TOP PAIR PRODUCTION AT THRESHOLD AND EFFECTIVE THEORIES *

André H. Hoang

Max-Planck-Institut für Physik, (Werner-Heisenberg-Institut)
Föhringer Ring 6, 80805 München, Germany

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I give an introduction to the effective field theory description of top pair production at threshold in e^+e^- annihilation. The impact of the summation of logarithms of the top quark velocity including most recent results at next-to-next-to-leading order is discussed.

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1. Introduction

The so-called "threshold scan" of the total cross section lineshape of top pair production constitutes a major part of the top quark physics program at a future e^+e^- collider [1]. In the Standard Model the top quark width $\Gamma_t \approx 1.5$ GeV is much larger than the typical hadronization energy $\Lambda_{\rm QCD}$. In contrast to the J/ψ or the Υ region, it is therefore expected that non-perturbative effects are strongly suppressed, and that the threshold lineshape is a smooth function of the c. m. energy. Throughout this talk I will therefore neglect non-perturbative effects associated with the hadronization scale $\Lambda_{\rm QCD}$. From the location of the rise of the cross section a precise measurement of the top quark mass will be possible, while from the shape and the normalization of the cross section one can extract the top quark Yukawa coupling y_t (for a light Higgs), the top width or the strong coupling. In the past, numerous studies have been carried out to assess the feasibility and precision for extracting various top quark properties from a threshold run (see *e.g.* Ref. [2] for a recent study).

In the threshold region, $\sqrt{s} \simeq 2m_t \pm 10\,\text{GeV}$, the top quarks move with nonrelativistic velocity in the c.m. frame. Let us define $m_t v^2 \equiv \sqrt{s} - 2m_t$. We see that parametrically $|v| \lesssim \alpha_{\rm s}$. Because in the loop expansion one

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encounters terms proportional to $(\alpha_s/v)^n$ $(n = 0, 1, \ldots)$ in the amplitude, it is necessary to count α_s/v of order one and to carry out an expansion in α_s as well as in v. Schematically one needs an expansion of $\sigma_{t\bar{t}} = \sigma(e^+e^- \to t\bar{t})$ of the form

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_k \left(\frac{\alpha_s}{v}\right) \times \left\{ 1 \text{ (LO)} ; \quad \alpha_s, v \text{ (NLO)} ; \alpha_s^2, \alpha_s v, v^2 \text{ (NNLO)} \right\}, \tag{1}$$

where α_s/v is counted as order one. The indicated terms are of leading order (LO), next-to-leading order (NLO), and next-to-next-to-leading order (NNLO). I call the expansion scheme in Eq. (1) *fixed-order perturbation theory*, although it involves the summations of the terms proportional to $(\alpha_s/v)^n$. It can be implemented systematically using the factorization properties and the power counting of non-relativistic QCD (NRQCD) [3]. The NNLO QCD corrections to the total cross section were calculated some time ago in Refs. [4–10]. Surprisingly, the corrections were found to be as large as the next-to-leading order (NLO) QCD corrections, and from the residual scale dependence in the NNLO result, the normalization of the cross section was estimated to have at least 20% uncertainty [11]. It was concluded that the top quark mass in a threshold mass scheme can be determined with a precision of 200 MeV or better from the shape of the cross section [11]. However, the large NNLO QCD corrections to the normalization of the cross section jeopardized competitive measurements of the top width, strong top coupling, or the top Yukawa coupling. Moreover, the large NNLO corrections seemed to indicate that, despite the perturbative nature of the $t\bar{t}$ system, high precision computations might not be feasible. The complete determination of higher orders would be necessary to clarify the feasibility of the fixed-order approach. Figure 3(a) shows the vector-current-induced cross section $\sigma(e^+e^- \to \gamma^* \to t\bar{t})$ at LO, NLO and NNLO in fixed-order perturbation theory for typical choices of parameters and renormalization scales. (Top threshold production mediated by the axial vector current is suppressed by v^2 and small and not relevant here for the discussions on the convergence of the perturbative expansion. See *e.g.* Refs. [11, 12].)

One way to understand the large scale uncertainties and the large size of the fixed-order NNLO QCD corrections is to recall that the perturbative $t\bar{t}$ threshold dynamics is governed by vastly different energy scales, the top mass ($m_t \sim 175$ GeV), the top three-momentum ($\boldsymbol{p} \simeq mv \simeq 25$ GeV) and the top kinetic or potential energy ($E \simeq mv^2 \simeq 4$ GeV). This hierarchy of scales is the basis of NRQCD factorization, which separates hard and soft effects, and which can be implemented in the fixed-order scheme shown above. However, in fixed-order perturbation theory NRQCD matrix elements involve logarithmic terms such as

$$\ln\left(\frac{\mu^2}{m_t^2}\right), \qquad \ln\left(\frac{\mu^2}{\boldsymbol{p}^2}\right), \qquad \ln\left(\frac{\mu^2}{E^2}\right), \tag{2}$$

which cannot be rendered small for a single choice of the renormalization scale μ. For example, since $E \sim 4\,\mathrm{GeV}$, one finds $\alpha_{\mathrm{s}}(m_t)\ln(m_t^2/E^2) \simeq 0.8$ for $\mu = m_t$ which is of order unity, and fixed-order perturbation theory becomes unreliable. Moreover, at the higher orders, fixed-order perturbation theory cannot distinguish the scale at which to evaluate α_{s}. Mistaking an $\alpha_{\mathrm{s}}(m_t)$ for an $\alpha_{\mathrm{s}}(m_t v^2)$ is a difference of a factor of two. Both problems cannot be addressed systematically within the framework of fixed-order perturbation theory. But they can be avoided in a framework that allows for renormalization group improved perturbative computations, where *all* logarithmic terms are summed through renormalization group equations. Thus a better expansion should have the schematic form

$$\begin{aligned} R \;=\; \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} &= v\sum_k \left(\frac{\alpha_{\mathrm{s}}}{v}\right)\sum_i (\alpha_{\mathrm{s}}\ln v)^i \\ &\times\Big\{\, 1\ (\mathrm{LL});\ \alpha_{\mathrm{s}}, v\,(\mathrm{NLL});\ \alpha_{\mathrm{s}}^2, \alpha_{\mathrm{s}} v, v^2\,(\mathrm{NNLL})\,\Big\}, \end{aligned} \tag{3}$$

where the indicated terms are of leading logarithmic (LL), next-to-leading logarithmic (NLL), and next-to-next-to-leading logarithmic (NNLL) order. I have used the expression "better expansion" because in QCD computations one should favor expansion schemes where logarithmic terms are summed into coefficients and not contained in isolated form in matrix elements. This is even true in cases where the non-logarithmic corrections are sizeable too, because possible cancellations between large logarithmic and large non-logarithmic terms can be unphysical. To accomplish a renormalization group improved calculation one needs to employ a more sophisticated effective field theory approach than the one that is used for the fixed-order computations.

In this talk I give a basic introduction to "velocity NRQCD" (vNRQCD) Ref. [12–15] (for a review see also Ref. [16]), which is an effective theory for heavy non-relativistic quark pairs that, through renormalization, sums all logarithms involving ratios of the scales m_t, $|\boldsymbol{p}|$ and E. The program is similar in spirit to let us say summing QCD logarithms of M_W/m_b for the electroweak Hamiltonian describing b-quark decays. However, in the non-relativistic case the program is more complicated because the structure of the relevant degrees of freedom in the effective theory action is more involved and because the two low energy scales $\boldsymbol{p} \sim mv$ and $E \sim mv^2$ are correlated through the heavy quark equation of motion $E = \boldsymbol{p}^2/m$. At first sight it seems impossible to keep this correlation of scales and to render the logarithms in the effective theory matrix elements shown in Eq. (2) small (which is equivalent to saying that all logarithms are summed into Wilson coefficients of the effective theory). In the effective theory this problem is dealt with by using two renormalization group scales in the effective Lagrangian: μ_S for soft gluons, which fluctuate at energies $\sim mv$, and μ_U for ultrasoft gluons, which fluctuate at energies $\sim mv^2$. The matrix elements of the effective theory then only contain the logarithms $\ln(\mu_S^2/\boldsymbol{p}^2)$ and $\ln(\mu_U^2/E^2)$. Both renormalization scales are related by $\mu_U = \mu_S^2/m_t = m_t\nu^2$, where ν is a dimensionless parameter. All renormalization group equations of the effective theory are expressed in terms of ν. Running from $\nu = 1$ to $\nu \sim v$, where v is the typical top quark velocity, sums all logarithms of v and minimizes both $\ln(\mu_S^2/\boldsymbol{p}^2)$ and $\ln(\mu_U^2/E^2)$ in the matrix elements. (Logarithms involving m_t are minimized by matching QCD onto the effective theory at $\nu = 1$.) In dimensional regularization the factors of μ_S^ϵ and μ_U^ϵ multiplying each operator in the effective theory action are uniquely determined from its mass dimension and v power counting. In this way the scheme indicated in Eq. (3) can indeed be achieved.

With this setup and after having formulated the effective theory action the program is similar the one for summing QCD logarithms for the electroweak Hamiltonian:

1. Matching computation of the coefficients $C_i(\nu)$ of the effective theory operators (including external currents) at $\nu = 1$ in a perturbation series in $\alpha_{\rm s}(m_t)$.

2. Computation of the anomalous dimensions of the operators of the effective action and scaling of $C_i(\nu)$ from $\nu = 1$ to $\nu = v_0 \simeq v \simeq C_F\alpha_{\rm s}$ using the renormalization group.

3. Computation of the cross section using the effective Lagrangian and the currents renormalized at the low scale $\nu = v_0$.

In this talk I show how the vNRQCD effective Lagrangian is constructed, and I also spend some time on the points 1.–3. described above including new recent results for the total top pair production cross section at threshold in e^+e^- annihilation. At the end I will briefly discuss the status of top pair production at threshold.

2. The vNRQCD Lagrangian

The physical system we wish to describe is that of a heavy quark and antiquark with mass m_t, and energies $E \sim mv^2$, and momenta $\boldsymbol{p} \sim mv$ in the c.m. system where $v \ll 1$. The degrees of freedom from which we have to build the effective action can be identified from the relevant momentum regions that can be found in an asymptotic expansion of non-relativistic scattering diagrams in the c.m. frame [17]. The regions include hard modes with momenta $(k_0, \boldsymbol{k}) \sim (m_t, m_t)$, soft modes with momenta $\sim (m_t v, m_t v)$, potential modes with momenta $\sim (m_t v^2, m_t v)$ and ultrasoft modes with momenta $\sim (m_t v^2, m_t v^2)$. Fluctuations with off-shell momenta are integrated out because they do not resonate, while for modes that can resonate fields are introduced from which we build the effective Lagrangian. Resonating heavy quarks can only live in the potential regime, while massless modes can live either in the soft or the ultrasoft regime. Thus the effective Lagrangian is build from heavy potential quarks and antiquarks ($\psi_{\boldsymbol{p}}$, $\chi_{\boldsymbol{p}}$), soft gluons, ghosts, and massless quarks (A_q^μ, c_q, φ_q) and ultrasoft gluons, ghosts, and massless quarks (A^μ, c, φ_{us}). The ultrasoft gluons are the gauge partners of momenta $\sim mv^2$, while soft gluons are the gauge partners of momenta $\sim mv$. This classification is, of course, only meaningful in the c.m. frame for a non-relativistic heavy quark-antiquark pair. Double counting is avoided since ultrasoft gluons reproduce only the physical gluon poles where $k^0 \sim \boldsymbol{k} \sim mv^2$, while soft gluons only have poles with $k^0 \sim \boldsymbol{k} \sim mv$. It is essential that both soft and ultrasoft gluons are included at all scales below m_t because the heavy quark equation of motion correlates the soft and ultrasoft scales. We will also see later in Sec. 3, when I show new results for the 3-loop (NNLL) anomalous dimension for the $t\bar{t}$ production current, that both soft and ultrasoft running can in general feed into anomalous dimensions induced by potential loops. The dependences on soft energies and momenta of potential and soft fields appear as labels on the fields, while only the lowest-energy ultrasoft fluctuations are associated by an explicit coordinate dependence. Formally this is achieved by a phase redefinition for the potential and soft fields [13]

$$\phi(x) \to \sum_k \mathrm{e}^{-ik\cdot x} \phi_k(x) \,, \tag{4}$$

where k denotes momenta $\sim mv$ and $\partial^\mu \phi_k(x) \sim mv^2 \phi_k(x)$. This means that the effective Lagrangian contains sums over fields with soft indices, which also build up potential and soft loop integrations when we renormalize the theory or compute matrix elements.

The effective vNRQCD Lagrangian for a $t\bar{t}$ angular momentum S-wave and color singlet state has terms [13–15]

$$\begin{aligned}\mathcal{L} &= \sum_{\boldsymbol{p}} \left\{ \psi_{\boldsymbol{p}}^\dagger \left[iD^0 - \frac{(\boldsymbol{p}-i\boldsymbol{D})^2}{2m} + \frac{\boldsymbol{p}^4}{8m^3} + \ldots \right] \psi_{\boldsymbol{p}} + (\psi \to \chi) \right\} - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} \\ &\quad -\mu_S^{2\epsilon} g_s^2 \sum_{\boldsymbol{p},\boldsymbol{p}',q,q',\sigma} \left\{ \frac{1}{2} \psi_{\boldsymbol{p}'}^\dagger [A_{q'}^\mu, A_q^\nu] U_{\mu\nu}^{(\sigma)} \psi_{\boldsymbol{p}} + (\psi \to \chi) + \ldots \right\} \\ &\quad + \sum_p \left| p^\mu A_p^\nu - p^\nu A_p^\mu \right|^2 + \ldots - \sum_{\boldsymbol{p},\boldsymbol{p}'} \mu_S^{2\epsilon} V(\boldsymbol{p},\boldsymbol{p}')\, \psi_{\boldsymbol{p}'}^\dagger \psi_{\boldsymbol{p}} \chi_{-\boldsymbol{p}'}^\dagger \chi_{-\boldsymbol{p}} \\ &\quad + \sum_{\boldsymbol{p},\boldsymbol{p}'} \frac{2i\mu_S^{2\epsilon} \mathcal{V}_c}{(\boldsymbol{p}'-\boldsymbol{p})^4} f^{ABC} (\boldsymbol{p}-\boldsymbol{p}').(\mu_U^\epsilon g_u \mathbf{A}^C) [\psi_{\boldsymbol{p}'}^\dagger T^A \psi_{\boldsymbol{p}} \chi_{-\boldsymbol{p}'}^\dagger \bar{T}^B \chi_{-\boldsymbol{p}}] + \ldots\,, \end{aligned} \tag{5}$$

where color and spin indices have been suppressed and $g_s \equiv g_s(m\nu)$, $g_u \equiv g_s(m\nu^2)$. All coefficients are functions of the renormalization parameter ν, and all explicit soft momentum labels are summed. The covariant derivative in the first line contains only the ultrasoft gluon field. There are 4-quark potential-like interactions of the form ($\boldsymbol{k} = (\boldsymbol{p}-\boldsymbol{p}')$)

$$V(\boldsymbol{p},\boldsymbol{p}') \;=\; \frac{\mathcal{V}_c}{\boldsymbol{k}^2} + \frac{\mathcal{V}_r(\boldsymbol{p}^2+\boldsymbol{p}'^2)}{2m_t^2\boldsymbol{k}^2} + \frac{\mathcal{V}_2}{m_t^2} + \frac{\mathcal{V}_s}{m_t^2}\boldsymbol{S}^2\,, \tag{6}$$

where $\boldsymbol{S}$ is the total $t\bar{t}$ spin operator. There is some freedom in the choice of the operator basis for the potential interactions shown in Eq. (6), which can affect the matching conditions and the anomalous dimensions. In Refs. [12, 18,19] we also used a potential interaction term of the form $1/(m|\boldsymbol{k}|)$, while in Ref. [20] the order $1/(m|\boldsymbol{k}|)$ potentials were implemented only by potential-like interactions with additional sums over intermediate indices as explained below. At NNLL order for the total cross section the coefficient $\mathcal{V}_c$ of the $1/\boldsymbol{k}^2$ potential has to be matched at two loops [21] because it contributes at the LL level, whereas the coefficients of the order $1/m_t^2$ potentials have to matched at the Born level [14]. The $1/(m|\boldsymbol{k}|)$-type potentials are of order α_s^2 and have to be matched at two loops [19]. There are also 4-quark interactions with the radiation of an ultrasoft gluon (last line) and interactions between quarks and soft gluons (second line). Due to momentum conservation at least two soft gluons are required. The potential terms shown in Eq. (6), and time-ordered products of soft interactions contribute to the potentials that describe the instantaneous interactions between the top quarks. For example, the time-ordered product at two loops of two soft interactions at leading order in the v-counting leads to the potential interaction [21]

$$\tilde{V}_{\rm soft}(\boldsymbol{p},\boldsymbol{q}) \;=\; -\,\frac{4\pi C_F\,\alpha_{\rm s}(\mu_S)}{\boldsymbol{k}^2}\,\Bigg\{\,\frac{\alpha_{\rm s}(\mu_S)}{4\pi}\,\bigg[\,-\,\beta_0\,\ln\Big(\frac{\boldsymbol{k}^2}{\mu_S^2}\Big)+a_1\,\bigg] + \Big(\frac{\alpha_{\rm s}(\mu_s)}{4\pi}\Big)^2\,\bigg[\beta_0^2\,\ln^2\Big(\frac{\boldsymbol{k}^2}{\mu_S^2}\Big)-\Big(2\,\beta_0\,a_1+\beta_1\Big)\,\ln\Big(\frac{\boldsymbol{k}^2}{\mu_S^2}\Big)+a_2\,\bigg]\,\Bigg\}\,, \quad (7)$$

where the β_i are the coefficients of the QCD beta-function and the a_i were determined some time ago in Ref. [22]. The sum of Eq. (7) and the first term in Eq. (6) agrees with the static potential[1] and constitutes the complete $1/\boldsymbol{k}^2$ potential at NNLL order. The Lagrangian also contains more complicated interactions describing 4-quark potential-like interactions and 4-quark interactions with soft or ultrasoft gluons that contain additional sums over intermediate indices. Their form is partly fixed by the renormalizability of the theory [19], but there is still some freedom in the choice of their form. In this talk I use the conventions of Ref. [20] were all $1/(m|\boldsymbol{k}|)$-type potentials are represented by such 4-quark sum operators. I do not display their expressions here and refer to Ref. [20] where they are collected in an appendix.

One might wonder why the effective theory contains both soft and ultrasoft degrees of freedom at the same time, since, naively, one might argue that for $v \ll 1$ the soft fields fluctuate at a much smaller length scale than the ultrasoft fields. That both soft and ultrasoft degrees of freedom have to be present at the same time is at the heart of the effective theory construction and is related to the fact that the dispersion relation of the heavy quarks, $E = \boldsymbol{p}^2/m$ introduces a correlation between the soft and ultrasoft scales. The presence of soft and ultrasoft degrees of freedom also allows for the simultaneous summation of logarithms of $\boldsymbol{p}$ and E, which I mentioned in the introduction, after the theory is renormalized. I will come back to this point in the next section.

An important point to mention is that the factors of μ_U^ϵ and μ_S^ϵ multiplying the operators in the effective Lagrangian are uniquely determined by the mass dimension and the v power counting in $d = 4-2\epsilon$ dimensions [15]. Each field in the action is assigned a certain scaling with v to keep its kinetic term of order v^0. This results for example in $\psi_{\boldsymbol{p}} \sim (mv)^{3/2-\epsilon}$, $A^\mu \sim (mv^2)^{1-\epsilon}$, and $A_q^\mu \sim (mv)^{1-\epsilon}$. Then, since $D^\mu \sim mv^2$, the renormalized combination $g_u A^\mu$ must be multiplied by $\mu_U^\epsilon \sim (m\nu^2)^\epsilon$ for $\nu \sim v$ so that this gluon

[1] The statement made in Refs. [16, 21] that the vNRQCD $1/\boldsymbol{k}^2$ potential at NNLL order differs from the static potential has shown to be incorrect in Ref. [19].

term also scales consistently as mv^2. Applying the same procedure to all interactions results in the factors of μ_U^ϵ and μ_S^ϵ shown in Eq. (5).

To incorporate the effect of the large top quark width we include in the effective Lagrangian the operators

$$\delta\mathcal{L} = \sum_{\boldsymbol{p}} \psi_{\boldsymbol{p}}^\dagger \,\frac{i}{2}\Gamma_t\, \psi_{\boldsymbol{p}} \;+\; \sum_{\boldsymbol{p}} \chi_{\boldsymbol{p}}^\dagger \,\frac{i}{2}\Gamma_t\, \chi_{\boldsymbol{p}}\,. \tag{8}$$

Here, Γ_t is the total on-shell top quark width and treated as a fixed number to be determined from experiment (or taken as an input). The typical energy of the top quarks at threshold is $E \sim mv^2 \sim 4\,\mathrm{GeV}$, and in the Standard Model one can take $\Gamma_t = 1.43\,\mathrm{GeV} \sim E$. Thus, the propagator for a single top (or antitop) with momentum $(p^0, \boldsymbol{p})$ is

$$\frac{i}{p^0 - \boldsymbol{p}^2/(2m) + i\Gamma_t/2 + i\epsilon}\,, \tag{9}$$

which gives a consistent NLO treatment of electroweak effects. [23] The complete NNLL order treatment of electroweak effects for the total cross section is currently unknown. Although it is not expected that the unknown electroweak corrections are beyond the few percent level [9], this has to be kept in mind when I discuss the NNLL QCD corrections.

Besides the interactions contained in the effective Lagrangian that describe the dynamics of the $t\bar{t}$ pair we also need external currents that describe the production of the top quarks. For e^+e^- annihilation these currents are induced by the exchange of a virtual photon or a Z boson At NNLL order we need the vector S-wave currents $\boldsymbol{J}_{\boldsymbol{p}}^v = c_1(\nu)\boldsymbol{O}_{\boldsymbol{p},1} + c_2(\nu)\boldsymbol{O}_{\boldsymbol{p},2}$, where

$$\begin{aligned}
\boldsymbol{O}_{\boldsymbol{p},1} \;&=\; \boldsymbol{p}^\dagger\, \boldsymbol{\sigma}(i\sigma_2)\,\chi_{-\boldsymbol{p}}^*\,, \\
\boldsymbol{O}_{\boldsymbol{p},2} \;&=\; \frac{1}{m^2}\,{\psi_{\boldsymbol{p}}}^\dagger\, \boldsymbol{p}^2\boldsymbol{\sigma}(i\sigma_2)\,\chi_{-\boldsymbol{p}}^*\,,
\end{aligned} \tag{10}$$

and the axial-vector P-wave current $\boldsymbol{J}_{\boldsymbol{p}}^a = c_3(\nu)\boldsymbol{O}_{\boldsymbol{p},3}$, where

$$\boldsymbol{O}_{\boldsymbol{p},3} \;=\; \frac{-i}{2m}\,{\psi_{\boldsymbol{p}}}^\dagger\,[\,\boldsymbol{\sigma}, \boldsymbol{\sigma}\cdot\boldsymbol{p}\,]\,(i\sigma_2)\,\chi_{-\boldsymbol{p}}^*\,. \tag{11}$$

Note that the currents have a soft momentum index. The currents $\boldsymbol{O}_{\boldsymbol{p},2}$ and $\boldsymbol{O}_{\boldsymbol{p},3}$ lead to contributions in the total cross section that are v^2-suppressed with respect to those of the current $\boldsymbol{O}_{\boldsymbol{p},1}$. Thus, at NNLL order, two-loop matching is needed for c_1 and Born level matching for c_2 and c_3. Note that the two-loop matching result for $c_1(\nu = 1)$ depends on the choice of the operator basis used in the effective Lagrangian.

3. Anomalous dimensions and renormalization group scaling

To sum the logarithmic terms $(\alpha_s \ln\nu)^i$ at NNLL order, as indicated schematically in Eq. (3), one needs to determine the anomalous dimensions of the potentials and currents that contribute to the cross section at the appropriate order. The computation of the anomalous dimensions in the effective theory works just as for the effective weak Hamiltonian. The coefficient of the $1/\boldsymbol{k}^2$ (Coulomb) potential, $\mathcal{V}_c$, needs to be determined at the three-loop level since it contributes already at leading order. The results are available in Ref. [19]. The anomalous dimensions of the $1/m_t^2$ potentials, $\mathcal{V}_{r,2,s}$ only need to be determined at one-loop because the $1/m_t^2$ potentials are suppressed by v^2 and they are only needed at LL order. The results have been given in Refs. [14, 19]. (See also Ref. [24].) For the $1/(m|\boldsymbol{k}|)$ potentials, on the other hand, the anomalous dimensions have to determined at two-loops. Also this work has been achieved [19]. For the current $\boldsymbol{O}_{\boldsymbol{p},1}$ one needs to compute the anomalous dimension at three loops and for $\boldsymbol{O}_{\boldsymbol{p},2}$ and $\boldsymbol{O}_{\boldsymbol{p},3}$, they are needed at the one-loop level. The LL anomalous dimensions of $\boldsymbol{O}_{\boldsymbol{p},2}$ and $\boldsymbol{O}_{\boldsymbol{p},3}$ were computed in Refs. [12,18]. I will not give any details on these results in this talk since they are already available in the literature for some time.

Here I want to discuss some aspects of the anomalous dimension of the current $\boldsymbol{O}_{\boldsymbol{p},1}$, which is currently only fully known to NLL order, but for which I have recently obtained a partial NNLL order result [20]. The NNLL order results I will discuss are also of conceptual interest as far as the construction and the renormalizability of the effective theory is concerned.

To obtain the anomalous dimension of $\boldsymbol{O}_{\boldsymbol{p},1}$, one needs to determine the renormalization constant of the current. Using dimensional regularization and the $\overline{\text{MS}}$ scheme one can define the unrenormalized Wilson coefficient as

$$c_1^0 \,=\, c_1 \,+\, \delta c_1 \,=\, Z_{c_1}\, c_1 \,, \tag{12}$$

and one can write the renormalization constant as

$$Z_{c_1} \,=\, 1 \,+\, \frac{\delta z_{c_1}^{\rm NLL}}{\epsilon} \,+\, \Big(\frac{\delta z_{c_1}^{\rm NNLL,2}}{\epsilon^2} \,+\, \frac{\delta z_{c_1}^{\rm NNLL,1}}{\epsilon} \Big) \,+\, \ldots \,. \tag{13}$$

At LL order there are no UV divergences that have to be absorbed into the current. This means that at LL order the Wilson coefficient c_1 does not run.

To determine the N^kLL order renormalization constant of the current $\boldsymbol{O}_{\boldsymbol{p},1}$ one has to compute the overall UV-divergences of quark-antiquark-to-vacuum *on-shell* matrix elements of spin-triplet S-wave currents at $\mathcal{O}(\alpha_s^{k+1})$. All lower order UV-subdivergences have to be subtracted by lower order counterterms. It is mandatory to use matrix elements where the external quarks are on-shell because for off-shell quarks one can obtain UV-divergences which do not belong to the current. This is because the current

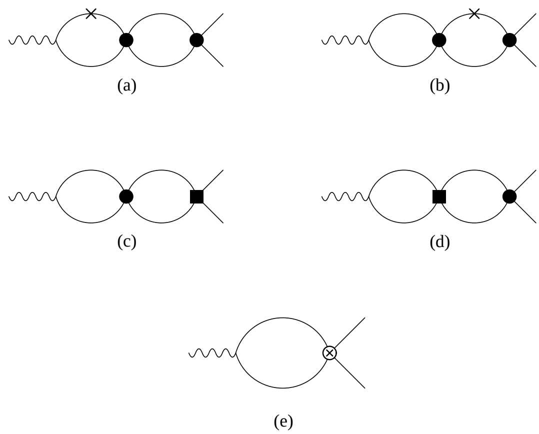

Fig. 1. Order α_s^2 vertex diagrams diagrams for the computation of the NLL anomalous dimension of c_1. The dot and box represent single insertions of the LL $1/\boldsymbol{k}^2$ and the $1/m_t^2$ suppressed potentials, respectively. The circled cross denotes a single insertion of the $1/(m|\boldsymbol{k}|)$-type potentials. Insertions of these potential involve only a one-loop diagram because the coefficients of the $1/(m|\boldsymbol{k}|)$-type potentials are of order α_s^2.

is supposed to describe the production of quarks that are fluctuating close to their mass-shell. (Such off-shell terms in fact exist at NNLL order.)

Let me first review the NLL result which was first obtained in Ref. [13]. The relevant order α_s^2 vertex diagrams are shown in Fig. 1. There are no one-loop subdivergences that have to be subtracted. The computation is complicated technically by the fact that there are IR-divergent Coulomb phases for on-shell external quarks which have to be distinguished from the UV-divergences that go into the renormalization of the current. At NLL order this distinction can be carried out easily because only $1/\epsilon$ singularities occur. From the UV-divergences one obtains

$$\begin{aligned}\delta z_{c_1}^{\rm NLL} \;=\; & -\,\frac{\mathcal{V}_c^{(s)}(\nu)}{64\pi^2}\left[\frac{\mathcal{V}_c^{(s)}(\nu)}{4}+\mathcal{V}_2^{(s)}(\nu)+\mathcal{V}_r^{(s)}(\nu)+\boldsymbol{S}^2\,\mathcal{V}_s^{(s)}(\nu)\right]\\ & +\alpha_s^2(m\nu)\left[\frac{C_F}{2}(C_F-2\,C_A)\right]+\alpha_s^2(m\nu)\left[3\mathcal{V}_{k1}^{(s)}(\nu)+2\mathcal{V}_{k2}^{(s)}(\nu)\right], \quad (14)\end{aligned}$$

where $\boldsymbol{S}^2=2$ is the squared quark total spin operator for the spin-triplet configuration. The terms in the second line arise from a single insertion of the $1/(m|\boldsymbol{k}|)$-type potentials. Using that c_1^0 is renormalization group invariant, one can derive the NLL order anomalous dimension of c_1,

$$\nu\frac{\partial}{\partial\nu}\ln[c_1(\nu)] \;=\; \gamma_{c_1}^{\rm NLL}(\nu)+\gamma_{c_1}^{\rm NNLL}(\nu)+\ldots\,,$$

$$\gamma_{c_1}^{\rm NLL}(\nu) \;=\; 4\,\delta z_{c_1}^{\rm NLL}\,. \tag{15}$$

To solve the NLL renormalization group equation one needs to know the LL solutions for all the couplings that appear on the RHS of Eq. (15) (*i.e.* which mix into c_1). From the fact that all higher order $1/\epsilon^n$ ($n=1,2,\ldots$) divergences have to cancel in the anomalous dimension (see *e.g.* Ref. [25]), one can also obtain the NNLL order $1/\epsilon^2$ coefficient $\delta z_{c_1}^{\rm NNLL,2}$ of the renormalization constant Z_{c_1} [20].

The determination of the NNLL order anomalous dimension proceeds along the same lines. One can distinguish between contributions from two classes. The first class arises from the terms in Eq. (14) due to the NLL order running of the couplings on the RHS of Eq. (15). The second class involves the computation of three-loop vertex diagrams with potential loops and either soft or ultrasoft loops that require new c_1 counterterms. I call this class "non-mixing contributions". It leads to genuinely new contributions in the anomalous dimension of c_1. By power counting there are no contributions from diagrams with three potential loops or which have both soft and ultrasoft loops. I have determined the non-mixing contributions recently in Ref. [20] and I will discuss the computation and the results in some detail in the following.

At NNLL order the determination of the UV-divergences from on-shell three-loop vertex diagrams is complicated because there are overlapping IR- and UV-divergences which are difficult to separate from each other. An efficient way to avoid these complication is to consider 4-loop current correlator graphs rather than the vertex diagrams. The correlator graphs are obtained from closing the external quark lines of the vertex diagrams with an additional insertion of the current. In Fig. 2 I have exemplarily displayed the four-loop correlator diagrams with an ultrasoft gluon (loop). The imaginary part of the correlator graphs is proportional to the squared matrix elements from which all IR-divergent Coulomb phases drop out automatically. The three-loop (NNLL) renormalization constant of the current is then obtained from the three-loop subdivergences of the correlator diagrams that remain after the one- and two-loop subdivergences have been subtracted. For dimensional reasons there are in fact no four-loop overall divergences in dimension regularization.

Due to lack of space I cannot give all the details of the computation and the full result here and refer to Ref. [20]. But I would like to discuss some interesting conceptual aspects of the results. First, the NNLL order $1/\epsilon^2$ coefficient $\delta z_{c_1}^{\rm NNLL,2}$ I have obtained by the computations agrees with the NLL prediction based on renormalization group invariance mentioned above. This is a non-trivial check for the consistency of the effective theory under renormalization, since it requires that the interactions between the

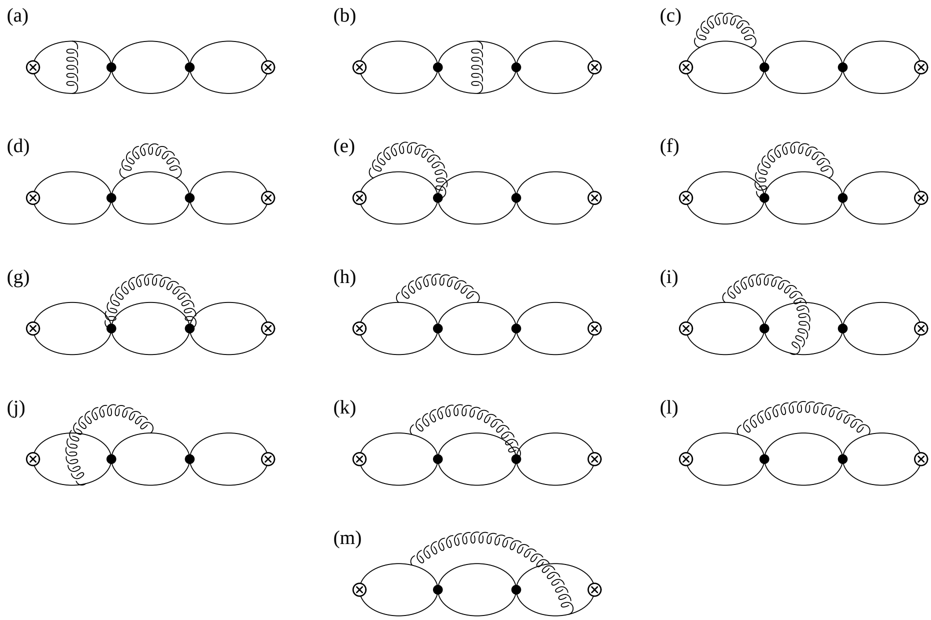

Fig. 2. Four-loop graphs with an ultrasoft gluon for the calculation of the ultrasoft non-mixing contributions of the NNLL anomalous dimension of c_1. The ultrasoft gluon is attached to the top quark or to potential-like 4-quark interactions.

relevant degrees of freedom are encoded correctly in the structure of the effective Lagrangian. This shows in particular, that the renormalizability of the effective theory requires that soft and ultrasoft degrees of freedom have to be present for all scales below the quark mass m_t. In addition, the results for the diagrams in Fig. 2, which involve the soft and the ultrasoft renormalization scales, μ_S and μ_U, show that the correlation $\mu_U = \mu_S^2/m_t$ I have mentioned previously is required by the renormalizability of the effective theory. As an example, let us consider the contribution to Z_{c_1} induced by the diagram Fig. 2(g) which, when the correlation between μ_S and μ_U is neglected, has the form

$$\frac{\alpha_{\mathrm{s}}(m_t\nu)^2\alpha_{\mathrm{s}}(m_t\nu^2)}{\pi}\frac{C_A^2 C_F}{4}\left[-\frac{1}{12\epsilon^2}-\frac{1}{6\epsilon}\left[\ln 2+\frac{7}{6}-\ln\left(\frac{m\,\mu_U}{\mu_S^2}\right)\right]\right]. \quad (16)$$

The dependence on $\ln\mu_S$ and $\ln\mu_U$ in the $1/\epsilon$ piece that contributes to the NNLL anomalous dimension of c_1 vanishes only if the correlation $\mu_U = \mu_S^2/m$ is accounted for. This shows that the correlation of the soft and ultrasoft scales in the effective theory is required by physical reasons and not just imposed by hand.

Although the computation of the NNLL anomalous dimension of c_1 is not completed yet, it is instructive to compare the numerical size of the non-mixing NNLL contributions with the NLL ones in the running of c_1. Let us write the solution of Eq. (15) for $\nu < 1$ as

$$\ln\left[\frac{c_1(\nu)}{c_1(1)}\right] \;=\; \xi^{\rm NLL}(\nu) + \Big(\,\xi^{\rm NNLL}_{\rm m}(\nu) + \xi^{\rm NNLL}_{\rm nm}(\nu)\,\Big) + \ldots\,. \tag{17}$$

where $\xi^{\rm NNLL}_{\rm nm}$ refers to the NNLL non-mixing contributions. In Table I the

TABLE I

Numerical values for $\xi^{\rm NLL}(\nu)$ and $\xi^{\rm NNLL}_{\rm nm}(\nu)$. The values for m are pole masses. The numbers are obtained by evaluation of the analytic results using four-loop running for $\alpha_{\rm s}$ and taking $\alpha_{\rm s}^{(5)}(175\,{\rm GeV}) = 0.107$ and $\alpha_{\rm s}^{(4)}(4.8\,{\rm GeV}) = 0.216$ as input.

	$m = 175\,{\rm GeV}$		$m = 4.8\,{\rm GeV}$	
ν	$\xi^{\rm NLL}(\nu)$	$\xi^{\rm NNLL}_{\rm nm}(\nu)$	$\xi^{\rm NLL}(\nu)$	$\xi^{\rm NNLL}_{\rm nm}(\nu)$
1.0	0.0000	0.0000	0.0000	0.0000
0.8	0.0069	0.0041	0.0308	0.0425
0.6	0.0157	0.0104	0.0712	0.1304
0.4	0.0274	0.0216	0.1335	0.4537
0.2	0.0435	0.0512		

values for $\xi^{\rm NLL}(\nu)$ and $\xi^{\rm NNLL}_{\rm nm}(\nu)$ are displayed for different ν for the top and the bottom quarks. For top quarks we find that the NNLL non-mixing contributions are of the same size as the NLL terms for the relevant region $\nu \sim v \simeq 0.2$. Here, the new NNLL order corrections shift c_1 by about $+5\%$, which is substantial considering that the total cross section contains c_1^2. The shift is dominated by the contributions coming from the ultrasoft diagrams shown in Fig. 2. However, it is not yet possible to draw phenomenological conclusions for the normalization of the top threshold cross section in e^+e^- collisions from this result, because the yet unknown NNLL mixing corrections could be sizeable as well. It is therefore an important future task to determine the mixing contributions to the NNLL anomalous dimension of c_1.

For bottom quarks the NNLL non-mixing contributions are several times larger than the NLL terms for the relevant region $\nu \sim v \approx 0.3$–0.4. This is not unexpected because for bottomonium systems the binding energy $\sim mv^2$ is already of order $\Lambda_{\rm QCD}$. Thus our result seems to affirm that for $b\bar{b}$ states non-perturbative effects have a rather strong influence, and that the vNRQCD description ceases to work even for the ground state [12].

4. Determination of the total cross section

The total cross section for $e^+e^- \to \gamma^*, Z^* \to t\bar{t}$ at threshold at NNLL order has the form

$$\sigma_{\rm tot}^{\gamma,Z}(s) = \frac{4\pi\alpha^2}{3s}\Big[\,F^v(s)\,R^v(s) + F^a(s)R^a(s)\Big]\,, \qquad (18)$$

where $F^{v,a}$ are trivial functions of the electric charges and the isospin of the electron and the top quark and of the weak mixing angle. At NNLL order the vector and axial-vector R-ratios have the form

$$R^v(s) \;=\; \frac{4\pi}{s}\,\mathrm{Im}\Big[\,c_1^2(\nu)\,\mathcal{A}_1(v,m,\nu) + 2\,c_1(\nu)\,c_2(\nu)\,\mathcal{A}_2(v,m_t,\nu)\,\Big]\,, \quad (19)$$

$$R^a(s) \;=\; \frac{4\pi}{s}\,\mathrm{Im}\Big[\,c_3^2(\nu)\,\mathcal{A}_3(v,m_t,\nu)\,\Big]\,, \qquad (20)$$

where the time-ordered products of the effective theory currents read ($\hat{q} \equiv (\sqrt{s}-2m_t,0)$)

$$\begin{aligned}
\mathcal{A}_1 &= i\sum_{\boldsymbol{p},\boldsymbol{p'}}\int d^4x\,\mathrm{e}^{i\hat{q}\cdot x}\,\Big\langle\,0\,\Big|\,T\,\boldsymbol{O}_{\boldsymbol{p},1}(x)\boldsymbol{O}^\dagger_{\boldsymbol{p'},1}(0)\,\Big|\,0\,\Big\rangle\,,\\
\mathcal{A}_2 &= \frac{i}{2}\sum_{\boldsymbol{p},\boldsymbol{p'}}\int d^4x\,\mathrm{e}^{i\hat{q}\cdot x}\,\Big\langle\,0\,\Big|\,T\,\Big[\boldsymbol{O}_{\boldsymbol{p},1}(x)\boldsymbol{O}^\dagger_{\boldsymbol{p'},2}(0) + \boldsymbol{O}_{\boldsymbol{p},2}(x)\boldsymbol{O}^\dagger_{\boldsymbol{p'},1}(0)\,\Big]\Big|\,0\,\Big\rangle\,,\\
\mathcal{A}_3 &= i\sum_{\boldsymbol{p},\boldsymbol{p'}}\int d^4x\,\mathrm{e}^{i\hat{q}\cdot x}\,\Big\langle\,0\,\Big|\,T\,\boldsymbol{O}_{\boldsymbol{p},3}(x)\boldsymbol{O}^\dagger_{\boldsymbol{p'},3}(0)\Big|\,0\,\Big\rangle\,. \qquad (21)
\end{aligned}$$

The correlators $\mathcal{A}_i$ can be written in a compact form in terms of non-relativistic zero-distance Greens functions. For the convention for the $1/(m|\boldsymbol{k}|)$-type potentials employed in Ref. [20] they have the form ($v = ((\sqrt{s}-2m_t+i\Gamma_t)/m_t)^{1/2}$)

$$\begin{aligned}
\mathcal{A}_1(v,m,\nu) \;=\; & 6\,N_c\,\Big[\,G^c(v,m,\nu) + \Big(\mathcal{V}_2(\nu)+2\mathcal{V}_s(\nu)\Big)\,\delta G^\delta(v,m,\nu)\\
& + \mathcal{V}_r(\nu)\,\delta G^r(v,m,\nu) + \delta G^{\rm kin}(v,m,\nu)\\
& - C_A C_F\,\alpha_{\rm s}^2(m\nu)\,\delta G^k_{\rm CACF}(v,m,\nu) + \frac{C_F^2}{2}\,\alpha_{\rm s}^2(m\nu)\,\delta G^k_{\rm CF2}(v,m,\nu)\\
& + \alpha_{\rm s}^2(m\nu)\mathcal{V}_{k1}(\nu)\,\delta G^{k1}(v,m,\nu) + \alpha_{\rm s}^2(m\nu)\mathcal{V}_{k2}(\nu)\,\delta G^{k2}(v,m,\nu)\,\Big]\,,\\
\mathcal{A}_2(v,m,\nu) \;=\; & v^2\,\mathcal{A}_1(v,m,\nu)\,, \quad \mathcal{A}_3(v,m,\nu) \;=\; \frac{4\,N_c}{m_t^2}\,G^1(a,v,m,\nu)\,. \qquad (22)
\end{aligned}$$

The Greens function G^c contains the sum of $1/\boldsymbol{k}^2$ potentials from Eqs. (7) and (6) and was computed numerically in Ref. [12,18]. (See also Ref. [26].) All other Greens functions were obtained analytically in dimensional regularization. The terms $\delta G^{\delta,r}$ arise from single insertions of the $1/m_t^2$ potentials in Eq. (6) and $\delta G^{\rm kin}$ from an insertion of the kinetic energy $\boldsymbol{p}^4/(8m_t^3)$. [12] The terms $\delta G^{\rm k,k1,k2}$ come from a single insertion of the $1/(m|\boldsymbol{k}|)$-type potentials and their expressions were obtained in Refs. [19, 20]. The P-wave Greens function G^1 was also obtained in Ref. [12]. The Greens functions contain UV-subdivergences which are removed by the renormalization constant of the current $\boldsymbol{O}_{\boldsymbol{p},1}$. They still contain overall divergences which, however, are not contained in their absorptive part for stable quarks.

I have displayed the result in the pole mass scheme. Since the pole mass is plagued by a renormalon ambiguity of order $\Lambda_{\rm QCD}$, which causes an instability in the prediction of the cross section, it is mandatory to switch to a "threshold mass" mass definition such as the kinetic, the PS of the 1S mass for phenomenological examinations [11]. I will use the 1S mass definition [9,28] for the discussions in the next section. The Greens functions also depend on the renormalization parameter ν. For ν of order $|v|$, the top quark velocity, the Greens function do not contain any large logarithmic terms and all logarithms are summed into the Wilson coefficients of the potentials and the currents by the renormalization group equations discussed in the previous section. Typically, one chooses $\nu \simeq 0.15 - 0.2$, which corresponds to a momentum scale $m_t\nu \simeq 25 - 35\,{\rm GeV}$ and an energy scale $m_t\nu^2 \simeq 4 - 7\,{\rm GeV}$.

Before my work in Ref. [20] appeared, the order $\alpha_{\rm s}^3 \ln \alpha_{\rm s}$ corrections to the heavy quarkonium partial width into a lepton pair in the fixed-order expansion were computed using an asymptotic expansion of QCD diagrams close to threshold [29]. (For a discussion of the behavior of the perturbative series in the fixed-order expansion I refer to Ref. [29].) Although a summation of logarithms is not contained in that work, the result can be used as a non-trivial cross check for the summations contained in the NNLL vector R-ratio R^v of Eq. (19). Expanding out the summations contained in the Wilson coefficients for energies on the bound state poles one can determine the same order $\alpha_{\rm s}^3 \ln \alpha_{\rm s}$ terms that were determined in Ref. [29]. After some errors were corrected in Ref. [29] agreement was found.

5. Discussion

Let me now turn to how the NNLL non-mixing contributions affect the vector-current-induced top threshold cross section R^v numerically. In Fig. 3(b) I have displayed $Q_t^2 R^v$ up to NNLL order. The curves show the LL (dotted blue lines), NLL (dashed green lines) and NNLL (solid red lines) cross section for $\nu = 0.15, 0.2$ and 0.3, where at NNLL order lower curves

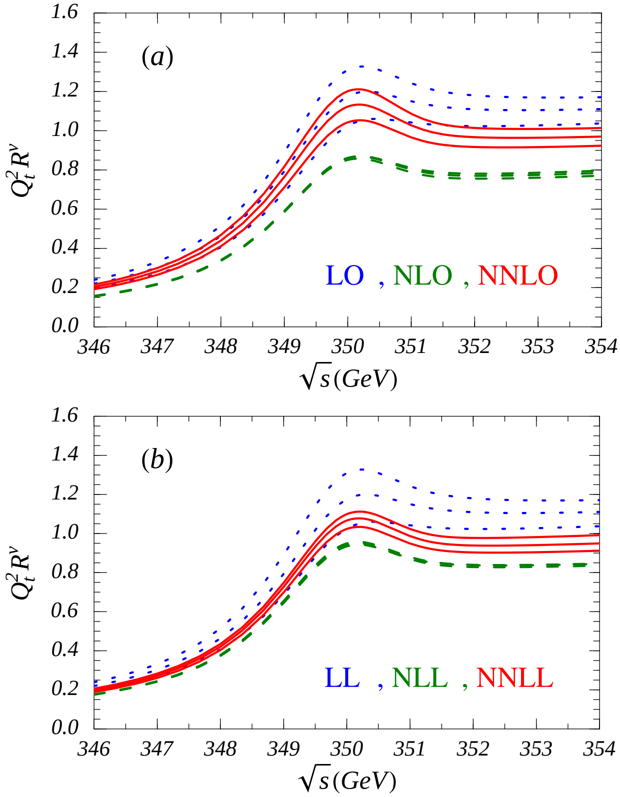

Fig. 3. Panel (a) shows the results for $Q_t^2 R^v$ with $M^{1S} = 175\,\mathrm{GeV}$ and $\Gamma_t = 1.43\,\mathrm{GeV}$ in fixed-order perturbation theory at LO (dotted lines), NLO (dashed lines) and NNLO (solid lines). Panel (b) shows the results for $Q_t^2 R^v$ with the same parameters in renormalization group improved perturbation theory at LL (dotted lines), NLL (dashed lines) and NNLL (solid lines) order. For each order curves are plotted for $\nu = 0.15$, 0.20, and 0.3. The effects of initial state radiation, beamstrahlung and the beam energy spread at a e^+e^- collider are not included.

correspond to larger values of ν. Figure 3(a) shows the corresponding results in fixed-order perturbation theory. Compared to earlier analyses where the NNLL non-mixing contribution were not yet accounted for [12, 18, 19], the NNLL cross section is shifted up by about +10%. On the other hand the scale variation of the NNLL result is about ±3% for the variation of ν I have made here and moderate. Compared to the fixed-order results with the same scales the improvement is substantial, particularly around the peak position and for smaller energies, but not as dramatic as concluded from our earlier analyses when the NNLL non-mixing contributions were not yet included.

Although it appears premature to me to draw any definite conclusions from this result for the determination of the top Yukawa coupling, the strong coupling or the top quark width, because the mixing contributions are still not computed, it appears prudent to say that the error estimate of $\pm 3\,\%$ made in Refs. [12, 18, 19] cannot be upheld at present and should be enlarged to $\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \simeq \pm 6\,\%$ due to the relatively large shift between the NLL and the NNLL order results. As far as the determination of the top quark mass from a threshold scan is concerned, the new NNLL order results are not expected to affect the prospects of a determination with $\delta m_t \sim 100$ MeV since already for the fixed-order NNLO and the earlier NNLL order results the conclusions for the top mass determination from simulation studies were quite similar. [2, 27]

6. Conclusion

In this talk I have discussed the ingredients needed to carry out a renormalization group improved QCD computation of the total cross section $\sigma(e^+e^- \to t\bar{t})$ in the threshold regime. In renormalization group improved perturbation theory QCD logarithms of the top quark velocity are summed up to all order in $\alpha_{\rm s}$ according to the schematic expansion shown in Eq. (3). The summations can be obtained within an effective theory were all degrees of freedom that can fluctuate close to their mass-shell are represented as fields and all off-shell fluctuations are integrated out. All logarithmic terms are associated to UV-divergences in the effective theory and the summation of logarithms is achieved by solving the renormalization group equations of the couplings and coefficients of the effective theory. At present all ingredients for a full NNLL order prediction of the total threshold cross section are known except for the NNLL order result of c_1, the coefficient of the dominant current that produces the top pair in e^+e^- annihilation, which is only fully known at NLL order. I have presented new results for the NNLL non-mixing contributions of the anomalous dimension of the coefficient c_1. Numerically, the NNLL non-mixing contributions to c_1 are of the same size as the NLL contributions and shift the total cross section by about +10%. Including the new results the present theoretical uncertainty for the normalization of the total cross section is about $\pm 6\%$. The new result does not affect the prospects for a precise determination of the top quark mass from a threshold scan based on earlier work, but it does affect the prospects for the determination of the top Yukawa coupling, the strong coupling and the top quark width. However, it is premature to draw definite conclusion as long as the NNLL mixing contributions have not yet been determined.

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