ARE THE CONTEMPORARY FINANCIAL FLUCTUATIONS SOONER CONVERGING TO NORMAL?

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Based on the tick-by-tick price changes of the companies from the U.S. and from the German stock markets over the period 1998–99 we reanalyse several characteristics established by the Boston Group for the U.S. market in the period 1994–95, which serves to verify their space and time-translational invariance. By increasing the time scales, in the region covered by the data, we find a significantly more accelerated departure from the power-law ($\alpha \approx 3$) asymptotic behaviour of the distribution of returns towards a Gaussian, both for the U.S. as well as for the German stock markets. In the latter case the crossover is even faster. Consistently, the corresponding autocorrelation functions of returns and of the time averaged volatility also indicate a faster loss of memory with increasing time. This route towards efficiency, as seen in a fixed time scale, may reflect a systematic increase of the quality of information processing when going from past to present.

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1. Introduction

Besides its obvious practical implications studying the nature of financial fluctuations proves extremely inspiring and productive for fundamental reasons [1]. The related contributions by Bachelier [2] and by Mandelbrot [3], and broad scientific consequences of these contributions, provide immediate examples. In financial dynamics, even though somewhat opposite, the two corresponding scenarios of uncorrelated random Gaussian [2], *versus* Lévy stable [3] fluctuations, turn out to be taking part and leaving their imprints. As documented by Stanley and collaborators [4–6], the central part of the distribution of returns falls within the Lévy stable regime, while larger fluctuations are governed by a power law with an exponent $\alpha \approx 3$, well outside the Lévy stable regime. At the same time the autocorrelation function for returns sampled at short time scales drops down very quickly and after about 20 min it reaches the noise level. Consequently, because of the central limit theorem, the convergence to a Gaussian distribution on longer time scales is expected. Quite surprisingly, such a convergence has been shown [4–6] to be extremely slow. In fact, for returns of up to approximately 4 days, the functional form of their distribution even is retained for both, the individual companies [6] as well as for the global stock market index [5]. Based on this analysis a visible crossover to a Gaussian takes place only after about 16 days. The volatility autocorrelation function, on the other hand, decays very slowly with time, largely according to a power law, and remains positive for many months. These higher order correlations can thus be considered responsible for such an ultraslow convergence to a Gaussian. These, at present, are the so called stylised empirical facts which constitute a reference for realistic theoretical models. In connection with the fact that the scaling range visible in the financial data typically extends over only 1–1.5 order of magnitude, one has to keep in mind that the stretched exponential distributions can also be considered reasonable candidates [7] for modeling the financial fluctuations. Other interesting related scenario is the one which corresponds to subordinated stochastic processes [8] where time itself is a stochastic process, or its multifractal [9] and elastic time [10] generalizations.

From the point of view of the central limit theorem an essential element is the speed of decay of correlations between the consecutive elementary events. The speed of such a decay can be expected to be related to the availability of information, opportunities to access it and quality of its processing. These definitely systematically increase when going from past to present which finds, for instance, evidence in a systematically increasing frequency of trading. A natural question thus is to what extent such elements can modify the dynamics of markets and, in particular, if they can influence the characteristics mentioned above.

In addressing the related issues on the empirical level, we systematically study the databases comprising the tick-by-tick price changes of the 30 companies included in the Dow Jones Industrial Average (DJIA) [11] for most of the time during the period 1998–99, and of the 30 companies included in the Deutsche Aktienindex (DAX) [12] for most of the time during the same period. This corresponds to a selection of stocks of similar market capitalization and thus their dynamics compatible within either of these two groups, respectively. Since we are dealing with a more recent history of the stock market dynamics than the one presented in previous systematic analysis for the American market by the Boston Group [6] (years 1994–95), by comparison, our present study can be oriented towards verifying the time translational invariance of the relevant characteristics, of primary interest being the probability of returns over varying time scales. Secondly, such a selection of stocks also allows to compare the two different stock markets in the same time intervals. Similarly as in Ref. [6] the data from the TAQ databases have been filtered to remove occasional spurious events.

2. 1998–1999 stock market fluctuations

When determining the distribution of returns, in order to obtain a reasonable statistics, we consider fluctuations of all the companies individually rather than those of the corresponding global index. The resulting sample size (for the price changes sampled every 5 min) then equals 30×39000 for the American market and 30×52000 for the German market. As it has been shown in Ref. [6], the fluctuations of the market and of its individual companies are typically governed by distributions of essentially the same functional form and the crossover to a Gaussian is even slower in the latter case (4 *versus* 16 days). For this reason the fluctuations of the companies are expected [6] to provide un upper bound for the distribution characterising fluctuations of the global index.

For the time series $P_i(t)$ representing the share price of i-th company we use the commonly accepted definition of returns as

$$G_i \equiv G_i(t, \Delta t) = \ln P_i(t + \Delta t) - \ln P_i(t) \,. \tag{1}$$

As another standard procedure, in order to make fluctuations of different companies comparable, we make use of the normalised returns $g_i \equiv g_i(t, \Delta t)$ defined as

$$g_i = \frac{G_i - \langle G_i \rangle_T}{v_i} \,, \tag{2}$$

where $v_i \equiv v_i(\Delta t)$ of company i is the standard deviation of its returns over the period T

$$v_i^2 = \langle G_i^2 \rangle_T - \langle G_i \rangle_T^2 \tag{3}$$

and $\langle \ldots \rangle_T$ denotes a time average. Since the distribution of return fluctuations is typically, to a good approximation, symmetric [5, 6] with respect to zero, in the present contribution we do not discuss such 'higher order' effects, and, in the following, by returns we simply mean the moduli of returns.

The cumulative distributions of such returns for the two sets of the companies specified above are shown in Fig. 1. The most relevant here is their asymptotic behaviour which, based on the previous study, is expected to obey a power-law

$$P(g > x) \sim x^{-\alpha}, \tag{4}$$

with $\alpha \approx 3$. The corresponding slope is indicated by the dash–dotted line in this figure. On the short time scales ($\Delta t = 5$ min and 30 min) the-DJIA-associated stock prices fluctuate according to such a law, indeed. However, a deviation towards a Gaussian (dashed line) can be seen starting already with $\Delta t = 120$ min and it systematically increases with increasing Δt. For the largest value of $\Delta t = 780$ min (two trading days for the DJIA) for which this characteristics has been calculated, no scaling regime exists. The corresponding transition in the case of the DAX companies turns out to occur even more rapidly. In fact, in this case, already at $\Delta t = 5$ min, the distribution significantly deviates from $\alpha = 3$ towards its larger value. This is to be compared to a study [13] based on the older DAX data which shows consistency with $\alpha = 3$ for much larger time scales. For the present data the fluctuations on the time scale of already one trading day (for DAX this corresponds to 510 min) assume functional form much closer to a Gaussian than to any scaling power-law.

A more global quantitative measure of distributions is in terms of the moments. For the normalised returns g these are defined as

$$\mu_k = \langle |g|^k \rangle, \tag{5}$$

and $\langle \ldots \rangle$ denotes here an average over all the normalised returns for all the bins. For both sets of returns the so-calculated spectrum of moments is shown in Fig. 2 for the same sequence of time scales as in Fig. 1. The moments can be seen to reflect basically the same tendency as it can be deduced from the distributions of returns, *i.e.*, a systematically increasing departure from the $\alpha = 3$ scaling law in the region covered by the actual data.

A question now arises: is the above observation consistent with some more dynamically oriented characteristics, like the autocorrelation function of returns or the time averaged volatility $v(\Delta t)$ on different time scales Δt? Indeed, an impressive consistency can be identified when inspecting these characteristics shown in Figs. 3 and 4, calculated here from the returns of the corresponding global indices, DJIA and DAX respectively, *versus* the behaviour of the distribution of returns from Fig. 1. The previous study [5,6] shows that correlations in returns drop down to the level of noise after about

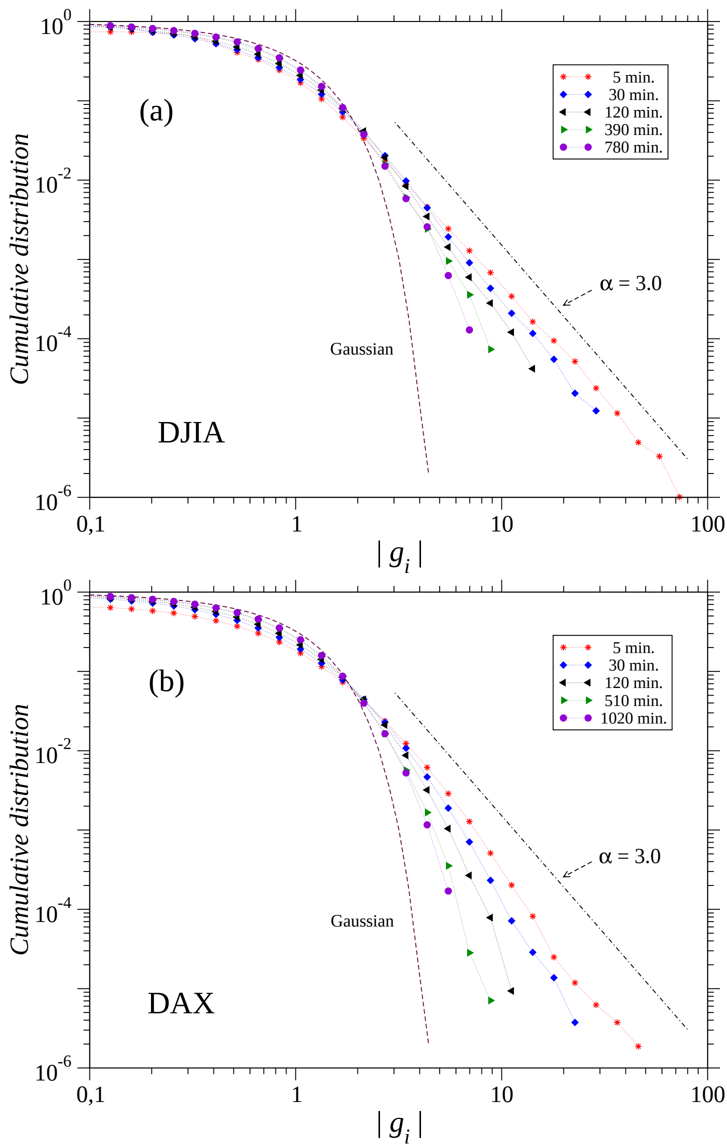


Fig. 1. Cumulative distributions of the moduli of normalised returns of the 30 companies which were included in the Dow Jones Industrial Average (a) and of the 30 companies which were included in the Deutsche Aktienindex (b) for most of the time during the same period 1998–99. Different lines correspond to varying time scales Δt starting from 5 min up to two trading days (780 min for DJIA and 1020 min for DAX).

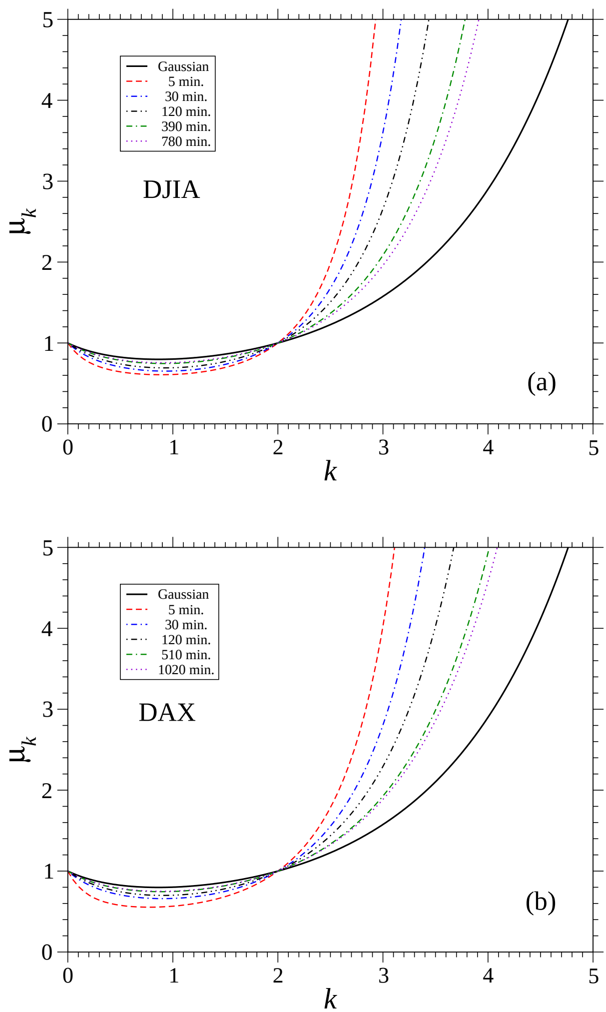


Fig. 2. Fractional moments for the normalised returns for the same cases and for the same time scales as in Fig. 1. The solid full line shows the Gaussian moments.

20 min. In our case, this time is clearly much shorter and equals about 5 min for both markets. This provides an independent evidence that in the period 1998–99 the stock market correlations cease to exist much faster than in the period 1994–95. Interestingly, even though reaching the noise level after about the same 5 min, the speed of disappearance of correlations is larger for the DAX than for the DJIA. This nicely correlates with the corresponding more abrupt transition with increasing Δt towards Gaussian

(Fig. 1) in fluctuations of the DAX companies than those of the DJIA. It is also at the same Δt of 5 min where $v(\Delta t) \sim \Delta t^{\delta}$ changes its slope from superdiffusive ($\delta > 0.5$) to normal ($\delta = 0.5$) for both markets. As consistent with behaviour of the autocorrelation function, the dynamics of DJIA is more superdiffusive ($\delta = 0.68$) in these initial 5 min than the one of the DAX.

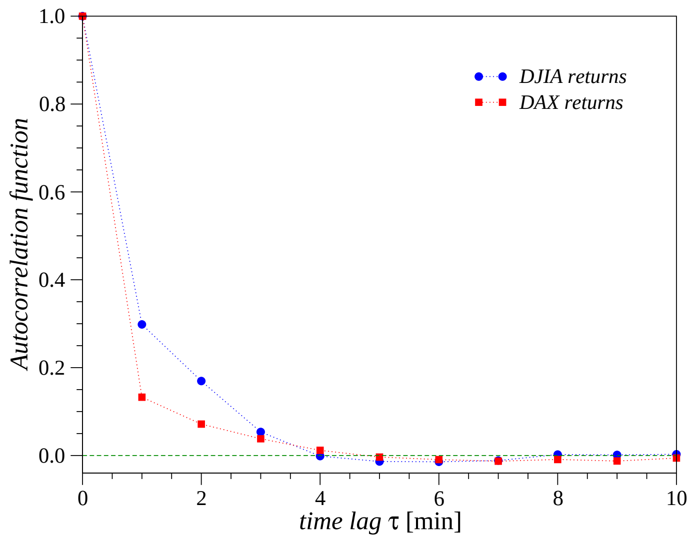


Fig. 3. Time-lag τ dependence of the autocorrelation functions computed from the returns of the DJIA index and from the returns of DAX index both sampled at a $\Delta t = 1$ min time scale within the time interval 1998-99.

In order to further illuminate on a possible origin of such a change of the stock market dynamics we split our 1998–99 time interval into two halves and for them separately calculate the autocorrelation functions of returns. As shown in Fig. 5, we again can see an amazing consistency for both markets: the more recent period of 1999 turns out to be associated with a visibly faster decay of correlations than 1998, and the autocorrelation functions for the whole period 1998–99, to a good approximation, constitute the averages of the ones calculated over the corresponding subintervals.

Finally, as an extra test of our analysis procedure and on the way towards identifying further correlations between the above observations and other measurable market characteristics, we select the three groups from the TAQ database, including 30 companies each, representing significantly different market capitalisations S. These include (a) $S \geq 90$, (b) $10 \leq S \leq 15$ and (c) $0.1 \leq S \leq 0.3$, (all in units of 10^9 USD), *i.e.*, the companies of the largest, medium and the lowest capitalisation, respectively. The first group

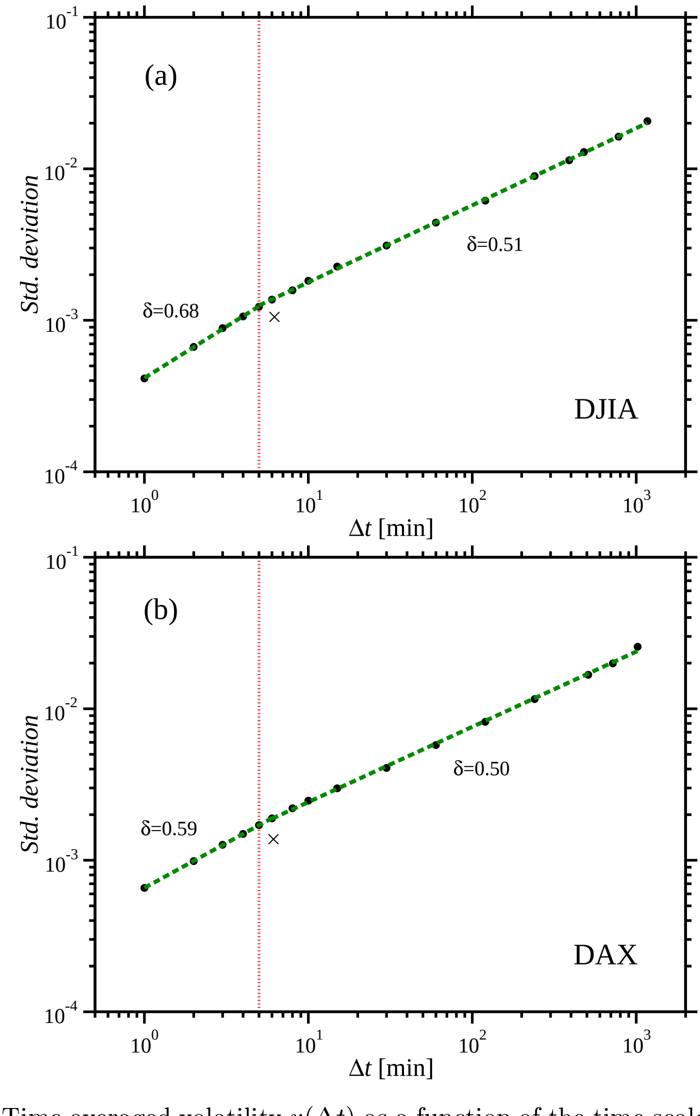
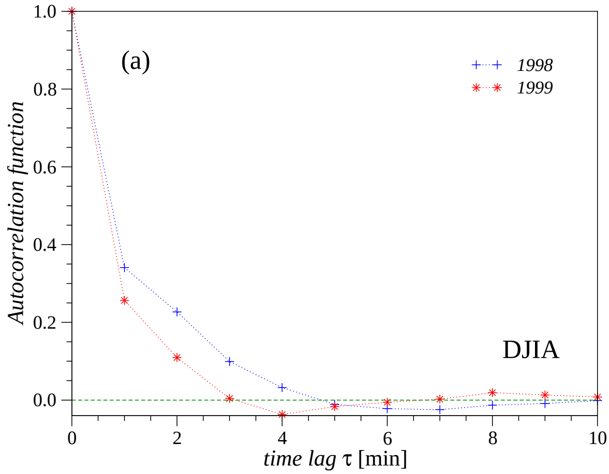


Fig. 4. (a) Time averaged volatility $v(\Delta t)$ as a function of the time scale Δt for the DJIA and (b) for the DAX within the same time interval. Dashed lines represent fits in terms of $v(\Delta t) \simeq \Delta t^{\delta}$. Vertical dotted lines indicate the crossover ($\times$) at around $\Delta t = 5$ min.

partially overlaps with the DJIA. The corresponding cumulative distributions of returns for the same different scales of time aggregation as before are shown in Fig. 6(a)–(c). As it can be clearly seen the case (a) follows the same tendency as the DJIA, the case (b) is somewhat less pronounced in



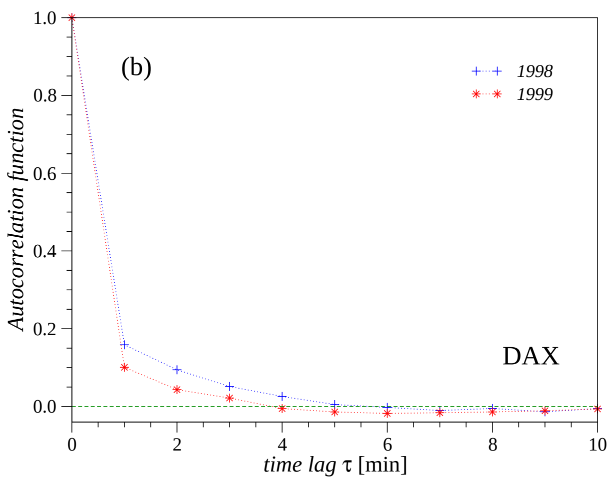
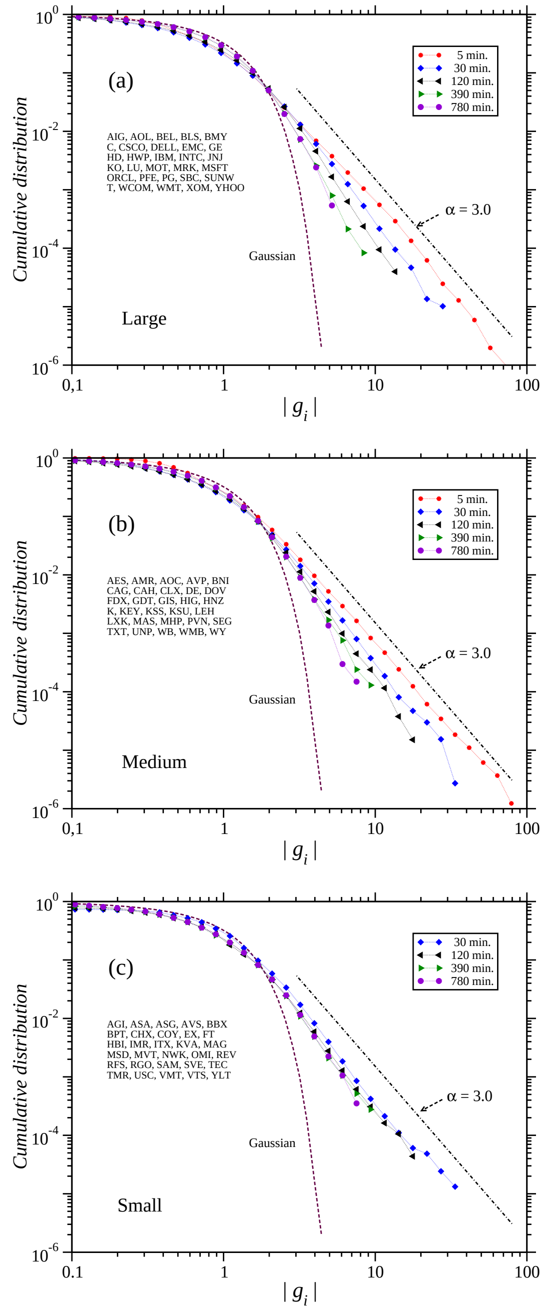


Fig. 5. Time-lag τ dependence of the autocorrelation functions of returns for the DJIA (a) and for the DAX (b) returns sampled at a $\Delta t = 1$ min time scale within the time interval 1998 and 1999, separately.

this respect but in the case (c) the slope of the distribution remains essentially preserved up to the largest time scales considered. Fig. 6(d)–(f) shows the time averaged volatilities $v(\Delta t)$ for each of the above three groups, correspondingly. $v(\Delta t)$ is here calculated from an "index" which is a sum of prices of the companies involved. Summing up the prices is in fact close to the price-weighted procedure of constructing the DJIA index. In the cases


(a)
Large
AIG, AOL, BEL, BLS, BMY
C, CSCO, DELL, EMC, GE
HD, HWP, IBM, INTC, JNJ
KO, LU, MOT, MRK, MSFT
ORCL, PFE, PG, SBC, SUNW
T, WCOM, WMT, XOM, YHOO
5 min.
30 min.
120 min.
390 min.
780 min.
α = 3.0
Gaussian
Cumulative distribution
| g_i |
(b)
Medium
AES, AMR, AOC, AVP, BNI
CAG, CAH, CLX, DE, DOV
FDX, GDT, GIS, HIG, HNZ
K, KEY, KSS, KSU, LEH
LXK, MAS, MHP, PVN, SEG
TXT, UNP, WB, WMB, WY
5 min.
30 min.
120 min.
390 min.
780 min.
α = 3.0
Gaussian
Cumulative distribution
| g_i |
(c)
Small
AGI, ASA, ASG, AVS, BBX
BPT, CHX, COY, EX, FT
HBI, IMR, ITX, KVA, MAG
MSD, MVT, NWK, OMI, REV
RFS, RGO, SAM, SVE, TEC
TMR, USC, VMT, VTS, YLT
30 min.
120 min.
390 min.
780 min.
α = 3.0
Gaussian
Cumulative distribution
| g_i |

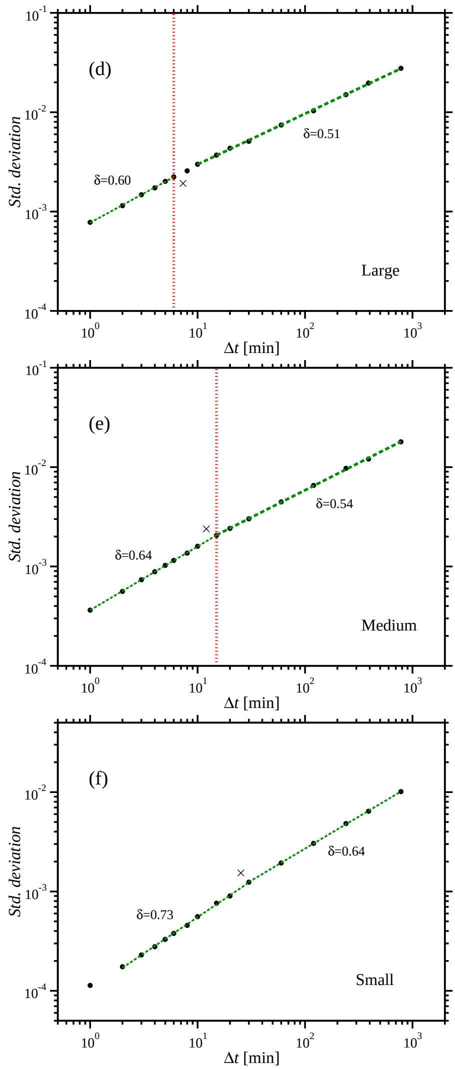


Fig. 6. (LEFT) Cumulative distributions of the moduli of normalised returns during the period 1998–99 of the three groups including 30 companies each, representing significantly different market capitalisations S. These include (a) the largest ($S \geq 90$), (b) medium ($10 \leq S \leq 15$) and (c) the lowest ($0.1 \leq S \leq 0.3$, all in units of 10^9 USD), available capitalisation, respectively. Different lines correspond to varying time scales Δt starting from 5 min up to 780 min (two trading days). In (c) the time scale of 5 min is omitted due to a too large number of zero returns occurring in this group of stocks. (RIGHT) Time averaged volatilities $v(\Delta t)$ for each of the three groups, correspondingly. In all these three cases $v(\Delta t)$ is calculated from an "index" which is a sum of split-adjusted prices of the 30 companies involved (d) and (e) and of 300 small companies (f).

(d) and (e) these are the same 30 companies listed in Fig. 6(a)–(b), while in the case (f), in order to resolve the dynamics down to the time scales of 1 min, the corresponding list of the small companies is extended up to 300 (the small companies are significantly less frequently traded which results in many zero 1 min "index" returns if a too small number of such companies is used). As one can see, in the case of the largest companies $v(\Delta t)$ behaves very similarly as for the DJIA itself (Fig. 4(a)), including the time scale (5–6 min) of the transition from superdiffusive to normal. For the medium size companies such a transition is somewhat delayed ($\sim$20 min) and even not to a fully normal diffusion (from $\delta = 0.64$ to $\delta = 0.54$). Continuing this way, for the small companies the dynamics remains superdiffusive over the whole interval of the time scales considered but still a transition from $\delta = 0.73$ to $\delta = 0.64$ can be seen at around Δt=30 min. Again all this looks rather consistent with the corresponding development of the distributions of returns.

The analysis presented in Fig. 6 provides thus a test of significance of the original (Figs. 1–4) results for the DJIA and for the DAX, since the numbers of data points used are the same in all those cases. Secondly, in view of the fact that an average frequency of transactions in the above three groups of the companies is about (a) 15/min, (b) 1.5/min and (c) 0.2/min per company, correspondingly, it points just to this physical parameter as the one which is directly related to the observed effects. However, as a visible difference between the DAX and the DJIA in approaching a limit of normal distributions shows, this definitely is not the only relevant parameter. For the DAX the average number of transactions per company is about 1/min and still it is DAX whose departure from scaling and the decline of correlations in time is the fastest among the cases considered here. A leading role of the DJIA in dictating direction of the global stock market development has recently been identified [14] by studying correlation between the DAX and the DJIA. Whether it is DAX which benefits from information already preprocessed by the DJIA is an interesting possibility to be considered in this connection.

3. Conclusions

These results provide quite a remarkable indication that the contemporary financial dynamics on average is more efficient in the sense of the efficient market hypothesis [15] in its weak form, as compared to a more distant history. From the practical point of view this may be considered good news for the conventional option pricing methods [16, 17] which assume a normal distribution of financial fluctuations. In a sense this result also provides some more arguments in favour of the standard extreme value theory [18] for estimating the value-at-risk for very low probability extreme events. The related literature assumes independent returns which implies the decreasing degree of fatness in the tails. There is still one more element that is to be kept in mind when trying to interpret the present observations. The world stock markets, including the two considered here, were experiencing more sizable increases during the period 1998–99 than during 1994–95. As shown in Ref. [19], such periods are typically more noisy and more competitive as far as correlations among the individual stocks are concerned. Just a time-translation is thus not the only element when relating those two periods of the stock market history. In any case, however, the issue of the so-called financial stylised facts needs to be revised and, possibly, generalised to incorporate an increasing access to information and ability to process it when going from past to present. All this provides further arguments for being time-adaptive, and even market-adaptive, when looking into the dynamics of the financial markets, which is especially important for an appropriate perception of the risk involved.

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