-out time $t^*_{\rm f.o.}$ (stars denote quantities in the comoving frame, which is the local rest frame of the gas and also the c.m.s here). The spatial coordinates x^{*1}, x^{*2}, x^{*3} are the parameters of the hypersurface in this case. Thus the normal vector reads $d\sigma^*_\mu = (d^3\vec{x}^*, 0, 0, 0)$. In the laboratory frame $d\sigma_\mu = (\gamma d^3\vec{x}^*, 0, 0, -\gamma v d^3\vec{x}^*)$, $\gamma = (1-v^2)^{-1/2}$, and since $u_\mu = \gamma(1, 0, 0, -v)$ in the considered case, the normal vector is proportional to the four-velocity, $d\sigma_\mu = u_\mu d^3\vec{x}^*$. This is the necessary condition for the invariant distribution expression of particles j to have the form (9), where f_j is calculated in the local rest frame of the gas (for more details, see [20]). Note also that since $u^\mu u_\mu = 1$, the normal vector is time-like, so the hypersurface consists only of a space-like part. Thus the conceptual problems with time-like parts of a hypersurface are avoided (for discussion of the subject, see *e.g.* [21, 22] and references therein). Now the invariant distribution of the particles of species j reads

$$E_j \frac{dN_j}{d^3\vec{p}} = \int\limits_{V^*} d^3\vec{x}^* (p \cdot u) \, f_j(p \cdot u) = V^* (p \cdot u) \, f_j(p \cdot u) \,, \tag{10}$$

where $p \cdot u = \gamma \cdot (E_j - p_z v) = E^*_j$ is the energy in the comoving frame. From (10) the multiplicity of the j-th particles can be obtained as

$$N_j = V^* \int \frac{d^3\vec{p}}{E_j} (p \cdot u) \, f_j(p \cdot u) = V^* \int d^3\vec{p}^* \, f_j(E^*_j) = V^* \cdot n^j \,, \tag{11}$$

where the last equality holds because distributions of particles depend here only on the magnitude of the three-momentum, so $f_j(E^*_j) = f_j(p^*)$. Similarly, the final transverse energy carried by the particles of species j is calculated as

$$E_{\mathrm{T}}^{j} = V^{*} \int \frac{d^{3}\vec{p}}{E_{j}} E_{j} \frac{p_{\mathrm{T}}}{p} (p \cdot u)\, f_{j}(p \cdot u) = V^{*} \int d^{3}\vec{p}^{*} E_{j} \frac{p_{\mathrm{T}}}{p} f_{j}(p^{*})\,, \tag{12}$$

where $E_j = \gamma \cdot (E_j^* + p_z^* v)$, $p_{\mathrm{T}} = p_{\mathrm{T}}^*$ and $p = \sqrt{E_j^2 - m_j^2}$. And finally, the transverse energy, the multiplicity of charged particles and the ratio (6) are given by the expressions

$$E_{\mathrm{T}} = \sum_{j \in A} E_{\mathrm{T}}^{j}\,, \tag{13}$$

$$N_{\mathrm{ch}} = \sum_{j \in B} N_{j}\,, \tag{14}$$

$$\frac{\frac{dE_{\mathrm{T}}}{d\eta}_{|\mathrm{mid}}}{\frac{dN_{\mathrm{ch}}}{d\eta}_{|\mathrm{mid}}} \equiv \frac{E_{\mathrm{T}}}{N_{\mathrm{ch}}}\,, \tag{15}$$

respectively. Note that the dependence on the volume V^* has disappeared in the ratio (15) and for $v = 0$ the formula (6) has been recovered.

The final results of calculations following from the formula (15) are presented in Table I. In the first column estimates of freeze-out parameters obtained from the analysis of particle ratios [4–9] are listed. In the second column corresponding values of $\frac{E_{\mathrm{T}}}{N_{\mathrm{ch}}}$ calculated with the use of (4) are placed. In the third column of Table I, the most realistic results are presented, namely the fact that for nucleons only the kinetic energy is measured [1] is taken into account. The appropriate $v_{\mathrm{c.m.s}}$ is put for each collider. For clearness, these results have been also depicted together with the data in Fig. 1.

Generally, very good agreement of the obtained predictions with the data has been reached. One can notice that the accuracy of theoretical evaluations rises with the collision energy. For AGS, the ratio $E_{\mathrm{T}}/N_{\mathrm{ch}}$ is within error bars of the experimental point only for the higher temperature freeze-out, whereas for RHIC both estimates fit very well. This could mean that AGS energy is the low limit of applicability of a statistical model. Alternatively, one could call the application of the same gas for both RHIC and AGS in question. This is because of a different baryon content of each case. The AGS gas should be much more rich with baryons than the RHIC one. If one considers contributions to primordial E_{T} (*i.e.* decays are not included in calculation of E_{T}) from constituents of the gas assumed here, nucleons weight most for AGS, but pions are the biggest fraction in the RHIC case. So, probably additional baryon resonances should be included in a gas in the AGS case. In fact, some preliminary estimates of primordial

TABLE I

Values of $\frac{E_{\rm T}}{N_{\rm ch}}$ calculated with the use of the formula (4) (second column) and the realistic version of the formula (4) with the kinetic energy instead of the total energy for nucleons (third column). In the first column estimates of freeze-out parameters obtained from the analysis of particle ratios [4–9] are listed. In the last column experimental data are given. The velocity $v_{\rm c.m.s}$ is the velocity of the center of mass of colliding nuclei with respect to the laboratory frame and equals respectively: 0 for RHIC, 0.994 for SPS and 0.918 for AGS.

	$E_{\rm T}/N_{\rm ch}$ [GeV] ($v_{\rm c.m.s}$ appropriate)		$\frac{dE_{\rm T}}{d\eta}/\frac{dN_{\rm ch}}{d\eta}$ [GeV]
		$E \to E - m_n$ for nucleons	
$T = 175$ MeV $\mu_B = 51$ MeV RHIC [7]	0.88	0.80	$0.8^{+0.08}_{-0.06}$ [1]
$T = 165$ MeV $\mu_B = 41$ MeV RHIC [8]	0.84	0.77	$0.8^{+0.08}_{-0.06}$ [1]
$T = 168$ MeV $\mu_B = 266$ MeV SPS [6]	0.76	0.75	0.8 ± 0.2 [2]
$T = 164$MeV $\mu_B = 234$ MeV SPS [9]	0.74	0.73	0.8 ± 0.2 [2]
$T = 130$ MeV $\mu_B = 540$ MeV AGS [4]	0.81	0.66	0.72 ± 0.08 [3]
$T = 110$ MeV $\mu_B = 540$ MeV AGS [5]	0.70	0.57	0.72 ± 0.08 [3]

$E_{\rm T}$ indicate that by adding more species into the gas one could increase the $E_{\rm T}/N_{\rm ch}$ ratio at the AGS freeze-out. Also if one takes the expansion of the gas into account one might improve the results for the AGS case. Roughly speaking, the expansion produces additional energy, so it could increase $E_{\rm T}$. These problems need much more detailed analysis and will be under further investigation.

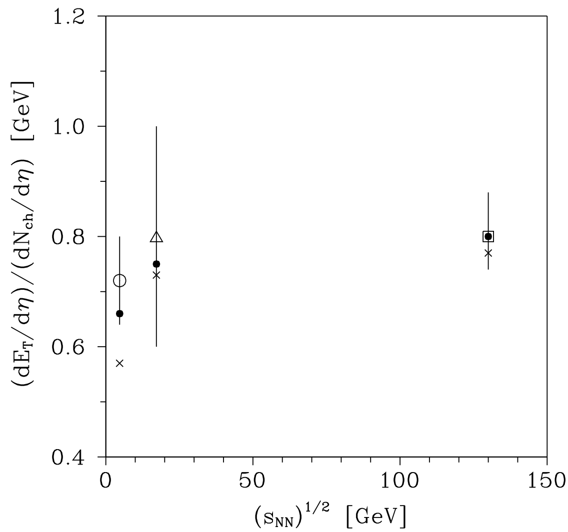

Fig. 1. Values of $E_{\mathrm{T}}/N_{\mathrm{ch}}$ from the third column of Table I. Black dots and crosses denote evaluations of the ratio at higher and lower temperature for a given collider, respectively (see the first column of Table I). Also data points for AGS [3] (circle), SPS [2] (triangle) and RHIC [1] (square) are depicted.

To conclude, a statistical model has been used to reproduce the ratio $\frac{dE_{\mathrm{T}}}{d\eta}/\frac{dN_{\mathrm{ch}}}{d\eta}$ measured at RHIC, SPS and AGS. The importance of presented analysis lies in the fact that the ratio is an independent observable, so it can be used as a new tool to verify the consistence of predictions of a statistical model for all colliders simultaneously. The point-like non-interacting hadron gas with 40 species has been used in final calculations. Decays and sequential decays of constituents of the gas have been taken into account. In spite of the simplicity of the model, theoretical predictions for $E_{\mathrm{T}}/N_{\mathrm{ch}}$ agree very well with the data for the wide range of collision energies starting from AGS up to RHIC. The predictions have been made at the previous estimates of freeze-out parameters obtained from the analysis of measured particle ratios for RHIC [7, 8], SPS [6, 9] and AGS [4, 5]. This means that the applicability of a statistical model to heavy-ion collisions has been confirmed strongly in an independent way.

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