# SPIN PROPERTIES OF TOP QUARK PAIRS PRODUCED AT HADRON COLLIDERS*

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We discuss the spin properties of top quark pairs produced at hadron colliders at next-to-leading order in the coupling constant $\alpha_s$ of the strong interaction. Specifically we present, for some decay channels, results for differential angular distributions that are sensitive to $t\bar{t}$ spin correlations.

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## 1. Introduction

The top quark is the heaviest fundamental particle discovered so far. Its interactions are still relatively unexplored and their experimental investigation may lead to exciting new discoveries. To mention only a few examples: due to its large mass, it is not excluded that the top quark decays into yet unobserved particles like charged Higgs bosons or supersymmetric particles — or, if these particles are heavier than the top quark, they may mediate top quark decay, leading also to new decay modes and/or branching ratios that differ from the Standard Model (SM) predictions (for overviews, see, *e.g.*,

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Refs. [1,2]). Specifically, the V–A structure of the top quark decay vertex is modified in many extensions of the Standard Model [3,4]. In the production of top quarks, the resonant production of heavy spin-0-particles could lead to interesting signatures [5–7]. Its large mass makes the top quark also a good probe of the electroweak symmetry breaking mechanism. In this context it will be important to check whether the Yukawa coupling of the top quark is as predicted by the Standard Model. Finally, the question whether the discrete symmetry CP is violated in top quark production and decay has been the subject of numerous investigations (*cf.* Ref. [8] for a review). The foreseen large data samples at the upgraded Tevatron ($\sim 10^3 - 10^4$ $t\bar{t}$/year) and at the LHC ($\sim 10^7$ $t\bar{t}$/year) will tell us more about these issues in the near future. Of help in this context will be another special feature of the top quark, namely that the spin properties of top quark pairs are predictable by perturbation theory. This is due to its large decay width, $\Gamma_t^{\rm SM} \sim 1.5$ GeV $\gg \Lambda_{\rm QCD}$, which serves as a cut-off for hadronization effects. In particular, the top quark decays so fast that the information about its polarization is not diluted by hadronization but transferred to the decay products. Thus observables related to the spins of the $t$ and $\bar{t}$ quarks can be constructed and used reliably for the detailed study of the dynamics of top quark production and decay [9]. Apart from searching for non-standard effects, the study of the $t$ and $\bar{t}$ spin properties is interesting even within the framework of the SM: They probe the 'quasi-free' nature of the top quark, thus allowing us to study properties of a 'bare' quark. Further, a measurement of spin correlations would provide a lower bound on $|V_{tb}|$ without assuming the existence of three quark generations [10].

Here we confine ourselves to the top quark pair production and decay at hadron colliders and investigate the top quark polarization and spin correlation phenomena that are induced by the strong interaction dynamics. We discuss predictions for the normal polarization of the top quarks and $t\bar{t}$ spin correlations at next-to-leading order (NLO) in the QCD coupling $\alpha_{\rm s}$.

## 2. Theoretical framework

Needless to say, for a correct interpretation of upcoming and future data on top quark production and decay at the Tevatron and the LHC, precise theoretical predictions within the SM are needed. We consider here the following reactions:

$$h_1 h_2 \to t\bar{t} + X \to \left\{ \begin{array}{lcl} \ell^+\ell'^- & + & X \\ \ell^+ j_{\bar{t}} & + & X \\ \ell^- j_t & + & X \\ j_t j_{\bar{t}} & + & X \end{array} \right. , \qquad (1)$$

where $h_{1,2} = p, \bar{p}$; $\ell = e, \mu, \tau$, and $j_t$ ($j_{\bar{t}}$) denotes jets originating from hadronic $t$ ($\bar{t}$) decays. Experimental analysis of the above processes requires predictions of the fully differential cross sections. The calculation of these cross sections at next-to-leading order QCD simplifies enormously in the leading pole approximation (LPA), which amounts to expanding the full amplitudes for the reactions listed in Eq. (1) around the complex poles of the top quark propagators. Only the leading pole terms are kept in this expansion, *i.e.*, one neglects terms of order $\Gamma_t/m_t \approx 1\%$. Within the LPA, the radiative corrections can be classified into factorizable and non-factorizable contributions. Here we consider only the factorizable corrections; for the non-factorizable contributions see Ref. [11]. Further we apply the on-shell approximation for the top quark propagators:

$$\lim_{\Gamma/m \to 0} \left| \frac{1}{k^2 - m^2 + im\Gamma} \right|^2 \to \frac{\pi}{m\Gamma} \delta(k^2 - m^2) \,. \tag{2}$$

The necessary ingredients at NLO QCD within this approximation are the differential cross sections for the following parton processes

$$q\bar{q} \to t\bar{t},\ gg \to t\bar{t} \quad (\text{to order } \alpha_s^3)\,, \tag{3}$$
$$q\bar{q} \to t\bar{t}g,\ gg \to t\bar{t}g \quad (\text{to order } \alpha_s^3)\,, \tag{4}$$
$$gq(\bar{q}) \to t\bar{t}q(\bar{q}) \quad (\text{to order } \alpha_s^3)\,, \tag{5}$$
$$t \to b\ell\nu, bq\bar{q}' \quad (\text{to order } \alpha_s)\,, \tag{6}$$

where we have to keep the full information on the $t$ and $\bar{t}$ spins.

## 3. Spin density matrix

The fully differential cross section $d\sigma_{\text{fact.}}$ for the production and subsequent decay of top quark pairs at the parton level can be written in terms of a spin density matrix $R$ and decay density matrices $\rho, \bar{\rho}$:

$$d\sigma_{\text{fact.}} = \text{Tr}_{t,\bar{t}\text{ spins}}\,(R\ \rho\ \bar{\rho})\,. \tag{7}$$

The unnormalized spin density matrix is explicitly given by the following expression ($i = q\bar{q}, gg, \ldots$):

$$R_{\alpha\beta,\alpha'\beta'} = \sum \langle t(k_t,\alpha)\bar{t}(k_{\bar{t}},\beta)X|\mathcal{T}|i\rangle \langle t(k_t,\alpha')\bar{t}(k_{\bar{t}},\beta')X|\mathcal{T}|i\rangle^*, \tag{8}$$

where the sum runs over all unobserved degrees of freedom. The decomposition of the spin density matrix with respect to the $t$ and $\bar{t}$ spin spaces reads:

$$R = A\ \mathbb{1} \otimes \mathbb{1} + \boldsymbol{B}^{+}\sigma \otimes \mathbb{1} + \mathbb{1} \otimes \sigma\ \boldsymbol{B}^{-} + C_{ij}\ \sigma^i \otimes \sigma^j\,. \tag{9}$$

The polarization of the top quark (antiquark) is encoded in $\boldsymbol{B}^{\pm}$, *e.g.*

$$\boldsymbol{P}_t \equiv 2\langle \boldsymbol{S}_t \rangle = \frac{\boldsymbol{B}^+}{A} = \frac{\mathrm{Tr}\,[R\sigma \otimes \mathbb{1}]}{\mathrm{Tr}\,[R]}\,, \tag{10}$$

where $\boldsymbol{S}_t$ denotes the top quark spin operator. The matrix $C$ encodes the spin correlations of the top quark and antiquark:

$$4\langle S_{t,i} S_{\bar{t},j} \rangle = \frac{\mathrm{Tr}\,[R\sigma_i \otimes \sigma_j]}{\mathrm{Tr}\,[R]} = \frac{C_{ij}}{A}\,. \tag{11}$$

### *3.1. Normal polarization*

If only strong interactions are taken into account then the polarization of $t$ and $\bar{t}$ in $pp, p\bar{p} \to t\bar{t}X$ can only be normal to the event plane due to parity invariance of QCD. Normal polarization requires absorptive parts in the scattering amplitude, *i.e.* one-loop diagrams with discontinuities. For the two initial states $i = q\bar{q}, gg$,

$$\boldsymbol{P}^i_t = \boldsymbol{P}^i_{\bar{t}} = b^i_3(y, \hat{s})\hat{\boldsymbol{n}}\,, \tag{12}$$

where $\hat{\boldsymbol{n}}$ is the unit vector normal to the event plane and $y = \cos\theta$ with $\theta$ denoting the scattering angle of the top quark in the parton center-of-mass frame. The normal polarization at the parton level is a percent effect [12, 13]. Several observables to study this effect were proposed and studied in Ref. [12]. At the Tevatron, and probably even at the LHC, the normal polarization of the top quark induced by QCD will be very difficult to observe. This means that it provides a sensitive probe of non-standard strong rescattering effects in hadronic top quark pair production.

### *3.2. Spin correlations at leading order*

The correlation between two observables $O_1$, $O_2$ is defined as

$$\mathrm{corr}(O_1, O_2) = \frac{\langle O_1 O_2 \rangle - \langle O_1 \rangle \langle O_2 \rangle}{\delta O_1 \delta O_2}\,, \tag{13}$$

with $\delta O_i = \sqrt{\langle O_i^2 \rangle - \langle O_i \rangle^2}$. Assuming parity invariance we have for hadronic production of top quark pairs at leading order (*i.e.*, no absorptive parts):

$$\langle S^i_t \rangle = \langle S^j_{\bar{t}} \rangle = 0 \tag{14}$$

and therefore

$$\mathrm{corr}(S^i_t, S^j_{\bar{t}}) = 4\langle S^i_t S^j_{\bar{t}} \rangle = \frac{C_{ij}}{A}\,. \tag{15}$$

For the process $q\bar{q} \to t\bar{t}$, the spin correlation matrix at LO is quite simple:

$$\frac{C_{ij}^{q\bar{q}}}{A^{q\bar{q}}} = \frac{1}{3}\delta_{ij} + \frac{2}{2-\beta^2(1-y^2)}\left[\left(\hat{d}_i\hat{d}_j - \frac{1}{3}\delta_{ij}\right) + \beta^2(1-y^2)\left(\hat{d}_i^\perp\hat{d}_j^\perp - \frac{1}{3}\delta_{ij}\right)\right] \tag{16}$$

with

$$\hat{\boldsymbol{d}} = \frac{1}{\sqrt{1+(\gamma^2-1)y^2}}\left[\gamma y\hat{\boldsymbol{k}}_t + \sqrt{1-y^2}\hat{\boldsymbol{k}}_t^\perp\right], \tag{17}$$

where $\beta = (1-4m_t^2/s)^{1/2}$, $\gamma = (1-\beta^2)^{-1/2}$, and $\hat{\boldsymbol{k}}_t$ denotes the direction of the top quark. Further, $\boldsymbol{k}^\perp = \hat{\boldsymbol{p}}_q - y\hat{\boldsymbol{k}}$ and $\boldsymbol{d}^\perp = \hat{\boldsymbol{p}}_q - (\hat{\boldsymbol{p}}_q \cdot \hat{\boldsymbol{d}})\hat{\boldsymbol{d}}$, where $\hat{\boldsymbol{p}}_q$ denotes the quark direction.

For $q\bar{q} \to t\bar{t}$ the direction $\hat{\boldsymbol{d}}$ is the optimal spin basis at leading order QCD, because at LO:

$$\frac{\hat{d}_i C_{ij}^{q\bar{q}}\hat{d}_j}{A^{q\bar{q}}} = 1\,. \tag{18}$$

For any $\beta$ and $y$, the $t\bar{t}$ spins are 100% correlated w.r.t. to this basis [14]. Following the nomenclature of Ref. [14] we shall call this basis the off-diagonal basis in the following. For the Tevatron, where this choice of spin axis is useful there is an equally efficient but simpler possibility. At threshold, the top quark pair is in a $^3S_1$ state and we have

$$\hat{\boldsymbol{d}} \xrightarrow{\beta\to 0} \hat{\boldsymbol{p}}_q\,. \tag{19}$$

This suggests that at the Tevatron the direction $\hat{\boldsymbol{p}}$ of, say, the proton beam is an equally good choice of spin axis.

At high energies, helicity is conserved and we have

$$\hat{\boldsymbol{d}} \xrightarrow{\beta\to 1} \hat{\boldsymbol{k}}_t\,. \tag{20}$$

For the process $gg \to t\bar{t}$, no optimal spin basis exists. The LO expression for the spin correlation matrix is quite lengthy, and we therefore discuss only the limiting cases. At threshold, the top quark pair is in a $^1S_0$ state and

$$\frac{C_{ij}^{gg}}{A^{gg}} \xrightarrow{\beta\to 0} -\delta_{ij}\,. \tag{21}$$

At high energies, we have

$$\frac{C_{ij}^{gg}}{A^{gg}} \xrightarrow{\beta\to 1} \frac{1}{3}\delta_{ij} + \frac{2}{1+y^2}\left[\left(\hat{k}_{t,i}\hat{k}_{t,j} - \frac{1}{3}\delta_{ij}\right) + (1-y^2)\left(\hat{k}_{t,i}^\perp\hat{k}_{t,j}^\perp - \frac{1}{3}\delta_{ij}\right)\right], \tag{22}$$

and helicity conservation is reflected by

$$\frac{\hat{k}_{t,i}C_{ij}^{gg}\hat{k}_{t,j}}{A^{gg}} \xrightarrow{\beta\to 1} 1\,. \tag{23}$$

## 4. Observing spin correlations

The $t\bar{t}$ spin correlations show up in certain angular distributions/correlations of the top decay products, *e.g.* for the dileptonic decay channel $t \to \ell^+\nu b$, $\bar{t} \to \ell'^-\bar{\nu}\bar{b}$ the following distribution is sensitive to the correlations:

$$\frac{1}{\sigma}\frac{d^2\sigma(h_1h_2 \to t\bar{t}X \to \ell^+\ell'^-X)}{d\cos\theta_+ \cos\theta_-} = \frac{1}{4}(1+B_1\cos\theta_+ + B_2\cos\theta_- - C\cos\theta_+\cos\theta_-). \tag{24}$$

In Eq. (24), $\theta_+, \theta_-$ are the angles of $\ell^\pm$ in the $t$ ($\bar{t}$) rest frame with respect to arbitrary spin quantization axes $\hat{\boldsymbol{a}}$, $\hat{\boldsymbol{b}}$, *e.g.*:

$$\begin{array}{ll} \hat{\boldsymbol{a}} = -\hat{\boldsymbol{b}} = \hat{\boldsymbol{k}}_t & \text{(helicity basis)}, \\ \hat{\boldsymbol{a}} = \hat{\boldsymbol{b}} = \hat{\boldsymbol{p}} & \text{(beam basis)}, \\ \hat{\boldsymbol{a}} = \hat{\boldsymbol{b}} = \hat{\boldsymbol{d}} & \text{(off-diagonal basis)}. \end{array} \tag{25}$$

The coefficient $C$ reflects the strength of the $t\bar{t}$ spin correlations for the chosen quantization axes, $-1 \le C \le +1$ [9]. Further we have $B_1^{\rm QCD} = B_2^{\rm QCD} = 0$ if $\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}$ are chosen to be in the production plane due to P invariance of QCD.

## 5. Spin correlations at NLO

Within the LPA, the coefficient $C$ of Eq. (24) factorizes:

$$C = \kappa_+\kappa_- D \tag{26}$$

with the $t\bar{t}$ double spin asymmetry

$$D = \frac{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) - \sigma(\uparrow\downarrow) - \sigma(\downarrow\uparrow)}{\sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow) + \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow)}\,. \tag{27}$$

In Eq. (27), $\sigma(\uparrow\uparrow)$ denotes the hadron cross section for top quark pairs with $t(\bar{t})$ spin parallel to the chosen spin quantization axis $\hat{\boldsymbol{a}}(\hat{\boldsymbol{b}})$ *etc.* The numbers $\kappa_\pm$ are the spin analyzing powers of charged leptons in decays $t(\bar{t}) \to b(\bar{b})\ell^\pm\nu(\bar{\nu})$. The decay distribution reads

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\vartheta_\pm} = \frac{1}{2}\left(1 \pm \kappa_\pm \cos\vartheta_\pm\right)\,, \tag{28}$$

where $\vartheta_\pm$ are the angles of $\ell^\pm$ w.r.t. the $t$ ($\bar{t}$) spin.

### *5.1. Spin analysing power of top quark decay products*

If the $t$ or $\bar{t}$ quark decays hadronically, one can use other decay products as spin analyzers. One defines in analogy to Eq. (28) for $t \to bW^+ \to b\ell^+\nu$ or $bq\bar{q}'$:

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\vartheta} = \frac{1}{2}\left(1 + \kappa_f \cos\vartheta\right). \tag{29}$$

The leading order results for $\kappa_f$ are given in Table I. In order to compute the

TABLE I

Leading order results for the spin analyzing power of top quark decay products. In the last column, '$q\bar{q}'$ jet' stands for the least energetic non-$b$-quark jet in hadronic $t$ decays [15] .

| $f$ | $\ell^+,\ \bar{d},\ \bar{s}$ | $\nu,\ u$ | $b$ | $W^+$ | $q\bar{q}'$ jet |
|---|---|---|---|---|---|
| $\kappa_f$ | 1 | $-0.31$ | $-0.41$ | 0.41 | 0.51 |

spin correlation coefficient $C$ in NLO in $\alpha_{\rm s}$, we need the QCD corrections to the spin analyzing power $\kappa_f$. For leptonic decays, the corrections are tiny [16]

$$\kappa_+ = \kappa_- = 1 - 0.015\alpha_{\rm s}\,, \tag{30}$$

implying that the charged lepton is a perfect analyzer of top quark spin. However, since only about 5% of the decays of $t\bar{t}$ are purely leptonic $(e, \mu)$, it is also important to compute the QCD corrections for hadronic decays $t \to bq\bar{q}'$. The results are [17] (using $\alpha_{\rm s}(m_t) = 0.108$):

$$\kappa_{\bar{d}} \;=\; 1 - 0.57\alpha_{\rm s} = 0.94\,, \tag{31}$$

$$\kappa_b \;=\; -0.41 \times (1 - 0.34\alpha_{\rm s}) = -0.39\,, \tag{32}$$

$$\kappa_j \;=\; +0.51 \times (1 - 0.65\alpha_{\rm s}) = +0.47\,, \tag{33}$$

where $\kappa_j$ is the analyzing power of least energetic non-$b$-quark jet. One observes that the $\bar{d}, \bar{s}$-jets have the highest analyzing power, but the reconstruction of their direction is very difficult due to the low efficiency of charm tagging. A better choice is to use the $b$-jet or the least energetic non-$b$-quark jet. The NLO results for $C$ in the single lepton channel are less sensitive due to the factor $\kappa_{j,b}$, but this loss in sensitivity is overcompensated by higher statistics, since one has about 30% $(e + \mu) + X$ single lepton $t\bar{t}$ decays. (Nonleptonic top quark decays at order $\alpha_{\rm s}$ were also analyzed in Ref. [18].)

### 5.2. Double spin asymmetries at NLO at the parton level

The spin dependent cross sections that enter the double spin asymmetry (27) are calculated as a convolution which reads schematically:

$$\sigma(\uparrow\uparrow) = \mathrm{PDF's} \otimes \hat{\sigma}(\uparrow\uparrow)\,, \ldots \tag{34}$$

We renormalize the top mass in the on-shell scheme, and $\alpha_{\mathrm{s}}$ in the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme. Factorization is performed in the $\overline{\mathrm{MS}}$ scheme, and we set $\mu_{\mathrm{F}} = \mu_{\mathrm{R}} = \mu$. The results at NLO QCD for the $\overline{\mathrm{MS}}$ subtracted parton cross sections $q\bar{q} \to t\bar{t}(g)$, $gg \to t\bar{t}(g)$, and $q(\bar{q})g \to t\bar{t}q(\bar{q})$ with $t\bar{t}$ spins summed over was computed already more than 10 years ago [19–21]. It can be written as follows:

$$\begin{aligned} \hat{\sigma}(\hat{s}, m_t^2) &= \hat{\sigma}(\uparrow\uparrow) + \hat{\sigma}(\downarrow\downarrow) + \hat{\sigma}(\uparrow\downarrow) + \hat{\sigma}(\downarrow\uparrow) \\ &= \frac{\alpha_{\mathrm{s}}^2}{m_t^2}\left\{ f^{(0)}(\eta) + 4\pi\alpha_{\mathrm{s}}\left[ f^{(1)}(\eta) + \tilde{f}^{(1)}(\eta)\ln\left(\frac{\mu^2}{m_t^2}\right)\right]\right\}, \end{aligned} \tag{35}$$

where $\eta = \hat{s}/(4m_t^2) - 1$. In our calculation of the spin-dependent cross sections Refs. [22,23] we obtain these cross sections as special cases and we find perfect agreement with the results of Refs. [19–21]. An analogous decomposition can be defined for the numerator of the double spin asymmetry:

$$\begin{aligned} \hat{\sigma}\hat{D} &= \hat{\sigma}(\uparrow\uparrow) + \hat{\sigma}(\downarrow\downarrow) - \hat{\sigma}(\uparrow\downarrow) - \hat{\sigma}(\downarrow\uparrow) \\ &= \frac{\alpha_{\mathrm{s}}^2}{m_t^2}\left\{ g^{(0)}(\eta) + 4\pi\alpha_{\mathrm{s}}\left[ g^{(1)}(\eta) + \tilde{g}^{(1)}(\eta)\ln\left(\frac{\mu^2}{m_t^2}\right)\right]\right\}. \end{aligned} \tag{36}$$

The following figures display our results for the functions $g^{(0)}, g^{(1)}$ and $\tilde{g}^{(1)}$ calculated for several spin quantization axes. In Eq. (36) the coupling $\alpha_{\mathrm{s}}$ denotes the six-flavor coupling $\alpha_{\mathrm{s}}^{f=6}$. Fig. 1 shows our results for the helicity basis. In Figs. 2 and 3 corresponding results for the beam basis and the off-diagonal basis are displayed.

It is natural to express the above partonic cross sections in terms of the $\overline{\mathrm{MS}}$ coupling $\alpha_{\mathrm{s}}^{f=6}$ in $f = 6$ flavor QCD. However, for the evaluation of hadronic observables, *e.g.* Eq. (34), the parameter change $\alpha_{\mathrm{s}}^{f=6} \to \alpha_{\mathrm{s}}^{f=5}$ using the standard $\overline{\mathrm{MS}}$ relation

$$\alpha_{\mathrm{s}}^{f=6}(\mu_{\mathrm{R}}) = \alpha_{\mathrm{s}}^{f=5}(\mu_{\mathrm{R}})\left[1 - \frac{1}{3\pi}\alpha_{\mathrm{s}}^{f=5}(\mu_{\mathrm{R}})\ln\left(\frac{m_t}{\mu_{\mathrm{R}}}\right) + \mathcal{O}(\alpha_{\mathrm{s}}^2)\right] \tag{37}$$

is necessary, in order to make contact with the physics and formalism incorporated in the PDF libraries. The evolution of $\alpha_{\mathrm{s}}^{f=5}$ is determined by the beta function with $n_f^{\mathrm{light}} = 5$ flavors.

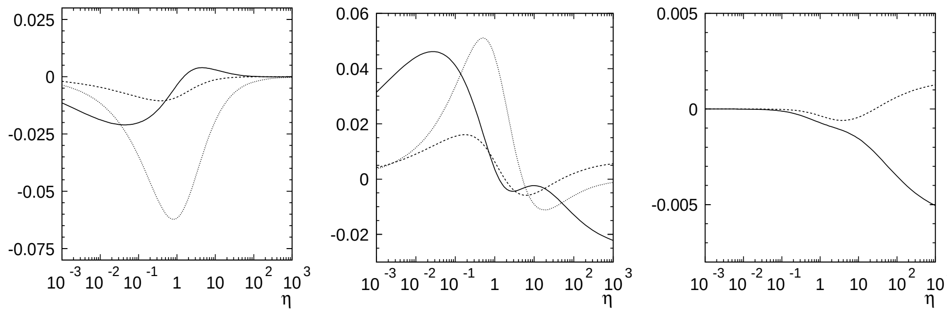


Fig. 1. Left: Scaling functions $g^{(0)}(\eta)$ (dotted), $g^{(1)}(\eta)$ (full), and $\tilde{g}^{(1)}(\eta)$ (dashed) in the helicity basis for the process $q\bar{q} \to t\bar{t}(g)$. Middle: The same for the process $gg \to t\bar{t}(g)$. Right: The functions $g^{(1)}(\eta)$ (full), and $\tilde{g}^{(1)}(\eta)$ (dashed) for the process $qg \to qt\bar{t}$ [22, 23].

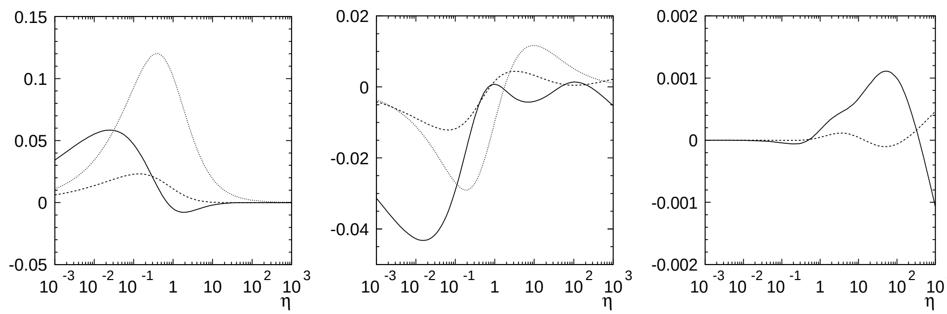


Fig. 2. The same as Fig. 1, but for the beam basis [22, 23].

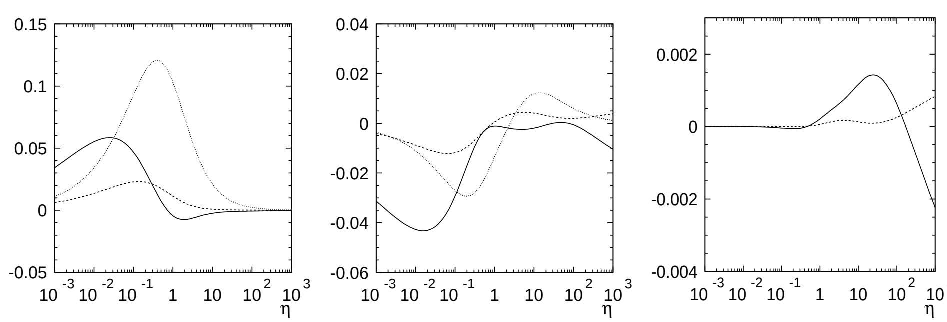


Fig. 3. The same as Fig. 1, but for the off-diagonal basis [22, 23].

### 5.3. NLO results for differential decay distributions

We now turn to the coefficient $C$ in the double angular distribution (24). For definiteness we discuss the spin correlation $C$ only for the dilepton channels. These results were obtained in Ref. [24]. The numbers in Table II are obtained for $\mu_{\rm F} = \mu_{\rm R} = m_t$ =175 GeV and using the PDF's CTEQ5L (LO) and CTEQ5M (NLO) [25]. At the Tevatron, the dilepton spin correlations

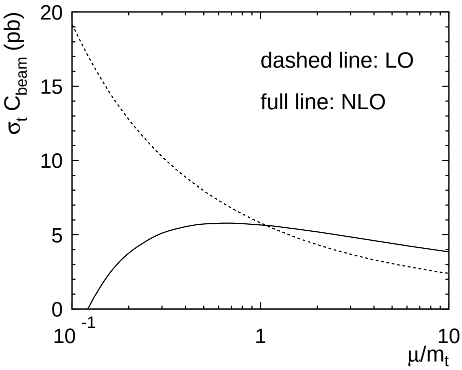
TABLE II

Coefficient $C$ of the double distribution (24) to leading and next-to-leading order in $\alpha_{\rm s}$ for the helicity basis, the beam basis (where the proton beam is taken as the spin quantization axis) and the off-diagonal basis. The parton distribution functions of Ref. [25] were used choosing the renormalization scale $\mu_{\rm R}$ equal to the factorization scale $\mu_{\rm F} = m_t = 175$ GeV.

| | $p\bar{p}$ at $\sqrt{s} = 2$ TeV | | $pp$ at $\sqrt{s} = 14$ TeV | |
|---|---|---|---|---|
| | LO | NLO | LO | NLO |
| $\mathrm{C_{hel.}}$ | $-0.456$ | $-0.389$ | 0.305 | 0.311 |
| $\mathrm{C_{beam}}$ | 0.910 | 0.806 | $-0.005$ | $-0.072$ |
| $\mathrm{C_{off.}}$ | 0.918 | 0.813 | $-0.027$ | $-0.089$ |

are large in the beam and the off-diagonal basis. Thus one sees there is practically no difference between these two choices as far as the sensitivity to QCD-induced spin correlations is concerned; yet the beam basis may be simpler to implement in the analysis of experimental data. The QCD corrections are about $-10\%$. At the LHC, the beam and off-diagonal bases are bad choices (due to the dominance of $gg \to t\bar{t}$). Here the helicity basis is a good choice, and the QCD corrections are small. The inclusion of the QCD corrections reduces the dependence of the $t\bar{t}$ cross section on the renormalization and factorization scales significantly. The same is true for the product $\sigma C$, as can be seen from Fig. 4.

Table III shows the dependence of the NLO results for $C$ on the scale $\mu$ (upper part) and on the choice of the PDFs (lower part). At the Tevatron the spread of results for different PDFs is larger than the scale uncertainty: The results for $C$ using the CTEQ5 and MRST98 distributions agree up to a few percent, but the difference between GRV98 and MRST98 at the Tevatron is about 10%. The main reason for this strong dependence on the PDFs is that the contributions from the $gg$ and the $q\bar{q}$ initial state enter with a different sign. This offers the interesting possibility to constrain the PDFs by measuring $t\bar{t}$ spin correlations.



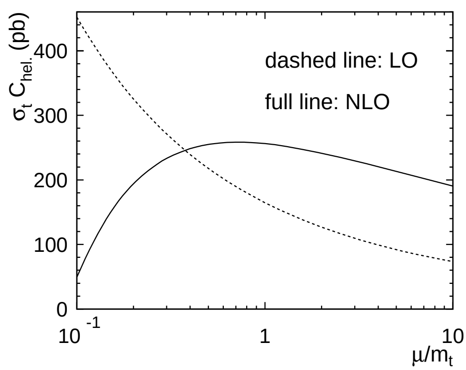




Fig. 4. Left: Dependence of $\sigma C_{\rm beam}$ at LO (dashed line) and at NLO (solid line) on $\mu = \mu_{\rm R} = \mu_{\rm F}$ for $p\bar{p}$ collisions at $\sqrt{s} = 2$ TeV, with PDF of Ref. [25]. Right: Same, but for $\sigma C_{\rm hel.}$ for $pp$ collisions at $\sqrt{s} = 14$ TeV.

TABLE III

Upper part: Dependence on the scale $\mu$ of the correlation coefficients $C$, computed at NLO with the PDF of Ref. [25]. Lower part: Correlation coefficients $C$ at NLO for $\mu = m_t$ and different sets of parton distribution functions: GRV98 [26], CTEQ5 [25], and MRST98 (c–g) [27].

| | Tevatron | | | LHC |
|---|---|---|---|---|
| $\mu$ | $C_{\rm hel.}$ | $C_{\rm beam}$ | $C_{\rm off.}$ | $C_{\rm hel.}$ |
| $m_t/2$ | $-0.364$ | 0.774 | 0.779 | 0.278 |
| $m_t$ | $-0.389$ | 0.806 | 0.813 | 0.311 |
| $2m_t$ | $-0.407$ | 0.829 | 0.836 | 0.331 |
| PDF | $C_{\rm hel.}$ | $C_{\rm beam}$ | $C_{\rm off.}$ | $C_{\rm hel.}$ |
| GRV98 | $-0.325$ | 0.734 | 0.739 | 0.332 |
| CTEQ5 | $-0.389$ | 0.806 | 0.813 | 0.311 |
| MRST98 | $-0.417$ | 0.838 | 0.846 | 0.315 |

In all results above we used $m_t = 175$ GeV. A variation of $m_t$ from 170 to 180 GeV changes the results for the Tevatron, again for $\mu = m_t$ and PDFs of Ref. [25] as follows: $C_{\rm hel.}$ varies from $-0.378$ to $-0.397$, $C_{\rm beam}$ from 0.790 to 0.817, and $C_{\rm off.}$ from 0.797 to 0.822. For the LHC, $C_{\rm hel.}$ changes by less than a percent.

The results above have been obtained without imposing any kinematic cuts. Standard cuts on the top quark transverse momentum and rapidity only have a small effect on $C$: For the Tevatron, demanding $|\boldsymbol{k}^{\rm T}_{t,\bar{t}}| > 15$ GeV and $|r_{t,\bar{t}}| < 2$ leads to the following results: $C_{\rm hel.} = -0.386, C_{\rm beam} = 0.815$, $C_{\rm off.} = 0.823$. For the LHC, when imposing the cuts $|\boldsymbol{k}^{\rm T}_{t,\bar{t}}| > 20$ GeV and $|r_{t,\bar{t}}| < 3$, we find $C_{\rm hel.} = 0.295$.

A tool for simulating these spin correlations at leading order in the QCD coupling exists [28]. Apart from the double differential distribution (24) there are also other distributions which are sensitive to spin correlations [29]. Finally let us mention that there has been a first measurement of spin correlations in the off-diagonal basis by the D0 collaboration [30]. It is based on six dilepton events from Run I. They find

$$C_{\rm off.} > -0.25 \quad @ \quad 68\% \text{ confidence level}\,. \tag{38}$$

This demonstrates that top quark spin correlations can be studied already at the Tevatron. It is expected that the correlations can be established at the $2\sigma$ level using Run II data.

## 6. Conclusions

QCD-induced spin correlations of top quark pairs produced at hadron colliders are large effects. They can be studied at the Tevatron and LHC. For the Tevatron the QCD corrections to the leading order predictions are sizeable but under control. The degree of correlation depends on the choice of the spin quantization axis. We have shown that using, at the Tevatron, the direction of one of the hadron beams as spin quantization axis is as efficient as the off-diagonal axis which has received much attention in the literature. Spin correlations are suited to study in detail the interactions of top quarks. As we have pointed out they should, in particular, be a useful tool for constraining PDFs. Taking the PDFs, once they have been determined with sufficient precision, as input, spin correlations will be an important tool for the search for new effects in top quark pair production. Future work on the theory side will include the implementation of the NLO matrix elements in an event generator, a study of non-factorizable corrections, and the resummation of Sudakov-type large logarithms for the spin-dependent cross sections.

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