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STATISTICAL HADRONIZATION AND MICROCANONICAL ENSEMBLE*

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We present a Monte Carlo calculation of the microcanonical ensemble of the of the ideal hadron-resonance gas including all known states up to a mass of 1.8 GeV, taking into account quantum statistics. The computing method is a development of a previous one based on a Metropolis Monte Carlo algorithm, with a the grand-canonical limit of the multi-species multiplicity distribution as proposal matrix. The microcanonical average multiplicities of the various hadron species are found to converge to the canonical ones for moderately low values of the total energy. This algorithm opens the way for event generators based for the statistical hadronization model.

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1. Introduction

The basic assumption of the statistical hadronization model is that, as a consequence of a dynamical QCD-driven process, the final stage of a high energy collision results in the formation a set of colourless massive extended objects, defined as *clusters* or *fireballs.* These are assumed to produce hadrons in a purely statistical manner, namely all multihadronic states within the cluster volume and compatible with its quantum numbers are equally likely. The set of states with fixed four-momentum and internal charges (disregarding angular momentum and parity conservation) is what is usually called the microcanonical ensemble. In the statistical model scheme, interactions among stable hadrons are partly taken into account by the inclusion of all resonances as independent states, which is the reason of the expression *ideal hadron-resonance gas* [1].

The motivation for a detailed study of the microcanonical ensemble is twofold: firstly, the need of a tool to hadronize final-state clusters in particle

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and heavy ion collisions with the statistical hadronization model and test observables which cannot be calculated analytically; secondly, the assessment of the validity of the calculations in the canonical ensemble, usually employed in the analysis of average multiplicities in high energy collisions.

The first point is quite clear: if we could compute microcanonical averages numerically in a practical way, we would be able to make predictions on many observables within the basic framework of the statistical model without invoking further assumptions or approximations just to obtain analytical expressions. Furthermore, the availability of a Monte Carlo integration algorithm is a fundamental requirement to design event generators based on the statistical hadronization models. The second point has to do with previous work in the statistical model. In all past analyses specific assumptions were to be invoked in order to allow the use of the canonical ensemble, which is far easier to handle as far as both analytical and numerical calculation is concerned. It is then necessary to verify whether the canonical ensemble was a good approximation by directly comparing it to the microcanonical ensemble.

In this paper we will just summarise a study of the microcanonical ensemble of the ideal hadron-resonance gas. More details about the analytical development and the numerical analysis can be found in Refs. [2, 3]

2. Microcanonical ensemble

In principle, any average on a given statistical ensemble can be calculated from the partition function. The *microcanonical partition function* of the hadron gas is best defined [2] as the sum over all multihadronic states localised within the cluster $|h_V\rangle$ with volume V, constrained with four-momentum and Abelian (*i.e.* additive) charge conservation:

$$\Omega = \sum_{h_V} \langle h_V | \delta^4(P - P_{\rm op}) \delta_{\boldsymbol{Q},\boldsymbol{Q}_{\rm op}} | h_V \rangle \,, \qquad (1)$$

where $\boldsymbol{Q} = (Q_1, \ldots, Q_M)$ is a vector of M integer Abelian charges (electric, baryon number, strangeness *etc.*) and P the four-momentum of the cluster. The microcanonical partition function is first decomposed into the sum of the phase space volumes with fixed numbers of particles for each species j (K in number):

$$\Omega = \sum_{N_j} \Omega_{N_j} \qquad N_j \equiv (N_1, \ldots, N_K)\,. \qquad (2)$$

It can be proved [2] that, for sufficiently large volumes V, Ω_{N_j} can be written as a multi-species cluster decomposition:

$$\Omega_{N_j} = \left[\prod_j \sum_{h_{n_j}} \frac{(\mp 1)^{N_j+H_j}}{\prod_{n_j=1}^{N_j} n_j^{4h_{n_j}} h_{n_j}!} \left[\prod_{l_j=1}^{H_j} \frac{V(2J_j+1)}{(2\pi)^3} \int d^3\mathrm{p}'_{l_j}\right]\right] \times \delta^4\left(P_i - \sum_{j,l_j=1}^{H_j} p'_{l_j}\right), \tag{3}$$

where, for a set of *partitions* $\{h_{n_1}\}, \ldots, \{h_{n_K}\}$ (such that $\sum_{n_j} n_j h_{n_j} = N_j$), the four-momenta p'_{l_j} are those of lumps of particles of the same species j (H_j in number) with mass $n_j m_j$ and spin J_j. The dominant term in Eq. (3), henceforth defined as $\Omega^c_{N_j}$, is that with the maximal power of V, *i.e.* with $H_j = N_j \ \forall j$. This term corresponds to the partitions $h_{n_j} = N_j, 0, \ldots, 0 \ \forall j$, namely to one particle per lump or N lumps overall, and reads:

$$\Omega^c_{N_j} = \frac{V^N}{(2\pi)^{3N}} \prod_j \frac{(2J_j+1)^{N_j}}{N_j!} \int d^3\mathrm{p}_1 \ldots d^3\mathrm{p}_N \ \delta^4\left(P - \sum_{i=1}^{N} p_i\right). \tag{4}$$

A nice feature of the cluster decomposition (3) is that every term of the expansion is a momentum integral like (4) with lumps replacing particles and this is advantageous from the point of view of numerical calculations because all the terms can be computed with the same routine.

3. Numerical calculation

In order to calculate efficiently and quickly the phase space volume for fixed multiplicities Ω_{N_j} quoted in Eq. (3), we adopt a Monte Carlo integration method proposed by Cerulus and Hagedorn in the '60s [4]. The Ω_{N_j} have been calculated with 1000 random samples for each channel, resulting in a statistical error of some percent. In principle, in order to calculate averages in the microcanonical ensemble, such as average hadronic multiplicities, in principle the sum over all possible channels should be performed. Since the number of channels allowed by conservation laws is huge (about 10^6 for a cluster with free charges and mass $M = 3$ GeV, with 271 light-flavoured hadrons and resonances), the exhaustive computation of all Ω_{N_j} is in practice impossible. Hence, one has to resort to Monte Carlo methods, in which the channel space is randomly sampled.

A Monte Carlo method based on the Metropolis algorithm has been proposed by Werner and Aichelin for this problem and used for a restricted set of hadrons [5]. We have developed this method (described in detail in Ref. [3]) using the product of Poissonian distributions as proposal matrix:

$$\Pi_{N_j} = \prod_{j=1}^{K} \exp[-\nu_j] \frac{\nu_j^{N_j}}{N_j!} \tag{5}$$

and extending the used set of hadronic species to all known light-flavoured resonances up to a mass of 1.8 GeV. The mean values ν_j in Eq. (5) are set to the average multiplicities in the canonical ensemble. The use of this multi-Poisson distribution as proposal matrix for the updating rule speeds up the Metropolis random walk in the multi-hadronic configuration space and altogether allows a better use of computing resources.

4. Results

In this paper we report only on the obtained results about the convergence between microcanonical and canonical average multiplicities; more results can be found in Ref. [3]. The comparison is shown in Fig. 1 for completely neutral clusters with increasing mass at an energy density of 0.4 GeV/fm^3. It can be seen that the relative difference between the two ensembles decreases, as expected, as mass increases and it is below 20% for nearly all particles at a mass of 12 GeV. This means that canonical analysis in previous works [6] is a good approximation, taking into account also the expected fluctuations of cluster volumes and energy densities which tend to make the system closer to the canonical model.

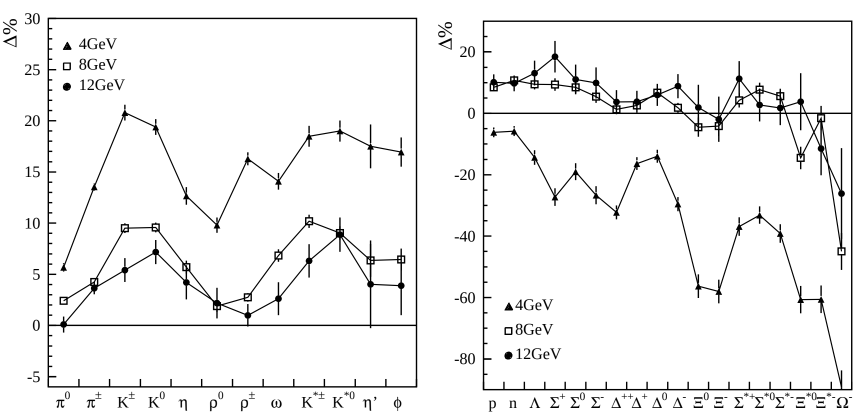

Fig. 1. Relative difference between microcanonical and canonical average multiplicities for neutral clusters with masses 4, 8, 12 GeV at an energy density of 0.4 GeV/fm^3.

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