COLLISIONAL EFFECTS IN ISOVECTOR RESPONSE FUNCTION OF NUCLEAR MATTER AT FINITE TEMPERATURE

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The dipole response function of nuclear matter at zero and finite temperatures is investigated by employing the linearized version of the extended TDHF theory with a non-Markovian binary collision term. Calculations are carried out for nuclear dipole vibrations by employing the Steinwedel– Jensen model and compared with experimental results for ¹²⁰Sn and ²⁰⁸Pb.

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Giant resonances, in particular giant dipole resonances (GDR), in medium-weight and heavy nuclei have been the subjects of extensive experimental and theoretical studies during the last decades [1]. A large amount of experimental information is now available concerning the properties of GDR built on the ground and the excited states of the nuclei revealing the properties of the collective motion of nuclear many-body systems at zero and finite temperatures. The mean resonance energy is observed not to change much with the excitation energy, or the temperature, but the recent experimental investigations show that the width of the resonance becomes broader as excitation energy increases with a possible saturation at high temperatures. This temperature dependence of the GDR width is still one of the open problems in the studies of nuclear collective response and its damping mechanisms at zero and finite temperatures [2–4].

The theoretical investigations of the nuclear collective response employing the random phase approximation (RPA) theory have been quite successful in describing the mean resonance energies [5]. However, the RPA theory is not suitable for describing the damping of collective excitations and therefore investigations based on the RPA theory have not been able to explain the increase of the width of GDR with temperature [6].

There are different mechanisms involved in the damping of the nuclear collective state. A part of the damping is due to the coupling of the collective mode to external degrees of freedom resulting in the cooling of the system by particle emission giving rise to escape width. Furthermore, the collective mode also acquires an intrinsic width as a consequence of its coupling to the internal degrees of freedom. The resulting spreading width which is thus due to the mixing of the collective mode with more complicated doorway states makes essentially the large part of the contribution to damping in medium-weight and heavy nuclei. There are essentially three different theoretical approaches for the calculation of the spreading widths. In the first case, the temperature dependence of the width is explained by the coherent mechanism due to adiabatic coupling of the giant resonance with thermal surface deformations [7,8], which is particularly important at low temperature. In the second approach, the mechanism of damping is due to the coupling with incoherent two particle-two hole (2p-2h) states resulting from the increasing rate of collisions between the nucleons with temperature which is usually referred to as the collisional damping [9, 10]. The collisional damping is relatively weak at low temperature, but its magnitude becomes large with increasing temperature. The last mechanism is the Landau damping which is due to the spreading of the collective mode on non-collective particle-hole (p-h) excitations. Most investigations of nuclear response that have been carried out so far are based on the coherent damping or the collisional damping mechanisms [11, 12].

In this work, we perform a linear response treatment of the nuclear collective mode by including the collisional damping. The small amplitude limit of the extended the time-dependent Hartree–Fock (TDHF) theory, in which the collisional damping due to the incoherent 2p–2h decay is included in the form of a non-Markovian collision term, provides an appropriate framework for investigating the damping widths of collective modes at zero and finite temperatures [10–13]. We employ the extended TDHF theory to study the isovector response function with collisional effects of nuclear matter at zero and finite temperatures in semiclassical approximation using a simplified effective Skyrme force.

The equation of motion of the single particle density matrix $\rho(t)$ in the extended TDHF approximation is given by a transport equation [13]

$$i\hbar\frac{\partial}{\partial t}\rho - [h(\rho), \rho] = K(\rho), \qquad (1)$$

where $h(\rho)$ is an effective mean-field Hamiltonian and the right-hand side

represents a non-Markovian collision term, which can be expressed in terms of the correlated part of the two-particle density matrix as $K(\rho) = \text{Tr}_2[v, C_{12}]$ with the effective residual interaction v. The correlated part of the twoparticle density matrix $C_{12} = \rho_{12} - \rho_1 \rho_2$ where $\rho_1 \rho_2$ represents the antisymmetrized product of the single-particle density matrices, is given by the second equation of the BBGKY hierarchy. In the extended TDHF theory, the hierarchy is truncated at the second level by retaining only the lowest-order terms in the residual interactions, thus neglecting three-body correlations. Hence, the correlated part of the two-particle density matrix C_{12} satisfies the equation

$$i\hbar \frac{\partial}{\partial t}C_{12} - [h(\rho), C_{12}] = F_{12},$$
 (2)

where the source term is given by

$$F_{12}(\rho) = (1 - \rho_1)(1 - \rho_2) \ v \ \widetilde{\rho_1 \rho_2} - \widetilde{\rho_1 \rho_2} \ v \ (1 - \rho_1)(1 - \rho_2) \ . \tag{3}$$

In order to study the isovector collective response of the system, we include an external perturbation $F(\vec{r}, t)$ into the equation of motion,

$$F(\vec{r},t) = \tau_3 F(\vec{r}) \left[e^{-i\omega t} + e^{i\omega t} \right] , \qquad (4)$$

where τ_3 is the third component of the isospin operator and the frequency of the one-body harmonic perturbation operator contains a small imaginary part with the prescription $\omega \to \omega + i\eta$ in accordance with the adiabatic hypothesis. We obtain a description for small density fluctuations $\delta\rho(t) = \rho(t) - \rho_0$ in linear response treatment by linearizing the extended TDHF theory around a finite temperature equilibrium state density ρ_0 , and this way we obtain

$$i\hbar\frac{\partial}{\partial t}\delta\rho - [h_0, \delta\rho] - [\delta h + F(\vec{r}, t), \rho_0] = \operatorname{Tr}_2[v, \delta C_{12}], \qquad (5)$$

where $\delta h = (\partial U/\partial \rho)_0 \,\delta \rho$ represents small deviations in the effective meanfield potential. Moreover, the small deviation of the two-body correlations $\delta C_{12}(t)$ satisfies

$$i\hbar \frac{\partial}{\partial t} \delta C_{12} - [\delta h + F(\vec{r}, t), C_{12}^0] - [h_0, \delta C_{12}] = \delta F_{12}.$$
(6)

We look for a solution of Eq. (5) and Eq. (6) of the form $\delta\rho(t) = \delta\rho(\omega)e^{-i\omega t} + h.c.$ where now the small density fluctuations are given in terms of the proton and neutron density matrices as $\delta\rho(t) = \rho_p(t) - \rho_n(t)$. We note that in the small amplitude limit, the collision term is also harmonic,

 $\delta K = tr_2[v, \delta C_{12}] = \delta K(\omega) e^{-i\omega t} + h.c.$, and in momentum representation we then obtain

$$\begin{bmatrix} \hbar\omega + \varepsilon \left(\vec{p} - \frac{\vec{k}}{2} \right) - \varepsilon \left(\vec{p} + \frac{\vec{k}}{2} \right) \end{bmatrix} \left\langle \vec{p} + \frac{\vec{k}}{2} \middle| \delta\rho(\omega) \middle| \vec{p} - \frac{\vec{k}}{2} \right\rangle$$
$$- \left[f \left(\vec{p} - \frac{\vec{k}}{2} \right) - f \left(\vec{p} + \frac{\vec{k}}{2} \right) \right]$$
$$\times \left\{ \left\langle \vec{p} + \frac{\vec{k}}{2} \middle| \delta h \middle| \vec{p} - \frac{\vec{k}}{2} \right\rangle + 2 \left\langle \vec{p} + \frac{\vec{k}}{2} \middle| F(\vec{r}) \middle| \vec{p} - \frac{\vec{k}}{2} \right\rangle \right\}$$
$$= \left\langle \vec{p} + \frac{\vec{k}}{2} \middle| \delta K(\omega) \middle| \vec{p} - \frac{\vec{k}}{2} \right\rangle, \tag{7}$$

where $f(\vec{q}) = 1/\{1 + e^{\beta[\varepsilon(\vec{q}) - \mu]}\}$ is the Fermi–Dirac occupation factor. For simplified Skyrme interaction, the density fluctuation $\delta n(\vec{r}, t)$ induces local changes in the mean field potential, therefore

$$\left\langle \vec{p} + \frac{\vec{k}}{2} \middle| \delta h \middle| \vec{p} - \frac{\vec{k}}{2} \right\rangle = 2V_0 \left\langle \vec{p} + \frac{\vec{k}}{2} \middle| \delta \rho(\omega) \middle| \vec{p} - \frac{\vec{k}}{2} \right\rangle , \qquad (8)$$

where in terms of local proton and neutron mean field potentials δh is expressed as $\delta h = U_p(\vec{r}, t) - U_n(\vec{r}, t) = 2V_0\delta n(\vec{r}, t)$. Moreover, since

$$\int \frac{d^3 p}{(2\pi\hbar)^3} \left\langle \vec{p} + \frac{\vec{k}}{2} \right| \delta\rho(\omega) \left| \vec{p} - \frac{\vec{k}}{2} \right\rangle = \delta n(\vec{k}, \omega) \tag{9}$$

and

$$\left\langle \vec{p} + \frac{\vec{k}}{2} \middle| F(\vec{r}) \middle| \vec{p} - \frac{\vec{k}}{2} \right\rangle = F(\vec{k})$$
(10)

we finally obtain

$$\delta n(\vec{k},\omega) - \left[V_0 \delta n(\vec{k},\omega) + F(\vec{k})\right] \Pi_1(\vec{k},\omega) = \left[V_0 \delta n(\vec{k},\omega) + F(\vec{k})\right] \Pi_2(\vec{k},\omega).$$
(11)

The function $\Pi_1(\vec{k},\omega)$, which is known as the unperturbed Lindhard function, is given by

$$\Pi_1(\vec{k},\omega) = \frac{2}{(2\pi\hbar)^3} \int d^3p \; \frac{f\left(\vec{p} - \frac{\vec{k}}{2}\right) - f\left(\vec{p} + \frac{\vec{k}}{2}\right)}{\hbar\omega - \varepsilon\left(\vec{p} + \frac{\vec{k}}{2}\right) + \varepsilon\left(\vec{p} - \frac{\vec{k}}{2}\right) + i\eta} \tag{12}$$

and the function $\Pi_2(\vec{k},\omega)$ is obtained from

$$[V_0 \delta n(\vec{k}, \omega) + F(\vec{k})] \Pi_2(\vec{k}, \omega) = \frac{2}{(2\pi\hbar)^3} \int d^3p \frac{\left\langle \vec{p} + \frac{\vec{k}}{2} \right| \delta K(\omega) \left| \vec{p} - \frac{\vec{k}}{2} \right\rangle}{\hbar\omega - \varepsilon \left(\vec{p} + \frac{\vec{k}}{2} \right) + \varepsilon \left(\vec{p} - \frac{\vec{k}}{2} \right) + i\eta}.$$
 (13)

The retarded response function which is defined by

$$\delta n(\vec{k},\omega) = \Pi_{\rm R}(\vec{k},\omega)F(\vec{k}) \tag{14}$$

is then obtained as

$$\Pi_{\rm R}(\vec{k},\omega) = \frac{\Pi_0(k,\omega)}{1 - V_0 \Pi_0(\vec{k},\omega)}$$
(15)

with $\Pi_0(\vec{k},\omega) = \Pi_1(\vec{k},\omega) + \Pi_2(\vec{k},\omega).$

From Eq. (6), it is possible to obtain a closed form expression for small deviation of two-body correlations δC_{12} , that is valid for collective vibrations (for details please refer to [10]),

$$\delta C_{12}(t) = -\frac{i}{\hbar} \int^{t} dt' \rho_{1}^{0} \rho_{2}^{0} e^{-ih_{0}(t-t')} [\delta \Phi(t'), v] \\ \times e^{ih_{0}(t-t')} (1-\rho_{1}^{0})(1-\rho_{2}^{0}) + \text{h.c.}, \qquad (16)$$

where $\delta \Phi(t)$ is the distortion function associated with the single-particle density matrix, and it is related to the small vibrations in the single-particle density matrix $\delta \rho(t)$ according to $\delta \rho(t) = [\delta \Phi(t), \rho_0]$. We then obtain the expression for the linearized collision term by evaluating the matrix element $\langle \vec{p} + \frac{\vec{k}}{2} | \delta K(\omega) | \vec{p} - \frac{\vec{k}}{2} \rangle$ in which we retain \vec{k} -dependence only in distortion function [10]. Then, the collisional response function $\Pi_2(\vec{k}, \omega)$ can be expressed as

$$\Pi_{2}(\vec{k},\omega) = \frac{1}{(2\pi\hbar)^{3}} \int d^{3}p_{1}d^{3}p_{2}d^{3}p_{3}d^{3}p_{4} \\
\times \left(\frac{\Delta Q}{2}\right)^{2} \frac{W(12;34)}{\pi} \frac{f_{1}f_{2}\overline{f}_{3}\overline{f}_{4} - \overline{f}_{1}\overline{f}_{2}f_{3}f_{4}}{\hbar\omega - \varepsilon_{3} - \varepsilon_{4} + \varepsilon_{1} + \varepsilon_{2} + i\eta}, \quad (17)$$

where $\Delta Q = Q_1 + Q_2 - Q_3 - Q_4$ with $Q_i = 1/\left[\hbar\omega - \varepsilon\left(\vec{p_i} + \frac{\vec{k}}{2}\right) + \varepsilon\left(\vec{p_i} - \frac{\vec{k}}{2}\right)\right]$, $\overline{f}_i = 1 - f_i$ and W(12;34) denotes the basic two-body transition rate, which

can be expressed in terms of the spin averaged proton–neutron scattering cross section as

$$W(12;34) = \frac{1}{(2\pi\hbar)^3} \frac{4\hbar}{m^2} \left(\frac{d\sigma}{d\Omega}\right)_{pn} \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4).$$
(18)

The strength distribution function is obtained from the imaginary part of the retarded response function [14]

$$S(\vec{k},\omega) = -\frac{1}{\pi} \operatorname{Im} \Pi_{\mathrm{R}}(\vec{k},\omega) \,. \tag{19}$$

In our calculations, we employ a simplified Skyrme interaction

$$v = t_0 (1 + x_0 P_\sigma) \delta(\vec{r}) + \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho^\alpha(\vec{R}) \delta(\vec{r})$$
(20)

with $\vec{r} = \vec{r_1} - \vec{r_2}$ and $\vec{R} = (\vec{r_1} + \vec{r_2})/2$. The local potential for protons is then given by

$$U_{p}(\vec{r},t) = t_{0} \left(1 + \frac{1}{2}x_{0}\right) \rho(\vec{r},t) - t_{0} \left(\frac{1}{2} + x_{0}\right) \rho_{p}(\vec{r},t) + \frac{1}{12}t_{3}\rho^{\alpha}(\vec{r},t) \left[(2+\alpha)\left(1 + \frac{1}{2}x_{3}\right)\rho_{p}(\vec{r},t) - 2\left(\frac{1}{2} + x_{3}\right)\rho(\vec{r},t) - \alpha\left(\frac{1}{2} + x_{3}\right)\frac{\rho_{p}^{2}(\vec{r},t) + \rho_{n}^{2}(\vec{r},t)}{\rho(\vec{r},t)} \right]$$
(21)

with a similar expression for neutrons. In linear response approximation, the coupling constant V_0 for dipole vibrations becomes

$$V_0 = -\frac{1}{2}t_0\left(\frac{1}{2} + x_0\right) - \frac{1}{12}t_3\rho_0^\alpha\left(\frac{1}{2} + x_3\right), \qquad (22)$$

where ρ_0 is the saturation density of nuclear matter. In our analysis we consider in particular the Skyrme SLy4 force with the parameters [15] $t_0 = -2488.91 \text{ MeV fm}^3$, $t_3 = 13777 \text{ MeV fm}^{7/2}$, $x_0 = 0.834$, $x_3 = 1.354$ and $\alpha = 1/6$, which results for V_0 in the value $V_0 = 85 \text{ MeV fm}^3$.

In order to apply our results to finite nuclei, we work within the framework of Steinwedel and Jensen model for nuclear dipole oscillations [6]. In this model neutrons and protons oscillate inside a sphere of radius R given by the expression

$$\rho_p(\vec{r},t) - \rho_n(\vec{r},t) = F \sin(\vec{k} \cdot \vec{r}) e^{i\omega t}, \qquad (23)$$

the total density remaining equal to the saturation density ρ_0 of nuclear matter and the wavenumber k is given by $k = \pi/2R$. We apply Steinwedel and Jensen model to GDR in ¹²⁰Sn and ²⁰⁸Pb, and we take R = 5.6 fm k = 0.28 fm⁻¹ for ¹²⁰Sn and R = 6.7 fm k = 0.23 fm⁻¹ for ²⁰⁸Pb according to $R = 1.13A^{1/3}$.

As a result of the approximate treatment, the collisional response function $\Pi_2(\vec{k},\omega)$ has a singular behavior arising from the pole of the distortion functions, $Q_i = 1 \left/ \left[\hbar \omega - \varepsilon \left(\vec{p_i} + \frac{\vec{k}}{2} \right) + \varepsilon \left(\vec{p_i} - \frac{\vec{k}}{2} \right) \right]$. We avoid this singular behavior by incorporating a pole approximation. In the distortion functions, we make the replacement $\omega \to \omega_D - i\Gamma/2$ where ω_D and Γ are determined from $1 - V_0 \Pi_1(\vec{k}, \omega) = 0$ at each temperature that is considered. Furthermore, we neglect the real part Re Π_2 of the function Π_2 in our calculations. In the calculations of the collisional response function, using conservation laws and symmetry properties, it is possible to reduce the twelve dimensional integrals to five fold integrals by incorporating the transformations into the total momenta $\vec{P} = \vec{p}_1 + \vec{p}_2$, $\vec{P'} = \vec{p}_3 + \vec{p}_4$, and relative momenta $\vec{q} = (\vec{p_1} - \vec{p_2})/2$, $\vec{q}' = (\vec{p_3} - \vec{p_4})/2$ before and after the collisions. The integral over \vec{P}' can be performed immediately. The delta function $\delta(\hbar\omega - \varepsilon' + \varepsilon)$ in Im $\Pi_2(\vec{k},\omega)$ where $\varepsilon = \vec{q}^2/m$ and $\varepsilon' = \vec{q}'^2/m$ are the energies of two particle system in the center of mass frame before and after the collision makes it possible the reduce the integrals further using familiar methods from the Fermi liquid theory [17]. Then, we evaluate the remaining five dimensional integrals numerically by employing a fast algorithm. In the evaluation of momentum integrals, we neglect the angular anisotropy of the cross sections and make the replacement $(d\sigma/d\Omega)_{pn} \to \sigma_{pn}/4\pi$ with $\sigma_{pn} = 40$ mb.

We show our results for the response function with and without the collision term in Fig. 1 for ¹²⁰Sn and in Fig. 2 for ²⁰⁸Pb as a function of experimental temperature T^* where we also present the comparison with the normalized experimental data [4]. The experimental temperature T^* is related to the temperature parameter in the Fermi–Dirac function $f(\varepsilon, T)$ as $T = T^* \sqrt{a_{\rm E}/a_{\rm F}}$, where $a_{\rm E}$ denotes the energy dependent empirical level density parameter and $a_{\rm F} = A\pi^2/4\varepsilon_{\rm F}$ denotes Fermi gas level density parameter [4,16]. In our calculations, we use the temperature values T in the Fermi–Dirac function that are related to the experimental temperatures in this manner.

We first note that, at the RPA level that is without the contribution of the collision term in calculation of the nuclear response, the position of the peak of the response functions do not change with temperature. As a matter of fact, for ²⁰⁸Pb the peak is at $\omega = 12.3$ MeV for $T^* =$ 1.34, 1.62, 1.85, 2.05 MeV while for ¹²⁰Sn it occurs at $\omega = 14.3$ MeV for $T^* = 1.24$, 2.02 MeV and at $\omega = 14.2$ MeV for $T^* = 2.61$, 3.12 MeV.



Fig. 1. The GDR strength function of 120 Sn. Solid and dashed lines show the response function without and with the collision term, respectively. The normalized data is taken from [4].

This behavior of the peak energy with temperature is in accordance with the experimental results where it is observed that the mean-energy of the dipole response is almost constant for ²⁰⁸Pb when T^* changes between 1.3 to 2.0 MeV while a decrease of 1.5 MeV is observed in ¹²⁰Sn when T^* changes from 1.2 to 3.1 MeV [4]. However, the average positions of the peak values of the strength functions are slightly below the experimental values, which are $\omega = 15.4$ MeV for ¹²⁰Sn and $\omega = 13.4$ MeV for ²⁰⁸Pb. This discrepancy may be due to the nature of the effective Skyrme force that we employ. Moreover, the value for $k = \pi/2R$ that is used in Steinwedel and Jensen model depends on the value of R_0 used in $R = R_0 A^{1/3}$, and changing k somewhat also produces a change in the position of the peak but in general the above conclusions are not affected. Furthermore, these results for the peak position of the strength functions are in accordance with the earlier RPA calculations.



Fig. 2. The GDR strength function of ²⁰⁸Pb. Solid and dashed lines show the response function without and with the collision term, respectively. The normalized data is taken from [4].

Dotted lines in figures 1, 2 show the strength functions including collisional damping mechanism. Since, we neglect the real part of the collisional response the peak values of the strength do not change, but the collisional mechanism introduces a spread, in particular at the high frequency side of the strength functions and this spread becomes more pronounced with increasing temperature. In order to illustrate the effect of the collision term more drastically, in figure 3, we show the response function with and without the collision term for ¹²⁰Sn and for ²⁰⁸Pb at temperatures T = 2, 4 MeV.

Rather simple description presented in this paper is able to explain certain aspects of giant dipole excitations in 120 Sn and 208 Pb, but do not produce a good description of the experimental strength functions as a function of temperature. One important element missing in the calculations is the coherent damping mechanism due to coupling dipole vibrations with low frequency collective surface modes [18–20]. This mechanism is especially



Fig. 3. The GDR strength function of ¹²⁰Sn and ²⁰⁸Pb at $T^* = 2$, 4 MeV. Solid and dashed lines show the response function without and with the collision term, respectively.

important for describing the details of the strength distributions at low temperature. In the continuation of this work, we plan to improve this simple description by incorporating the coherent damping mechanism in Thomas– Fermi approximation.

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