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NNLO QCD CALCULATIONS OF RARE $\boldsymbol{B}$ DECAYS*

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Present status of the NNLO QCD calculations of the rare decays $\bar{B} \to X_s l^+ l^-$ and $\bar{B} \to X_s \gamma$ is summarized.

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1. Introduction

A variety of rare B-meson decays can be studied using large data samples that have been collected so far at the B factories. However, only a limited number of them are sensitive to new physics and theoretically clean at the same time. Among such modes, the inclusive decays $\bar{B} \to X_s l^+ l^-$ $(l = e$ or $\mu)$ and $\bar{B} \to X_s \gamma$ are of particular interest. For appropriately chosen kinematical cuts, their widths are well approximated by the perturbatively calculable widths of the b-quark decays $b \to X_s^{\rm parton} l^+ l^-$ and $b \to X_s^{\rm parton} \gamma$, respectively. The estimated non-perturbative effects and the expected experimental uncertainties are smaller than the perturbative Next-Next-to-Leading-Order (NNLO) QCD corrections. Consequently, calculating such corrections is essential for tightening constraints on extensions of the Standard Model (SM) that can be derived from the measurements of $\bar{B} \to X_s l^+ l^-$ and $\bar{B} \to X_s \gamma$. Actually, these two decay modes are the only rare B-decays for which the NNLO QCD corrections are of phenomenological interest.

Large QCD logarithms $\alpha_{\rm s} \ln(M_W^2/m_b^2)$ are treated as quantities of order unity in the renormalization-group-improved calculations of b decays. In consequence, the perturbative series for the decay amplitudes take the following form

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$$A(b \to sl^+l^-) = \frac{1}{\alpha_s(m_b)} f_{\rm LO}(\eta) + f_{\rm NLO}(\eta) + \alpha_s(m_b) f_{\rm NNLO}(\eta) + \ldots, \quad (1)$$

$$A(b \to s\gamma) = g_{\rm LO}(\eta) + \alpha_s(m_b) g_{\rm NLO}(\eta) + \alpha_s^2(m_b) g_{\rm NNLO}(\eta) + \ldots, \quad (2)$$

where the functions f and g are computed exactly in $\eta = \alpha_s(M_W)/\alpha_s(m_b)$, *i.e.* $\eta - 1$ is treated as a quantity of order unity.

One can see that there is an essential difference between the two series (1) and (2). Only the first of them contains the $\mathcal{O}(1/\alpha_s)$ term. If η was formally expanded in powers of α_s, the function $f_{\rm LO}(\eta)$ would become a quantity of order α_s, and the considered term would reproduce a logarithm $\ln(M_W^2/m_b^2)$ that occurs in the purely electroweak $b \to sl^+l^-$ amplitude (Fig. 1). On the other hand, since the electroweak one-loop $b \to s\gamma$ amplitude (Fig. 2) is free such logarithms, there is no $\mathcal{O}(1/\alpha_s)$ term in the series (2).

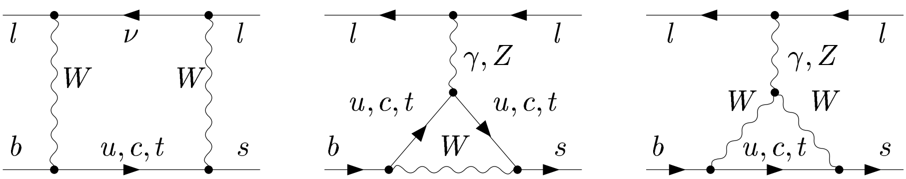

Fig. 1. Leading-order Feynman diagrams for $b \to sl^+l^-$ in the SM.

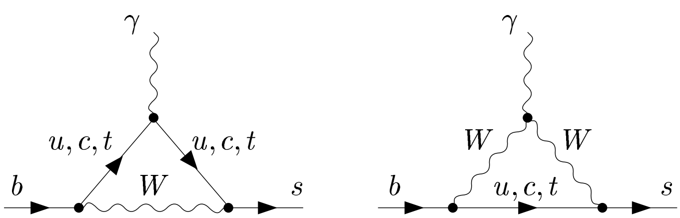

Fig. 2. Leading-order Feynman diagrams for $b \to s\gamma$ in the SM.

The function $f_{\rm LO}(\eta)$ turns out to be very small for the actual value of $\eta \simeq 0.56$. In effect, the first two terms in Eq. (1) are close in size. Consequently, the NNLO term in this equation is numerically as important as the Next-to-Leading-Order (NLO) QCD corrections in many other processes. Theoretical uncertainties in the SM prediction for BR$[\bar{B} \to X_s l^+l^-]$ get reduced below 10% only after this term is included[1].

In the case of BR$[\bar{B} \to X_s\gamma]$, theoretical uncertainties are brought down to the ~10% level already after including the NLO corrections. However,

[1] The necessary kinematical cuts will be discussed in the next section.

since the experimental errors will soon become significantly smaller than 10%, a calculation of $g_{\rm NNLO}(\eta)$ is necessary. Such a calculation is currently underway.

In the present paper, status of the perturbative QCD calculations of the two considered processes is summarized. Section 2 is devoted to the rare semileptonic decay for which the NNLO QCD calculations have been recently completed. The rare radiative decay is discussed in Section 3. Section 4 contains the conclusions.

2. The rare semileptonic decay

Theoretical predictions for the dilepton invariant mass spectrum in $\bar{B} \to X_s l^+ l^-$ are presented in Fig. 3. The dashed curve corresponds to the perturbative $b \to s l^+ l^-$ decay. The solid line includes non-perturbative contributions from intermediate $c\bar{c}$ states that have been calculated using "naive" factorization and dispersion relations [1]. The shape of the solid curve tells us that the perturbative methods fail in the region of intermediate ψ and ψ' resonances. On the other hand, the perturbative calculations are believed to work fairly well for low values of $m_{l^+l^-}$ (below 2 or even 2.5 GeV) [2].

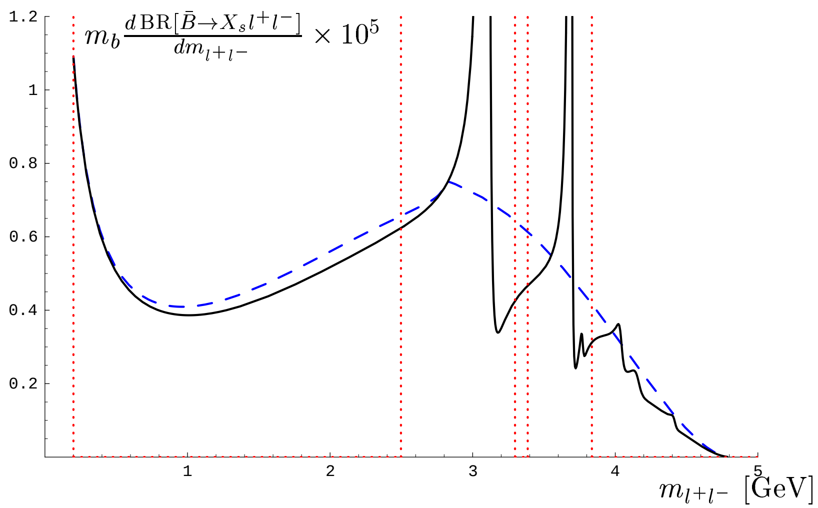

Fig. 3. Perturbative (dashed) and non-perturbative (solid) dilepton mass spectrum in $\bar{B} \to X_s l^+ l^-$ (see the text). The dotted vertical lines indicate cuts imposed by Belle [2] in the $l = e$ case — the vetoed regions are around the ψ and ψ' peaks, as well as for very low $m_{l^+l^-}$.

[2] Provided $\bar{B} \to \psi X_s^{(1)}$ followed by $\psi \to X^{(2)} l^+ l^-$ is treated as background.

The first measurement of the inclusive $\bar{B} \to X_s l^+ l^-$ branching ratio was announced in August 2002 by the Belle collaboration [2]. Cuts on $m_{l^+l^-}$ that were applied in their analysis are indicated by the dotted lines in Fig. 3. The perturbative (dashed) curve was used to extrapolate the measured spectrum to the range $m_{l^+l^-} \in [0.2 \text{ GeV}, m_B - m_K \simeq 4.8 \text{ GeV}]$. After averaging over electrons and muons, the following result for the "total" branching ratio was obtained:

$$\mathrm{BR}[\bar{B} \to X_s l^+ l^-]_{\mathrm{exp}} = (6.1 \pm 1.4 \,{}^{+1.4}_{-1.1}) \times 10^{-6}. \qquad (3)$$

It agrees within 1σ with the SM phenomenological analysis of Ali *et al.* [3] who have found

$$\mathrm{BR}[\bar{B} \to X_s l^+ l^-]_{\mathrm{SM}} = (4.2 \pm 0.7) \times 10^{-6} \qquad (4)$$

for the same kinematical cuts.

The uncertainty of the SM prediction (4) can be reduced in the future by reanalyzing the charm-quark mass dependence along the same lines as it is usually done in the determinations of V_{cb}. Furthermore, one can get rid of sizeable the non-perturbative uncertainties by restricting to the domain

$$\hat{s} \equiv \left(\frac{m_{l^+l^-}}{m_b} \right)^2 \in [0.05, 0.25] \qquad (5)$$

that has been used in several NNLO QCD analyses [4,5]. It corresponds to $m_{l^+l^-} \in [1.05, 2.35]$ GeV. The upper bound of this domain is determined by the requirement of not getting too close to the intermediate ψ resonance. The lower bound has been introduced in order to exclude intermediate photons of low virtuality and, in consequence, increase sensitivity to such new physics effects that are not yet constrained by $\bar{B} \to X_s \gamma$. In the following, we shall restrict our discussion to the interval (5).

The standard framework for $\bar{B} \to X_s l^+ l^-$ analyses is set by the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{QCD} \times \mathrm{QED}}(u, d, s, c, b, e, \mu) + \frac{4 G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i \,. \qquad (6)$$

The operators O_i and the numerical values of their Wilson coefficients at the scale $\mu = m_b$ are as follows:

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & i = 1,2, & |C_i(m_b)| \sim 1, \\ (\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma'_i q), & i = 3,4,5,6, & |C_i(m_b)| < 0.07, \\ \frac{em_b}{16\pi^2}\bar{s}_{\rm L}\sigma^{\mu\nu}b_R F_{\mu\nu}, & i = 7, & C_7(m_b) \sim -0.3, \\ \frac{gm_b}{16\pi^2}\bar{s}_{\rm L}\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}, & i = 8, & C_8(m_b) \sim -0.15, \\ \frac{e^2}{16\pi^2}(\bar{s}_{\rm L}\gamma_\mu b_{\rm L})(\bar{l}\gamma^\mu l), & i = 9 & |C_9(m_b)| \sim 4, \\ \frac{e^2}{16\pi^2}(\bar{s}_{\rm L}\gamma_\mu b_{\rm L})(\bar{l}\gamma^\mu\gamma_5 l), & i = 10 & |C_{10}(m_b)| \sim 4. \end{cases} \tag{7}$$

Here, Γ and Γ' stand for various products of the Dirac and color matrices (see *e.g.* [4, 5]).

Calculations of the $b \to X_s^{\rm parton} l^+ l^-$ decay amplitude are usually performed in three steps:

- **Matching:** Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and effective theory Green's functions at the leading order in (external momenta)/M_W.
- **Mixing:** Deriving the effective theory Renormalization Group Equations (RGE) and evolving $C_i(\mu)$ from μ_0 down to $\mu_b \sim m_b$.
- **Matrix elements:** Evaluating the on-shell amplitudes at $\mu_b \sim m_b$.

The Wilson coefficients $C_i(\mu_b)$ are perturbatively expanded as follows:

$$\begin{aligned} C_i(\mu_b) \;&=\; \delta_{i9}\frac{4\pi}{\alpha_{\rm s}(\mu_b)}C_9^{(-1)}(\mu_b) + C_i^{(0)}(\mu_b) + \frac{\alpha_{\rm s}(\mu_b)}{4\pi}C_i^{(1)}(\mu_b) \\ &\quad + \left(\frac{\alpha_{\rm s}(\mu_b)}{4\pi}\right)^2 C_i^{(2)}(\mu_b) + \dots\,, \end{aligned} \tag{8}$$

where $C_i^{(n)}(\mu_b)$ depend on $\alpha_{\rm s}$ only via the ratio $\eta \equiv \alpha_{\rm s}(\mu_0)/\alpha_{\rm s}(\mu_b)$. The origin of the $\mathcal{O}(1/\alpha_{\rm s})$ term in C_9 has been already explained in the introduction. Its presence implies that a calculation in which $C_9^{(0)}$ is included is called a NLO one. On the other hand, since $C_9^{(0)}(\mu_b) \simeq 2.2$ and

$$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \quad \Rightarrow \quad \frac{4\pi}{\alpha_{\rm s}(m_b)}C_9^{(-1)}(m_b) \simeq 2\,, \tag{9}$$

the NLO contribution is not smaller (but rather larger) than the LO one. Consequently the first actual QCD *correction* to the amplitude is the NNLO contribution that includes $C_i^{(1)}(\mu_b)$.

The coefficients $C_9^{(-1)}(m_b)$ and $C_{10}^{(0)}$ are known since 1989 when the LO calculations were completed [6]. The coefficients $C_9^{(0)}$ and $C_{10}^{(1)}$ were found in Refs. [7,8] and [9,10], respectively. The calculation of $C_9^{(1)}$ involves 2-loop matching and 3-loop mixing. Two-loop matching results for all the operators (7) were found in Ref. [4]. The necessary 3-loop mixing calculation (Fig. 4(a)) is currently being completed. Preliminary results have already been announced [11]. Their numerical effect on the branching ratio does not exceed 2%.

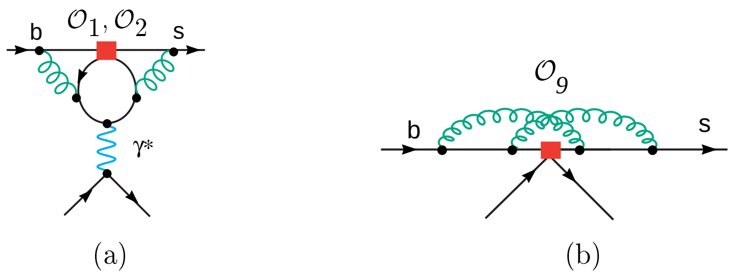

Fig. 4. Sample diagrams for (a) 3-loop mixing of $O_{1,2}$ into O_9 (b) 2-loop matrix element of O_9.

As far as the matrix elements are concerned, most of the necessary tree-level and one-loop ones were included already at the LO and NLO [6–8]. One-loop diagrams with insertions of O_9 and O_{10} (and the corresponding bremsstrahlung corrections) were read out from the $b \to X_u e\bar{\nu}$ decay calculations of Jeżabek and Kühn [12].

The most involved terms that arise at the NNLO are the 2-loop matrix elements of the 4-quark operators $O_{1,2}$ (Fig. 5). They were completed in

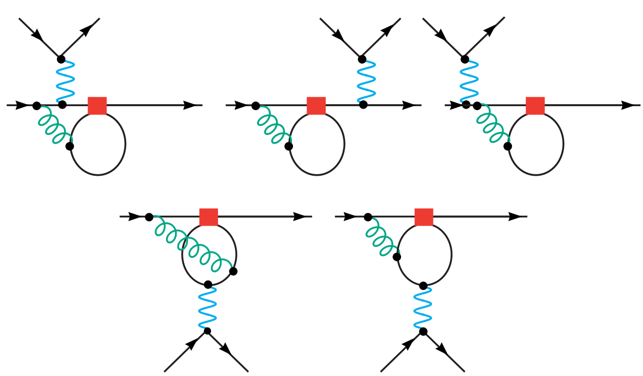

Fig. 5. Sample diagrams for 2-loop matrix elements of the 4-quark operators.

2001–2002 by two independent groups. The first of them [5] applied Mellin–Barnes transforms to the Feynman-parameter integrals. The other one [13] used numerical integration after reduction to the so-called sunrise topologies.

Two elements are still missing in the complete NNLO calculation of $b \to X_s^{\rm parton} l^+ l^-$:

- Two-loop matrix element of O_9 (Fig. 4b) and the corresponding bremsstrahlung correction. Its integral over $m_{l^+l^-}$ can be read out from the $b \to X_u e\bar{\nu}$ results of van Ritbergen [14]. Implementing the necessary cuts would require a new calculation. However, the considered contribution is proportional to the small Wilson coefficient $C_9^{(-1)}(m_b)$ (9). Thus, its numerical effect on the decay width is expected to be very small.

- Two-loop matrix elements of the so-called penguin operators O_3, …, O_6. Since the corresponding Wilson coefficients are small (see Eq. (7)), this contribution can hardly exceed 1%.

Thus, the existing NNLO results can be called "practically complete". Uncertainties due to the uncalculated higher-order corrections have already been significantly reduced. It is illustrated in Figs. 6(a) and (b) that originate from Refs. [5] and [13], respectively.

Fig. 6(a) presents the $\hat{s}$-dependence of

$$R_{\rm quark}(\hat{s}) \equiv \frac{d\Gamma[b \to X_s^{\rm parton} l^+ l^-]/d\hat{s}}{\Gamma[b \to X_c^{\rm parton} e\bar{\nu}_e]} \approx \frac{d\Gamma[\bar{B} \to X_s l^+ l^-]/d\hat{s}}{\Gamma[\bar{B} \to X_c e\bar{\nu}_e]} \tag{10}$$

for three different values of μ_b: 2.5, 5 and 10 GeV. The solid lines correspond to the current NNLO results. The dashed lines show what one would obtain

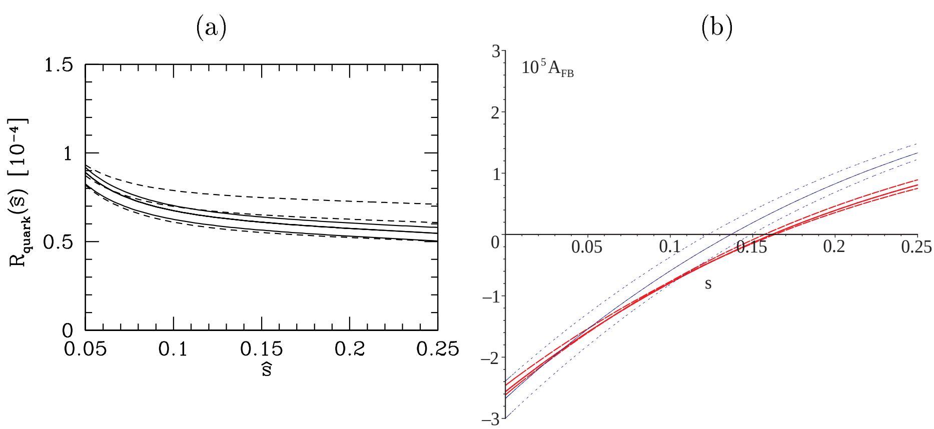

Fig. 6. (a) $R_{\rm quark}(\hat{s})$ and (b) $A_{\rm FB}(\hat{s})$ for different values of μ_b (see the text).

before including the NNLO matrix elements of the 4-quark operators. The weaker scale-dependence of the new results is clearly seen.

Fig. 6(b) shows the forward–backward asymmetry

$$A_{\rm FB}(\hat{s}) \equiv \frac{1}{\Gamma[\bar{B} \to X_c e \bar{\nu}_e]} \int\limits_{-1}^{1} d\cos\theta \frac{d^2\Gamma[\bar{B} \to X_s l^+ l^-]}{d\hat{s}\; d\cos\theta} {\rm sgn}(\cos\theta) \tag{11}$$

as a function of $\hat{s}$. The three lower curves describe the current NNLO results, while the three upper curves refer to the old NLO case. In both cases, the middle solid line corresponds to $\mu_b = 5$ GeV and the two dashed lines correspond to the two other values of μ_b (2.5 and 10 GeV). Again, it is clearly seen that the scale-dependence has shrinked after including the NNLO corrections.

The integrated BR$[\bar{B} \to X_s l^+ l^-]$ over the domain (5) reads [15][3]

$$\mathrm{BR} = (1.36 \pm 0.08_{\rm scale}) \times 10^{-6}\,, \tag{12}$$

where the scale-dependence uncertainty has been determined using the same variation of μ_b as above. This scale dependence serves us as an estimate of the yet uncalculated higher-order corrections. The remaining uncertainties have been analyzed so far only in Ref. [3] where other kinematical cuts were used and the m_c-dependence errors were overestimated. Thus, a detailed analysis of theoretical uncertainties in the domain (5) is still awaited. Several groups plan to perform such an analysis once the final results on the 3-loop anomalous dimensions [11] are published.

3. The rare radiative decay

The effective Lagrangian that governs the $\bar{B} \to X_s \gamma$ decay is the same as in Eq. (6), except for that O_9 and O_{10} do not need to be included at the leading order in $\alpha_{\rm em}$. Thus, at the LO, one includes only $C_i^{(0)}(\mu_b)$ in Eq. (8), at the NLO — also $C_i^{(1)}(\mu_b)$, and at the NNLO — also $C_i^{(2)}(\mu_b)$. The history of the LO and NLO calculations together with appropriate references has

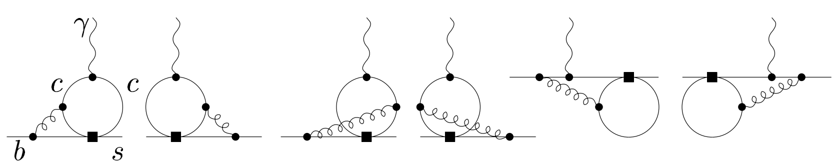

Fig. 7. Two-loop $b \to s\gamma$ matrix elements of the operators O_1 and O_2.

[3] The non-perturbative $\mathcal{O}(\Lambda^2/m_{c,b}^2)$ corrections are included in this number.

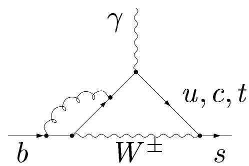

Fig. 8. Sample diagram for the NLO matching.

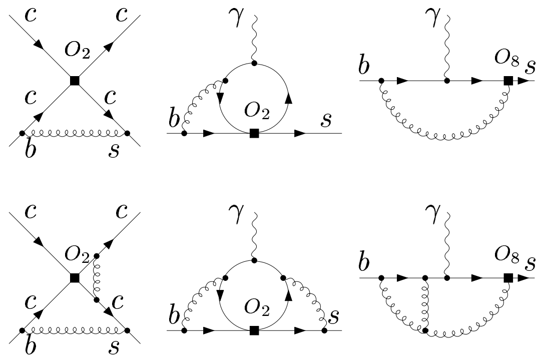

Fig. 9. Sample diagrams for the mixing.

been summarized in Ref. [16]. Figs. 7, 8 and 9 show examples of Feynman diagrams that have been evaluated for the matrix elements, matching and mixing, respectively.

Fig. 10 presents a comparison of the experimental determinations of $\mathrm{BR}[\bar{B} \to X_s\gamma]$ with some of the theoretical calculations. It is interesting to notice that the central value of the current world average [17]

$$\mathrm{BR}[\bar{B} \to X_s\gamma, \quad (E_\gamma > \tfrac{1}{20} m_b)] = (3.34 \pm 0.38) \times 10^{-4} \tag{13}$$

practically overlaps with the central value of the first SM prediction [32] in which most of the leading-logarithmic QCD effects were taken into account. The visible shrinking of the theoretical uncertainties in 1996 was due to (practical) completion of the NLO QCD calculations by that time. The central value of the SM prediction got modified of in 2001/2002 as a result of changing the parameter $\frac{m_c}{m_b}$ from the ratio of pole masses to $m_c(\mu)^{\overline{\mathrm{MS}}}/m_b^{1S}$ (see below)[4].

The largest NLO QCD correction (in the $\overline{\mathrm{MS}}$ scheme) originates from the 2-loop matrix elements of O_1 and O_2 (Fig. 7) that involve charm-quark loops. These on-shell diagrams were calculated first with the help of Mellin–Barnes transform of Feynman-parameter integrals [26]. The results had a form of a series in powers of $\frac{m_c}{m_b}$ and $\ln \frac{m_c}{m_b}$. They were later confirmed with the help of asymptotic expansions [33].

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[4] Here, m_b^{1S} stands for the b-quark mass in the so-called "1S-scheme". It is defined as half of the perturbative contribution to the Υ mass.

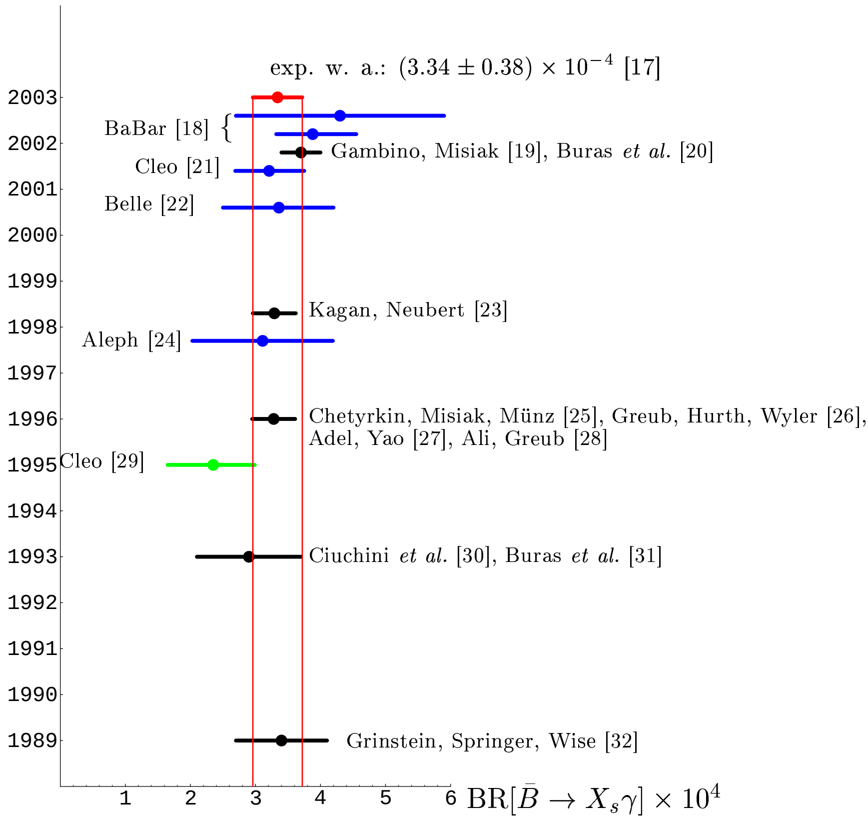

Fig. 10. Measurements and (some of the) theoretical calculations of $\mathrm{BR}[\bar{B} \to X_s\gamma]$.

Recently, two-loop matrix elements of all the four-quark operators (not only O_1 and O_2) have been found [20]. This was the very last element in the NLO QCD program for $\bar{B} \to X_s\gamma$. It required calculating the diagrams from Fig. 7 with q-quark loops for $m_q \in \{m_b, m_c, 0\}$. In practice, analytic expressions were obtained for arbitrary values on m_q/m_b. Apart from formally completing the NLO QCD calculation, these results allow us to study the behaviour of $\mathrm{BR}[\bar{B} \to X_s\gamma]$ for arbitrary values of m_c, which is going to be very useful in the following discussion.

The main uncertainty in the present SM prediction [19, 20]

$$\mathrm{BR}[\bar{B} \to X_s\gamma,\quad (E_\gamma > 1.6 \text{ GeV})] = (3.57 \pm 0.30) \times 10^{-4}\,, \tag{14}$$

$$\mathrm{BR}[\bar{B} \to X_s\gamma,\quad (E_\gamma > \tfrac{1}{20} m_b)] = 3.70 \times 10^{-4} \tag{15}$$

originates from the 2-loop matrix elements of O_1 and O_2 (Fig. 7). These diagrams with charm quark loops are the only source of m_c-dependence of the $b \to s\gamma$ amplitude. Since the NNLO QCD corrections are unknown yet, the renormalization scheme for m_c remains arbitrary, at least within a certain class of "reasonable" schemes that do not artificially enhance the unknown corrections. As argued in Ref. [19], the uncertainty in Eq. (14) stemming from this scheme-dependence can be accounted for by setting $m_c/m_b = m_c(\mu)^{\overline{\rm MS}}/m_b^{1S}$ and varying the scale μ between m_c and m_b. Such a variation is the dominant source of the error in Eq. (14).

The considered uncertainty should be removed because the errors in Eqs. (13) and (14) are close in size, while prospects for improvement on the experimental side are bright. Thus, the NNLO QCD corrections $b \to X_s^{\rm parton}\gamma$ should be calculated. Some of the diagrams that need to be evaluated can be obtained from Figs. 7, 8 and 9 by adding one more gluon.

The NNLO matching conditions for O_1–O_6 are already known [4]. The 3-loop matching conditions for O_7 and O_8 are currently being calculated [34]. Our preliminary results imply that the effect of all the NNLO matching conditions on $\mathrm{BR}[\bar{B} \to X_s\gamma]$ is negative and amounts to around -1.5% (in the $\overline{\rm MS}$ scheme).

As far as the NNLO mixing is concerned, all the 3-loop contributions should soon be known [11]. Computer algebra algorithms for evaluating the necessary 4-loop diagrams exist [35]. However, no calculation has yet begun.

Evaluation of the matrix elements is technically much more difficult than the matching or mixing because no expansion in external momenta can be applied. Finding the 2-loop on-shell matrix elements of O_7 and O_8 as well as the corresponding bremsstrahlung contributions is in the plans of the Bern group [36]. The diagrams with fermionic loops have already been calculated [37]. There remain 10 two-loop 1PI diagrams for O_7 and 34 such diagrams for O_8. The quasi-numerical approach of Ref. [13] might be applied for their evaluation.

The 3-loop matrix elements of O_1 and O_2 are the most problematic[5]. The massive on-shell 3-loop diagrams that one obtains from Fig. 7 by inserting a fermion loop on the gluon line have been already found [37]. However, if a new gluon line is added instead, the presently known techniques fail. The number of such 3-loop diagrams is too large (around 200) to follow the "manual" approach of Ref. [37]. On the other hand, algorithmic procedures for such diagrams are not well developed even at the 2-loop level.

A method of estimating contributions from such 3-loop diagrams can be found by studying charm-mass dependence of $\mathrm{BR}[\bar{B} \to X_s\gamma]$. It is shown in Fig. 11 where m_c is varied between 0 and 40 GeV, while all the other SM parameters are set to their measured values. The two dotted vertical lines

[5] The remaining 4-quark operators have so small Wilson coefficients that their NNLO matrix elements can safely be neglected.

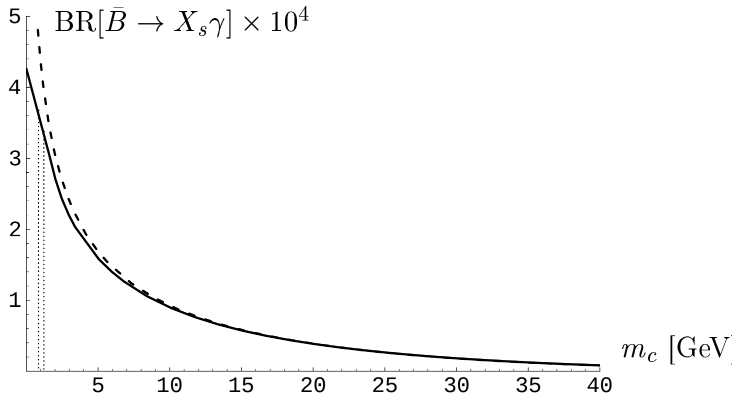

Fig. 11. Dependence of $\mathrm{BR}[\bar{B} \to X_s\gamma]$ on m_c

indicate the "measured" values of $m_c(\mu = m_c)$ and $m_c(\mu = m_b)$. The solid curve is found from the complete NLO formulae for $\mathrm{BR}[\bar{B} \to X_s\gamma]$. The dashed one corresponds to the asymptotic behaviour at $m_c \gg m_b$, *i.e.* all the functions of $\frac{m_c}{m_b}$ are replaced by $(\mathrm{const.})_1 + (\mathrm{const.})_2 \ln \frac{m_c}{m_b}$.

The large-m_c behaviour of the branching ratio is qualitatively explained by that it should vanish for $m_c = m_t$, up to small $\mathcal{O}(V_{ub}^2/V_{cb}^2)$ effects. It is interesting that the asymptotic large-m_c expression (dashed curve) remains a good approximation even for relatively small values of m_c. A reasonable approximation of the NLO results at realistic values of m_c can be found by following the asymptotic curve down to $m_c = \frac{1}{2}m_b$ and then performing a linear extrapolation. This approach would give even better results if the asymptotic formula was supplemented by higher-order terms in the $\frac{m_b}{m_c}$-expansion, because such an expansion turns out to be convergent down to the threshold $m_c = \frac{1}{2}m_b$.

If a similar approach worked at the NNLO, the 3-loop matrix element calculation would become technically feasible, because the large-m_c NNLO expressions could be found using an expansion in external momenta. The success of the considered extrapolation at the NLO (no matter whether accidental or not) implies that at least the effects related to the renormalization of m_c could be taken into account with reasonable accuracy. Such effects at the NNLO are proportional to the derivative of the NLO amplitude with respect to m_c. At present, the proportionality coefficient remains unknown. It could be found at $m_c = \frac{1}{2}m_b$ using the large-m_c expansion, and then extrapolated down to the realistic values of m_c.

Such a method of estimating the NNLO matrix elements of O_1 and O_2 is going to be used soon [34]. Even though the procedure is rather rough, we will definitely know more than we know now, *i.e.* more than just an expected order of magnitude of the NNLO corrections. The question whether an on-shell calculation might be feasible for $m_c = 0$ is currently under investigation. If it was, the extrapolation in m_c would become an interpolation, which would definitely improve our control over the final result and its uncertainty. However, no definite statement concerning the $m_c = 0$ case can be made yet.

4. Summary

The NNLO QCD calculations of rare B decays are of phenomenological interest only for the inclusive modes $\bar{B} \to X_s l^+ l^-$ and $\bar{B} \to X_s \gamma$. Sensitivity of these decays to new physics, relatively good control over non-perturbative effects and prospects for small experimental errors at the B-factories make the NNLO enterprise inevitable. In fact, the NNLO corrections for $\bar{B} \to X_s l^+ l^-$ are known since more than a year. Order α_s^2 calculations for $\bar{B} \to X_s \gamma$ have only just started. However, it is realistic to expect their completion within a year or so given the number of research groups that have undertaken complementary tasks.

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