$B \to \pi\pi$ DECAY FROM QCD LIGHT-CONE SUM RULES*

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We calculate the contribution to the $B \to \pi\pi$ decay from gluonic penguin operator, O_{8g}, including both hard $O(\alpha_s)$ and soft gluon effects. As expected, the soft effects enter at order $1/m_b^2$, but are numerically of the same order as the hard ones, indicating the necessity of taking them into account.

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1. Introduction

Understanding CP violation within the Standard Model requires information on the angles of the unitarity triangles of the Cabibbo–Kobayashi–Maskawa mixing matrix (CKM) [1]. In the area of B physics, which is currently extensively investigated in experiments [2], the process $B \to \pi\pi$ provides means of finding $\sin 2\alpha$, which is related to the asymmetry [3]

$$a_{\rm CP} = \frac{\Gamma(B_d^0(t) \to \pi^+\pi^-) - \Gamma(\overline{B_d^0}(t) \to \pi^+\pi^-)}{\Gamma(B_d^0(t) \to \pi^+\pi^-) + \Gamma(\overline{B_d^0}(t) \to \pi^+\pi^-)}\,. \tag{1}$$

The nonleptonic B decays are described by an effective Hamiltonian. While the Wilson coefficients appearing in this Hamiltonian can be computed perturbatively, the main problem in evaluating the transition probabilities are the matrix elements of the operators mediating the decays. These are essentially non-perturbative quantities and must be found using other methods,

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e.g. factorization, lattice or sum rules. The possibility of deducing information on $B \to \pi\pi$ decay from the lattice seems remote, while factorization and sum rules offer interesting ways of estimating the pertinent matrix elements.

Recently, new factorization theorems have been proved for non-leptonic B decays [4], which turns factorization into a systematic scheme, in particular allowing estimation of errors related to higher order terms. In the limit of infinite b quark mass, exact predictions can be made using this approach. The corrections to such predictions originating from the finiteness of the real quark mass are, however, worth examining in a quantitative way. It is not clear whether the formally power suppressed terms are numerically significant. The light-cone sum rules (LCSR) method is an excellent model to reliably compare the impact of factorizable and non-factorizable effects, even though it is inherently of limited accuracy. The main advantage of this approach is that both hard and soft gluon effects are evaluated in the same scheme.

The method of light-cone sum rules [5–7] for $B \to \pi\pi$ decays has been proposed in Ref. [8]. The results presented in this paper were calculated in Ref. [9]. We compute the contribution of the gluonic penguin operator to the $B \to \pi\pi$ decay. Apart from being interesting phenomenologically in its own right, this operator, O_{8g}, gives us the opportunity to apply the LCSR method without having to do two-loop computations, as is the case for the remaining operators.

In the next section, we will briefly present the method of light-cone sum rules as applied to the B decay to two pions. Then we will show the result for the matrix element of O_{8g} for this decay analytically and then discuss the numerics. For comparison, we quote the result from the factorization method. We conclude with a summary and outlook.

2. Method

The effective Hamiltonian to describe $B \to \pi\pi$ decays can be written

$$\begin{aligned} H_W \;=\; & \frac{G_\mathrm{F}}{\sqrt{2}}\Bigg\{\lambda_u\left[\left(c_1(\mu)+\frac{c_2(\mu)}{3}\right)O_1(\mu)+2c_2(\mu)\widetilde{O}_1(\mu)\right] \\ & +\ldots+\lambda_t c_{8g}(\mu)O_{8g}(\mu)\Bigg\}\,, \end{aligned} \tag{2}$$

where $\lambda_u = V_{ub}V_{ud}^*$, $\lambda_t = V_{tb}V_{td}^*$ and we have displayed only the most important operators and the gluonic penguin operator O_{8g}, which is of our interest in this paper:

$$O_1 = (\bar{d}\Gamma_\mu u)(\bar{u}\Gamma^\mu b)\,, \quad \widetilde{O}_1 = \left(\bar{d}\Gamma_\mu\frac{\lambda^a}{2}u\right)\left(\bar{u}\Gamma^\mu\frac{\lambda^a}{2}b\right)\,, \tag{3}$$

$$O_{8g} = \frac{m_b}{8\pi^2}\bar{d}\sigma^{\mu\nu}(1+\gamma_5)\frac{\lambda^a}{2}g_s G^a_{\mu\nu}b\,. \tag{4}$$

where $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$ and m_b is the b quark mass. The contribution of the operator O_1 in the emission topology is the main one in the case of $B \to \pi^+\pi^-$ decays and is referred to as the factorizable contribution although it must be stressed that other operators appearing in the effective Hamiltonian (2) are accessible in the framework of QCD factorization. However, it makes sense to express the size of the remaining contributions in terms of this leading naively factorized one because it is often the dominant one.

In order to find the matrix element $\langle B|O_{8g}|\pi\pi\rangle$, we define the correlation function

$$\begin{aligned} F_\alpha(p,q,k) \;=\; & -\int d^4x\, d^4y\, \mathrm{e}^{-i(p-q)x+i(p-k)y} \\ & \times\langle\, 0 \mid T\left\{ j^{(\pi)}_{\alpha 5}(y) O_{8g}(0) j^{(\mathrm{B})}_5(x)\right\} \mid \pi^-(q)\rangle\,, \end{aligned} \tag{5}$$

where the quark currents $j^{(\pi)}_{\alpha 5} = \bar{u}\gamma_\alpha\gamma_5 d$ and $j^{(\mathrm{B})}_5 = m_b\bar{b}i\gamma_5 d$ interpolate π and B mesons, respectively. This correlation function is evaluated in the spacelike region of the variables $s_1 = (p-k)^2$, $s_2 = (p-q)^2$, and $Q^2 = (p-q-k)^2$. Then the result is analytically continued and matched with the double dispersion relation to give a sum rule for the desired matrix element, see [8,9] for details. After Borel transformations in the variables s_1 and s_2, the final formula for the matrix element is [8]

$$\begin{aligned} & \langle \pi^-(p)\pi^+(-q) \mid O_{8g} \mid B(p-q)\rangle \\ & = \frac{-i}{\pi^2 f_\pi f_{\mathrm{B}} m^2_{\mathrm{B}}}\int_0^{s_0^\pi} ds \mathrm{e}^{-s_1/M_1^2}\int_{m_b^2}^{s_0^{\mathrm{B}}} ds_2 \mathrm{e}^{(m^2_{\mathrm{B}}-s_2)/M_2^2}\mathrm{Im}_{s_1}\mathrm{Im}_{s_2}F(s_1,s_2,m^2_{\mathrm{B}}), \end{aligned} \tag{6}$$

where F is the part of the correlation function F_α (5) proportional to the momentum $(p-k)$:

$$F_\alpha = (p-k)_\alpha F + q_\alpha\tilde{F}_1 + k_\alpha\tilde{F}_2 + \epsilon_{\alpha\beta\lambda\rho}q^\beta p^\lambda k^\rho\tilde{F}_3\,, \tag{7}$$

and the thresholds s_0^π and s_0^{B} are fitted from other sum rules, while f_π and f_{B} are the pion and B meson decay constants, respectively, and m_{B} stands for the B meson mass. M_1 and M_2 are the Borel parameters.

3. Results

The correlation function defined in Eq. (5) is evaluated in the spacelike region of the variables s_1, s_2, and Q^2, see [9] for details. The different terms emerging from this calculation can be traced to three kinds of physical effects: the hard gluon, soft gluon, and quark condensate contribution. They can be ordered according to the powers of α_s and $1/m_b^2$, and thus the calculation includes the twist-2 and 3 hard gluon effects as well as soft gluon effects of twist 3 and 4. The latter are suppressed by the quark mass but not by α_s so that they become numerically of the same order as the hard gluonic ones. Also we include the quark condensate contribution as a check on the validity of the twist expansion. The calculation was done with the help of FORM [10].

The result for the matrix element is,

$$\begin{aligned} A^{(O_{8g})}(\bar{B}_d^0 \to \pi^+\pi^-) &\equiv \langle \pi^-(p)\pi^+(-q)|O_{8g}|B(p-q)\rangle \\ &= A^{(O_{8g})}_{\mathrm{hard}} + A^{(O_{8g})}_{\mathrm{soft}} + A^{(O_{8g})}_{\langle \bar{q}q\rangle}\,, \end{aligned} \tag{8}$$

where

$$\begin{aligned} A^{(O_{8g})}_{\mathrm{hard}} &= i\frac{\alpha_s C_F}{2\pi} m_b^2 \left(\frac{1}{4\pi^2 f_\pi} \int_0^{s_0^\pi} ds\; \mathrm{e}^{-s/M_1^2} \right) \frac{m_b^2 f_\pi}{2 m_B^2 f_B} \\ &\quad \times \int_{u_0^B}^1 \frac{du}{u}\, \mathrm{e}^{m_B^2/M_2^2 - m_b^2/uM_2^2} \left(\frac{\varphi_\pi(u)}{u} + \text{twist } 3 \right), \end{aligned} \tag{9}$$

$$\begin{aligned} A^{(O_{8g})}_{\mathrm{soft}} &= -i m_b^2 \left(\frac{1}{4\pi^2 f_\pi} \int_0^{s_0^\pi} ds\; \mathrm{e}^{-s/M_1^2} \right) \left(\frac{f_\pi}{m_B^2 f_B} \int_{u_0^B}^1 \frac{du}{u}\, \mathrm{e}^{m_B^2/M_2^2 - m_b^2/uM_2^2} \right. \\ &\quad \left. \times \left(1 + \frac{m_b^2}{u m_B^2} \right) \left[\varphi_\perp(1-u,0,u) + \widetilde{\varphi}_\perp(1-u,0,u) \right] \right), \end{aligned} \tag{10}$$

$$\begin{aligned} A^{(O_{8g})}_{\langle \bar{q}q\rangle} &= i\frac{\alpha_s C_F}{3\pi} m_b^2 \left(\frac{-\langle \bar{q}q\rangle}{f_\pi m_b} \right) \left(\frac{m_b^2 f_\pi}{2 m_B^2 f_B} \int_{u_0^B}^1 \frac{du}{u} \right. \\ &\quad \left. \times\, \mathrm{e}^{m_B^2/M_2^2 - m_b^2/uM_2^2} \left(\frac{\varphi_\pi(u)}{u} + \text{twist } 3 \right) \right), \end{aligned} \tag{11}$$

where $u_0^B = m_b^2/s_0^B$. The twist-2 distribution amplitude φ_π as well as the omitted terms of higher twist can be found in [9]. The parameter δ_π^2 determines the normalization of the twist-4 quark–antiquark–gluon distribution amplitude and $\langle \bar{q}q\rangle$ is the quark condensate density.

For the sake of better numerical accuracy, rather than working directly with the decay amplitude we will consider the following ratio of this amplitude to the factorizable one:

$$r^{(O_{8g})}(\bar{B}_d^0 \to \pi^+\pi^-) = \frac{A^{(O_{8g})}(\bar{B}_d^0 \to \pi^+\pi^-)}{A_{\rm E}^{(O_1)}(\bar{B}_d^0 \to \pi^+\pi^-)}\,. \tag{12}$$

Notably, the above ratio does not depend on $f_{\rm B}$. The parameters entering the numerical evaluation are $f_\pi = 132$ MeV, $s_0^\pi = 0.7$ GeV^2, and the Borel parameter range $M_1^2 = 0.5 \div 1.5$ GeV^2. The parameters for the B channel are $m_b = 4.7 \pm 0.1$ GeV, $s_{\rm B} = 35 \mp 2$GeV^2, $M_2^2 = 8 \div 12$ GeV^2 for the second Borel parameter, and $\mu_b = \sqrt{m_{\rm B}^2 - m_b^2} \simeq 2.4$GeV for the renormalization scale of the pion distribution amplitudes and of $\alpha_{\rm s}$, where we take the two-loop running with $\Lambda^{(4)} = 280$ MeV. However, we give an estimate of the effect of varying this renormalization scale within the range $\mu_b/2 \div 2\mu_b$. For the quark condensate density, we take $\langle \bar{q}q \rangle$ (1GeV) $= (-240 \pm 10\text{MeV})^3$, or equivalently $\mu_\pi(1\text{ GeV}) = 1.59 \pm 0.2$ GeV, and the normalization parameter of the twist-4 DA $\delta_\pi^2(1\text{GeV}) = 0.17 \pm 0.05$ GeV^2.

In evaluating the decay amplitude, one must decide on some form of the pion distribution amplitudes. They are given in detail in Ref. [9]. In general, one may first approximate the distribution amplitudes with their asymptotic forms. Doing this, we find that the result is stable against variation of both Borel parameters and that the soft gluon contribution amounts to about 50% of the hard-gluon contribution with an opposite sign. This is due to a factor of 20 coming from the twist-4 DA and compensating for the suppression factor δ_π^2/m_b^2. The quark-condensate contribution is about 30%.

The inclusion of non-asymptotic corrections brings no essential change to the hard gluon and condensate contribution, but diminishes dramatically the role of the soft gluon contribution, down to about $-10\% \div +20\%$ of the hard effects. This change is due to the fact that in the kinematics of the soft-gluon process the gluon carries the dominant fraction of the momentum, see [9] for a detailed explanation.

Our estimate for the ratio of the gluonic-penguin and factorizable amplitudes is, including non-asymptotic effects,

$$r^{O_{8g}}(\bar{B}_d^0 \to \pi^+\pi^-) = 0.035 \pm 0.015. \tag{13}$$

The uncertainty above does not include the variation of the renormalization scale $\mu = \mu_b/2 \div 2\mu_b$, which adds another 20% to the error given above.

As for the validity of the heavy quark limit, it is interesting to note that the value of $r^{O_{8g}}$ given above for the physical b quark mass varies between 0.6 and 1.1 of its heavy quark limit.

To put the gluonic penguin decay amplitude in the right perspective, let us compare its role with that of the factorizable contribution, which is dominant in the $\bar{B}_d \to \pi^+\pi^-$ decay. We also include the emission topology contribution from $\tilde{O}_1$:

$$\begin{aligned}&\mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-) \equiv \langle \pi^+\pi^- | H_W | \bar{B}_d^0 \rangle \\ &= i\frac{G_\mathrm{F}}{\sqrt{2}} f_\pi f_{B\pi}^+(0)\ m_\mathrm{B}^2 \Bigg\{ \lambda_u \left[c_1(\mu) + \frac{c_2(\mu)}{3} + 2c_2(\mu) r_\mathrm{E}^{(\widetilde{O}_1)} \left(\bar{B}_d^0 \to \pi^+\pi^-\right)\right] \\ &+ \ldots + \lambda_t c_{8g}(\mu) r^{(O_{8g})} \left(\bar{B}_d^0 \to \pi^+\pi^-\right) \Bigg\}, \end{aligned} \tag{14}$$

with

$$r_\mathrm{E}^{(\widetilde{O}_1)} \left(\bar{B}_d^0 \to \pi^+\pi^-\right) \equiv \frac{A_\mathrm{E}^{(\widetilde{O}_1)} \left(\bar{B}_d^0 \to \pi^+\pi^-\right)}{A_\mathrm{E}^{(O_1)}(\bar{B}_d^0 \to \pi^+\pi^-)}, \tag{15}$$

where $A_\mathrm{E}^{(\widetilde{O}_1)}$ is the $\bar{B}_d^0 \to \pi^+\pi^-$ matrix element of $\widetilde{O}_1$ in the emission topology. The hard contribution to this ratio is taken from QCD factorization [8] while the soft part was found in [8]. We obtain,

$$\begin{aligned}\mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-) &= i\frac{G_\mathrm{F}}{\sqrt{2}} f_\pi\ m_\mathrm{B}^2 (0.28 \pm 0.05) \Bigg\{ \lambda_u \Big(1.03 \\ &+ [-0.004 \div 0.024 + 0.025i] \Big) + \ldots - \lambda_t [0.003 \div 0.008] \Bigg\}, \end{aligned} \tag{16}$$

where, for consistency, the Wilson coefficients [11] are taken at the scale μ_b. In Eq. (16), the first bracket contains the contribution of $\tilde{O}_1$ while the second that of the gluonic penguin operator. It is seen that both non-factorizable contributions are negligibly small, but remaining such contributions must be computed before one concludes that the decay $\bar{B}_d \to \pi^+\pi^-$ is dominated by the factorizable amplitude.

The prediction of the QCD factorization for the ratio $r^{O_{8g}}$ is,

$$r^{(O_{8g})}(\bar{B}_d^0 \to \pi^+\pi^-)_{\mathrm{QCDfact.}} = \frac{\alpha_\mathrm{s} C_\mathrm{F}}{2\pi N_c} \left(\int_0^1 du \frac{\varphi_\pi(u)}{1-u} + \frac{2\mu_\pi}{m_b} \right), \tag{17}$$

together with the $O(\mu_\pi/m_b)$ correction which is the only $1/m_b$ effect retained in QCD factorization [4] because of the large numerical value of μ_π. For $m_b \to \infty$, this result coincides with the heavy quark limit of LCSR.

Subleading terms differ, however. Apart from the factor of 2 of difference between the quark condensate contribution, the soft gluon corrections in LCSR are suppressed by $1/m_b^2$ and thus absent from the prediction of the QCD factorization. While this omission is consistent on the grounds of power-counting, it has been shown to matter numerically.

4. Conclusion

We have presented the results of the calculation of the gluonic penguin operator contribution to $B \to \pi\pi$ decay, done in Ref. [9]. Using LCSR made it possible to include the soft and hard gluon effects systematically within the same scheme. In this way the finite quark mass effects were seen to be important. On the other hand, the overall size of the result suggests that the gluonic penguin operator does not play significant role in the $\bar{B}_d \to \pi^+\pi^-$ decay.

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