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INELASTIC FINAL-STATE INTERACTIONS IN B DECAYS*

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We present the results of an effective approach to rescattering in B decays to two pseudoscalar mesons, where all inelastic Zweig-rule-satisfying SU(3)-symmetric final-state interactions are taken into account. It is shown how such rescattering corrections lead to a simple redefinition of the amplitudes, permitting the use of a simple diagram-based description, in which, however, weak phases may enter in a modified way. An estimate of how these modifications might affect the extracted value of unitarity triangle angle γ is given. It is pointed out that substantial shifts in the value of γ cannot be excluded on the basis of the low experimental bound on the $B^0_d \to K^+K^-$ branching ratio alone.

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1. Introduction

The goal of the current experimental and theoretical efforts in B physics is to extract and overconstrain the parameters of the Standard Model (SM), and in particular to determine the three angles of the unitarity triangle. It is often hoped that discrepancies between various independent extractions of these angles in the SM framework will (if found) point towards New Physics.

However, before assigning any of the possible discrepancies to New Physics one has to ensure that old physics is properly taken into account. In fact, several of the proposed extraction methods are based on the analysis of data on $B \to PP$ decays (P — light pseudoscalar meson), for which it is not clear what SM predictions are. Indeed, the majority of the popular SM approaches assume pure short-distance (SD) dynamics, while completely neglecting final-state interactions (FSI). On the other hand, it is well known

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that final-state interaction effects are very important in nonleptonic kaon decays, in which $\pi\pi$ rescattering determines relative phases between the relevant amplitudes. It has been argued by several physicists [1, 2, 6] that, at B-meson mass, such effects (although probably smaller) should be still important. Thus, the intermediate states in $B \to PP$ decays should include not only many PP states, but possibly also a plethora of inelastic states, which might eventually rescatter into PP.

The size of elastic and quasi-elastic FSI transitions can be estimated. Calculations show that these effects should be quite important (see *e.g.* [4, 5]). Much more problematic is the issue of contributions from inelastic rescattering. While it is possible that most inelastic states created in the short-distance stage of B decay do not rescatter into the final PP pair, various arguments and explicit calculations show that such rescatterings may be important [1, 3]. However, with the many possible inelastic intermediate states, a reliable calculation of their contribution is not feasible.

2. Simplified description of inelastic FSI

Since inelastic FSI effects are presumably incalculable, the best one can do is to parametrize them in some way, while minimizing the number of parameters used. In order to make things feasible, several simplifications need to be introduced. The most important simplifying assumptions of the approach developped in [7, 8] are given below.

(1) First, Refs. [7, 8] accept that FSI are oblivious of the original short-range decay mechanism and cannot change its overall probability. Consequently, the set of all FSI-corrected weak decay amplitudes $\{W(B \to PP)\}$ (gathered into vector $\boldsymbol{W}$) may be expressed as:

$$\boldsymbol{W} = \boldsymbol{S}^{1/2}\boldsymbol{w} \equiv \boldsymbol{w} + \Delta\boldsymbol{W} \, , \tag{1}$$

where $\boldsymbol{w}$ represents the set of all short-distance decay amplitudes $\{w(B \to PP)\ \}$, $\boldsymbol{S}$ is the strong interaction S-matrix, and $\Delta\boldsymbol{W}$ describes the rescattering correction. (For the one-channel case, $\boldsymbol{S}^{1/2}$ reduces to the Watson phase factor $\mathrm{e}^{i\delta}$, and amplitude W differs from w by a phase only.)

(2) In order to reduce the number of the necessary parameters, final-state interactions are assumed to be SU(3) symmetric. As a result, FSI in B^+, B^0_d, B^0_s decays are related.

(3) All intermediate inelastic states are represented by quasi-two-body states, as the short-range decay process always produces two quark-antiquark pairs at the most. Apart from the decay of states composed of a single $q\bar{q}$ pair into two such pairs, all other strong interaction effects are treated as part of the rescattering process.

(4) Short-distance amplitudes $w_X(B \to M_1M_2)$, corresponding to a given quark diagram X ($X \to$ penguin P, tree T, *etc.*) and a given final state M_1M_2, are assumed to be proportional to the short-distance $w_X(B \to PP)$ amplitudes for a diagram of the same topology (*i.e.*, $X \to$ penguin P, tree T, *etc.*) with a similar flavour composition of the PP state, with the (unknown) proportionality coefficients $\eta(M_1M_2)$ depending only on the M_1M_2 state produced:

$$w_X(B \to M_1M_2) = \eta(M_1M_2)w_X(B \to PP)\,. \tag{2}$$

Complete SD amplitudes are given as sums over the contributing diagrams:

$$\begin{aligned} w(B \to PP) &= \sum_X w_X(B \to PP)\,, \\ w(B \to M_1M_2) &= \sum_X w_X(B \to M_1M_2)\,. \end{aligned} \tag{3}$$

Rescattering through a single intermediate M_1M_2 state into a given final PP state leads then to the following correction to $w(B \to PP)$:

$$\Delta W(B \stackrel{M_1M_2}{\longrightarrow} PP) = f(M_1M_2 \to PP)w(B \to M_1M_2)\,, \tag{4}$$

where FSI amplitudes $f(M_1M_2 \to PP)$ are also unknown. The total FSI-induced correction is equal to the sum over all intermediate states:

$$\Delta W(B \to PP) = \sum_{M_1M_2} f(M_1M_2 \to PP)\eta(M_1M_2)w(B \to PP)\,. \tag{5}$$

Bose symmetry requires that the PP state should form a symmetric state. For simplicity, let us consider PP states composed of SU(3)-octet states $P_{\mathbf{8}}$ only. Then, if only symmetric octets were allowed for both the M_1M_2 and PP states, the sum $\sum_{M_1M_2} f(M_1M_2 \to PP)\eta(M_1M_2)$ could be replaced with a single parameter $R(\mathbf{8}_s)$, a counterpart of factor $\mathrm{e}^{i\delta}-1$ in the elastic case. In fact, however, since the two-particle states may belong to different representations of SU(3) built in different ways from singlet and octet mesons M_1 and M_2, the sum $\sum_{M_1M_2} f(M_1M_2 \to PP)\eta(M_1M_2)$ from Eq. (5) becomes a non-diagonal matrix

$$\boldsymbol{R} = \begin{bmatrix} R(\mathbf{27}) & 0 & 0 & 0 & 0 & 0 \\ 0 & R(\mathbf{8}_s) & R(\mathbf{8}_{(8,1)}) & R(\mathbf{8}_{\mathrm{a}}) & 0 & 0 \\ 0 & 0 & 0 & 0 & R(\mathbf{1}_{\{8,8\}}) & R(\mathbf{1}_{\{1,1\}}) \end{bmatrix} \tag{6}$$

so that $\boldsymbol{\Delta W} = \boldsymbol{Rw}$. The rows and columns in Eq. (6) correspond to different SU(3) couplings for the $P_{\mathbf{8}}P_{\mathbf{8}}$ and M_1M_2 states respectively, with parameters

$R(...)$ denoting the relevant transition terms of the form $\sum_{M_1M_2} f(M_1M_2 \to PP)\eta(M_1M_2)$. There are three rows as the $P_{\mathbf{8}}P_{\mathbf{8}}$ state may be in $\mathbf{27}$, $\mathbf{8}_s$, or $\mathbf{1}_{\{8,8\}}$ representation of SU(3). For the M_1M_2 states (with M_i singlet or octet), all different ways of their couplings to the three SU(3) representations have to be considered. In particular, the antisymmetric octet $\mathbf{8}_{\rm a}$, absent among the $P_{\mathbf{8}}P_{\mathbf{8}}$ states by virtue of Bose symmetry, may be constructed here, as M_1 and M_2 are not identical bosons in general. Since SU(3) does not impose any connections between R's, FSI interactions for the $B \to P_{\mathbf{8}}P_{\mathbf{8}}$ decays are parametrized by six complex parameters of Eq. (6).

With the above assumptions, the FSI-corrected amplitudes $\boldsymbol{W}$ are given in terms of quark-diagram SD amplitudes P, T, *etc.*, as well as parameters R. For example, the FSI-corrected $B^+ \to K^+\bar{K}^0$ amplitude is given by:

$$W(B^+ \to K^+\bar{K}^0) = -P(1+R(\mathbf{27})) - \frac{1}{5}(T\Delta_1 + P\Delta_2 + C\Delta_3), \qquad (7)$$

where Δ_i are linear combinations of R's. One can also check that, independently of the values of parameters R, the FSI-corrected amplitudes satisfy various triangle relations discussed in the literature [9], for example:

$$W(B^+ \to \pi^+\pi^0) = \frac{1}{\sqrt{2}}W(B_s^0 \to \pi^+K^-) + W(B_s^0 \to \pi^0\bar{K}^0). \qquad (8)$$

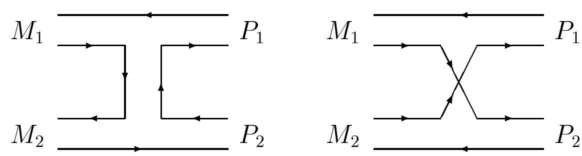
With several amplitudes connected by such triangle relations, the number of independent and — in principle — measurable data in all $B \to PP$ decays turns out to be too small for the determination of rescattering parameters R.

(5) The above discussion of the general SU(3) case shows that further assumptions have to be made. For example one may (a) — assume the relative sizes and phases of SD quark-diagram amplitudes, (b) — neglect some terms, and/or (c) — assume additional symmetry so that parameters R become related. Zweig rule, an important feature of strong interactions, presumably constitutes the most important additional and necessary ingredient here.

Zweig rule and nonet SU(3) symmetry significantly limit the number of rescattering parameters. There are only two possible topologies of quark-line diagrams (Fig. 1), to which only three possible SU(3) structures may be assigned. Indeed, the uncrossed FSI diagrams of Fig. 1(a) are parametrized with the help of two parameters u_+ and u_-, corresponding to the two SU(3)-invariant forms admissible:

$$\begin{aligned} &\mathrm{Tr}(\{M_1^\dagger, M_2^\dagger\}\{P_1,P_2\})\ u_+\,, \\ &\mathrm{Tr}([M_1^\dagger, M_2^\dagger]\{P_1,P_2\})\ u_-\,. \end{aligned} \qquad (9)$$

The first (second) structure describes transitions in which the product of



(u) (c)

Fig. 1. Types of rescattering diagrams: (u) uncrossed, (c) crossed.

charge conjugation parities of mesons M_1 and M_2 is positive (negative), *i.e.* $C_{M_1}\ C_{M_2} = +1(-1)$, respectively. For the crossed diagrams of Fig. 1(b), only one SU(3) structure is possible:

$$\mathrm{Tr}(M^{\dagger} P_1 M_2^{\dagger} P_2 + M_1^{\dagger} P_2 M_2^{\dagger} P_1)\ c\,. \tag{10}$$

The other structure, with a "$-$" sign in between the two terms above, is not symmetric under the $P_1 \leftrightarrow P_2$ interchange, being therefore inconsistent with the requirements of Bose symmetry for the final PP state. Consequently, six R's get replaced by only three rescattering parameters:

$$\begin{aligned} u &\equiv \frac{u_+ + u_-}{2}\,, \\ d &\equiv u_+ - u_-\,, \\ c&. \end{aligned} \tag{11}$$

For SU(3) symmetric rescattering, the FSI-corrected amplitudes $\boldsymbol{W}$ may be also expressed in terms of quark-diagram amplitudes. These quark-diagram amplitudes include all FSI corrections, however. They are denoted here as $\tilde{T}$ (tree), $\tilde{C}$ (colour-suppressed), $\tilde{P}$ (penguin), $\tilde{S}$ (singlet penguin), $\tilde{A}$ (annihilation), $\tilde{E}$ (exchange), and $\tilde{PA}$ (penguin annihilation). Their relation to the input SD amplitudes T, C, P, and S turns out to be [8]:

$$\begin{aligned} \tilde{T} &= T + C \cdot 2c\,, \\ \tilde{C} &= C + T \cdot 2c\,, \\ \tilde{P} &= P + S \cdot (2c + 2u) + (T + 3P + S) \cdot d\,, \\ \tilde{S} &= S + P \cdot 2c\,, \\ \tilde{A} &= C \cdot 2u\,, \\ \tilde{E} &= T \cdot 2u\,, \\ \tilde{PA} &= 2P \cdot 2u\,. \end{aligned} \tag{12}$$

Such formulas hold both for strangeness-conserving and strangeness-changing sectors.

3. Specific results

We proceed to the discussion of some conclusions following from Eqs. (12).

(1) It is known that in the strangeness-changing sector the data on $B \to K\eta'$ seem to require an effective singlet penguin amplitude $S'_{\rm eff} \approx 0.5 P'_{\rm eff}$ (with primes denoting strangeness-changing amplitudes) [10,11]. The primed counterparts of formulas (12) show that the effective singlet penguin may be due to the rescattering corrections: with SD amplitude $S' \approx 0$, one may still have $|\tilde{S}'| \approx 0.5|\tilde{P}'|$, provided $|c|$ is around 0.25.

(2) Several authors have argued that the size of the rescattering may be gleamed from the $B^0_d \to K^+K^-$ decays [6]. The relevant FSI-corrected amplitude is

$$W(B^0_d \to K^+K^-) = \tilde{E} + \tilde{PA} = (T + 2P)\ 2u \tag{13}$$

and it vanishes for vanishing FSI. The experimental bound on the size of the $B^0_d \to K^+K^-$ branching ratio (BR $< 0.6 \times 10^{-6}$) clearly limits the size of u. However, it does not say anything about the size of the remaining two parameters, d and c. A small value for u may mean that the contributions from the $C_{M_1}C_{M_2} = +1$ and $C_{M_1}C_{M_2} = -1$ intermediate states cancel in the expression for u, while they may add up in the expression for d. Consequently, even when the rate for $B^0_d \to K^+K^-$ is completely negligible, the FSI effects may be important and may affect the determination of the CP-violating SM parameters.

(3) One of the methods proposed for the future determination of angle γ [12] is based on the measurement of ratios

$$\begin{aligned}
R_d &= \frac{\Gamma(B^0_d \to \pi^-K^+) + \Gamma(\bar{B}^0_d \to \pi^+K^-)}{\Gamma(B^+ \to \pi^+K^0) + \Gamma(B^- \to \pi^-\bar{K}^0)}\,,\\
R_s &= \frac{\Gamma(B^0_s \to \pi^+K^-) + \Gamma(\bar{B}^0_s \to \pi^-K^+)}{\Gamma(B^+ \to \pi^+K^0) + \Gamma(B^- \to \pi^-\bar{K}^0)}\,,\\
A_d &= \frac{\Gamma(B^0_d \to \pi^-K^+) - \Gamma(\bar{B}^0_d \to \pi^+K^-)}{\Gamma(B^+ \to \pi^+K^0) + \Gamma(B^- \to \pi^-\bar{K}^0)}
\end{aligned} \tag{14}$$

which may be expressed in terms of $r \equiv |T'/P'|$, γ, and the difference δ of P' and T' strong phases as

$$\begin{aligned}
R_d &= 1 + r^2 + 2r\cos\gamma\cos\delta\,,\\
R_s &= \lambda^2 + (r/\lambda)^2 - 2r\cos\gamma\cos\delta\,,\\
A_d &= -2r\sin\gamma\sin\delta\,,
\end{aligned} \tag{15}$$

where $\lambda \approx \tan\theta_C \approx 0.22$.

When FSI's are taken into account, the relevant amplitudes are modified by the presence of a potentially important term in the $B_s^0 \to \pi^+ K^-$ amplitude:

$$\begin{aligned}
W(B^+ \to \pi^+ K^0) &= -\bar{P}' , \\
W(B_d^0 \to \pi^- K^+) &= \bar{P}' + \bar{T}' , \\
W(B_s^0 \to \pi^+ K^-) &= -\bar{P} - \bar{T} + 2Td , \qquad (16)
\end{aligned}$$

where $\bar{T}' = T'(1+3d)$, $\bar{P}' = P'(1+3d)$ *etc.* Please note the presence of the $2Td$ term in the third of Eqs. (16). Analogous terms proportional to $T'd$ in the first two equations of (16) can be safely neglected, since $|T'/P'| \ll 1$. The $2Td$ term modifies expressions (15) and affects the extraction of γ (note that in Eq. (12) with $S \approx 0$, the modified penguin amplitude $\tilde{P} \equiv \bar{P} + Td$ depends on both β and γ weak angles, while the SD penguin depends on β only). One can estimate the size of $|d|$ for a quasi-elastic rescattering $PP \to PP$. In a model with leading Regge exchanges one obtains $|d| \approx 0.05$. For γ around 60°, the resulting shift in the value of γ turns out to be of the order of 5° [8].

(4) The elastic FSI's are not described with the diagrams of Fig. 1. Such a rescattering consists in an exchange of a flavor-singlet Pomeron between the outgoing pseudoscalar mesons PP (*i.e.*, no quark lines are exchanged). Since the Pomeron exchange amplitude is predominantly imaginary and equal for all SU(3) representations available for the PP state, the elastic rescattering cannot affect the relative sizes and phases of the $B \to PP$ amplitudes in the SU(3) limit. When the leading Reggeon exchanges between the final PP mesons are considered alongside the Pomeron, one can estimate phase differences $\delta_1 - \delta_8$, $\delta_8 - \delta_{27}$, $\delta_1 - \delta_{27}$ between $B \to PP$ amplitudes in singlet, octet, and 27-plet SU(3) channels. At energy corresponding to B-meson mass, these differences may be of the order of 10° (see *e.g.* [5]). Large phase differences (*e.g.* 50°–100° as claimed in [13]) cannot be due to the elastic rescattering.

4. Conclusions

The presented approach, developed specifically for the description of all of the inelastic FSI's in the $B \to PP$ decays, permits parametrization of such rescattering effects in terms of three parameters only. Furthermore, if the experimental branching ratio for the $B_d^0 \to K^+K^-$ is negligible, one of these parameters may be set to zero. The determination of the remaining two parameters may require a fit to all available $B \to PP$ branching ratios. Thus, the size of FSI effects cannot be estimated on the basis of the $B_d^0 \to K^+K^-$ decay rate. In particular, substantial inelastic FSI effects may explain a part of the singlet penguin amplitude needed in $B \to K\eta'$. An estimate of the influence of quasi-elastic rescattering effects on the method of γ extraction considered here leads to shifts in the value of γ being of the order of 5° [14]. Further development along the lines discussed here should take into account SU(3) breaking in both SD amplitudes and in FSI, as well as a possible relevance of the charming penguins [15].

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