

INTRODUCTION TO LOW x PHYSICS AND SATURATION*

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The idea of saturation of parton densities in small x physics is briefly introduced. Some aspects of saturation are described, mainly focusing on the status of our knowledge on the non-linear equations describing the high parton density regime. Implications of saturation ideas on the description of nuclear collisions at the Relativistic Heavy Ion Collider are discussed.

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1. Introduction: small x and saturation

The BFKL equation [1] resums gluon ladders taking into account all leading contributions $[\bar{\alpha}_s \ln(1/x)]^n$ ($\text{LL}1/x$), with $\bar{\alpha}_s = \alpha_s N_c/\pi$ and x the fraction of momentum of the hadron carried by the parton. Although it was originally an attempt to compute the high-energy asymptotics of QCD and to justify Regge Theory, it turned out to be an evolution equation in $1/x$ for the so-called unintegrated parton distributions (used in k_T -factorization [2] to compute inclusive particle production at scale Q , $\Lambda_{\text{QCD}} \ll Q \ll E_{\text{cm}}$), which behave $\propto x^{-\bar{\alpha}_s 4 \ln 2}$. Experimentally [3] $xG(x, Q^2) \propto x^{-\lambda}$, with $\lambda \sim 0.3$ sizeably smaller than predicted by BFKL for reasonable values of α_s .

Both k_T -factorization and the BFKL equation are valid in a low parton density, linear regime. At small x (equivalent to large energies for a fixed scale) parton densities become high. Then the idea of saturation of parton densities (see [4]) becomes unavoidable, as parton fusion balances parton splitting if $A_{\mu\nu} \propto 1/g$. It can be alternatively formulated in terms of the S -matrix at fixed impact parameter b becoming black, $|S(b)| = 0$. Saturation constitutes a new regime of QCD, in which old ideas (pomeron interaction, multiple scattering, coherence arguments, ...) are expressed in a new, QCD

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language. It also offers a link between small x physics and heavy ion collisions: in nuclear collisions at high energies large partonic densities are expected, both due to the high energy and to the $A^{1/3}$ -enhancement coming from the nuclear size. The understanding of the initial state in a nuclear collision is crucial to get a coherent picture of the eventual thermalization of the system and creation of Quark Gluon Plasma.

Our present understanding of the different regimes of QCD is summarized in Fig. 1. At low Q^2 we have the confinement region. At large Q^2 and not too high $1/x$ we are in the low-density region, where the usual evolution equations can be used: BFKL in $\ln(1/x)$ and DGLAP in $\ln Q^2$. For both large $\ln(1/x)$ and $\ln Q^2$ we are in the DLL regime, where a first non-linear correction has been proposed, the GLRMQ equations [5]. Finally in the high-density regime, separated from the low-density one by a line which defines the saturation scale $Q_s(x)$, a non-linear generalization of BFKL is the Balitsky–Kovchegov (BK) equation [6, 7].

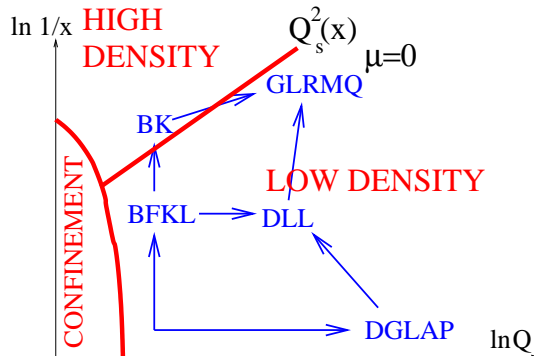


Fig. 1. Regimes of QCD in the $\ln(1/x)$ - $\ln Q$ plane.

I will present in the next section the phenomenological models implementing saturation and our current theoretical understanding of this new regime. In Section 3 I will analyze properties of the solutions of the BK equation, and in Section 4 the application of saturation ideas to high-energy nuclear collisions, presently studied at the Relativistic Heavy Ion Collider (RHIC) at BNL. Finally I will draw some conclusions. Due to space limitations, many interesting topics like NLL developments¹, non-linear equations in the collinear approach, relations with other realizations of collectivity or many examples of observables analyzed, will not be discussed (see *e.g.* [4, 8, 9] and references therein).

¹ NLL effects are expected to be important at larger rapidities, $Y > 1/\alpha_s^{5/3}$, than unitarity corrections, important for $Y > \ln(1/\alpha_s)/\alpha_s$.

2. Present realizations of saturation

2.1. Phenomenological models

The most compelling indirect evidence of saturation comes from the phenomenological success of some models containing saturation ideas when confronted to HERA data. The most commonly used is the GBW model [10]. It is formulated within the color dipole model, in which the interaction of the virtual photon with the hadron or nucleus is described as a convolution of the probability that the photon fluctuates into a $q\bar{q}$ pair of fixed transverse size r with the dipole-hadron cross section². For the latter, the GBW model provides an ansatz for the scattering amplitude N :

$$\frac{\sigma^{q\bar{q}-h}(r, b)}{2} = N(r) = 1 - e^{-Q_s^2 r^2/4}, \quad 2 \int d^2b \equiv \sigma_0, \quad Q_s^2 = \left(\frac{x_0}{x}\right)^\lambda, \quad (1)$$

which implements the unitarity limit, $\sigma^{q\bar{q}-h}(r, b) = 2 \operatorname{Re}[1 - S(r, b)] \leq 2$, in a very simple way. With $\sigma_0 \sim 20$ mb, $x_0 \sim 3 \cdot 10^{-4}$ and $\lambda \sim 0.3$, this model gives a reasonable description of all HERA data on F_2 of the proton for $x < 0.01$ ($Q^2 < 450$ GeV²). It has been extended to include DGLAP evolution and to describe diffraction at HERA [11], and is widely used for phenomenology [12]. It implies a $[\tau = Q^2/Q_s^2(x)]$ -scaling [13] observed at HERA and also searched in nuclear data [14], but it is unclear if the expectation $Q_{sA}^2 \propto AT_A(b) \propto A^{1/3 \div 2/3}$ [4] is fulfilled at present x , Q^2 [15].

2.2. Theoretical developments

After earlier studies [5, 16], a milestone in the theoretical development of saturation ideas in QCD was the MV model [17]. This model treats classical radiation from color sources moving ultrarelativistically through a large nucleus. With a form for the color correlators in the target and taking into account the non-abelian gluon interaction, it gives an explicit form for the nuclear gluon distribution in the transverse phase space,

$$\frac{dN_g^A}{d^2b d^2l} \propto \int \frac{d^2r}{r^2} e^{-il \cdot r} [1 - e^{-Q_s^2 r^2 \ln[(A_{\text{QCD}} r)^{-2}]/4}]. \quad (2)$$

Later on, gluon radiation (quantum evolution) of color sources was introduced, leading to a renormalization group-type equation (the so-called Color Glass Condensate [18]). The rescattering of the projectile in the nucleus is described through Wilson lines whose average on target configurations gives the S -matrix. The key point is that the fields (occupation numbers) become

² The fluctuation length of the photon into the $q\bar{q}$, $\propto 1/x$, becomes at small x larger than the hadron size, so its interaction is coherent.

large ($F_{\mu\nu}, f_g \propto 1/g^2$) but the coupling is small, so classical arguments and perturbative methods are applicable. A hierarchy of coupled equations for n -gluon correlators appears [6, 18].

3. The Balitsky–Kovchegov equation

In the framework outlined in the previous section, the BK equation [6, 7] is the evolution equation in $1/x$ for the 2-gluon correlator, decoupled from the hierarchy in the $N_c \rightarrow \infty$ limit (and with correlations neglected [19])³. As BFKL, it is infrared stable, mixes all twists and α_s is fixed. For $N(x_1, x_2) \equiv N_{x_1 x_2}$ given by a target average of the Wilson lines of a q and a \bar{q} located at transverse positions x_1 and x_2 respectively, it reads

$$\frac{\partial N_{x_1 x_2}}{\partial Y} = \bar{\alpha}_s \int \frac{d^2 z}{2\pi} \frac{(x_1 - x_2)^2}{(x_1 - z)^2 (z - x_2)^2} [N_{x_1 z} + N_{z x_2} - N_{x_1 x_2} - N_{x_1 z} N_{z x_2}], \quad (3)$$

with $Y = \ln(x_{\text{initial}}/x)$. Taking into account just the linear terms this equation reduces to BFKL. In the DLL limit, GLRMQ is recovered [7]. (3) is the most simple tool we have at our disposal to study the high-density, non-linear regime. Its analytical solutions are unknown. Only some theoretical estimates exist [22–24]. So, numerical methods have been developed.

These numerical methods usually deal with the situation $|b| = |x + y|/2 \gg |r| = |x - y|$ ⁴, either in position space [26, 27] or in momentum space [21, 28–30] (in this latter case we define $\phi(k) = \int \frac{d^2 r}{2\pi r^2} \exp(ik \cdot r) N(r)$). Using different techniques two very interesting properties of the solutions have been found. First, function $h(k) = k^2 \nabla_k^2 \phi(k)$ gets a constant shape in $\ln k$ at large Y , moving to the right with constant velocity [21]. Second, identifying the position of the maximum of h with the saturation scale Q_s , the solutions show scaling, *i.e.* $\phi(k) = \phi(k/Q_s)$ [27, 29] (linked to that in τ discussed in the previous Section, sometimes called geometrical scaling). These two features are illustrated in Fig. 2. The velocity of the solution has been computed, $Q_s^2 \propto e^{\bar{\alpha}_s \bar{\lambda} Y}$, with $\bar{\lambda} \simeq 4.1 \div 4.6$ [21, 27–30]. As a consequence of these features, the contribution from the low- k region is reduced with increasing Y , thus offering a possibility to avoid the infrared problem of BFKL. These features are independent of the initial condition (MV [30], BFKL-like [21, 29], DGLAP-like [27], Gaussian [28] or GBW [29, 30]). Finally, the shape of the solutions above Q_s has been examined [30], favoring a log-corrected shape [23] over a pure power in k [22].

³ It was deduced in the color dipole model [7], and also (to my knowledge) in an eikonal approach [20] and in BFKL summing fan diagrams in the large N_c limit [21].

⁴ The solution with full b dependence has also been studied numerically [25] and turns out to be of great importance to compute the behavior of the total cross section at very high energies in this approach.

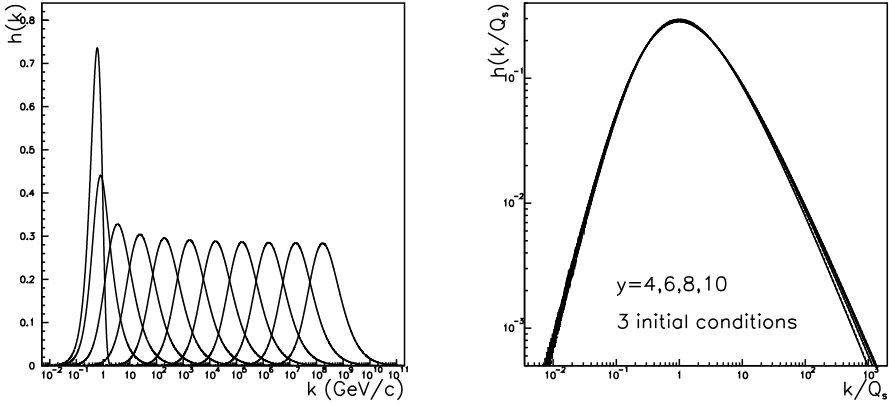


Fig. 2. Left: function h versus k for different $y = \bar{\alpha}_s Y = 0, 1, 2, \dots, 10$. Right: the same function for $y = 4, 6, 8, 10$ and three different initial conditions, versus k/Q_s . See [30] for further explanations.

4. Phenomenology at RHIC

4.1. Multiplicities and pseudorapidity distributions

For midrapidity at $\sqrt{s_{NN}} = 200$ GeV, $0.2 < m_T/\text{GeV} < 10$ means $0.002 < x < 0.1$, so x at RHIC may well be too high to apply safely saturation ideas. Assuming that x is small enough, which we will do in the following, multiplicities and their evolution with centrality and pseudorapidity can be computed in saturation, usually within a factorized approach

$$\frac{dN_{AB}}{dy d\mathbf{p}_T d^2b} \propto \frac{1}{p_T^2} \int d\mathbf{k}_T f^{Q_A}(y, k_T) f^{Q_B}(y, \mathbf{p}_T - \mathbf{k}_T), \quad (4)$$

with $Q_{A,B}$ the saturation momentum of hadrons A and B at some given centrality, and p_T the transverse momentum. It is still unclear to what extent this factorized ansatz is exact and which one is the function f which should be introduced into this equation [31]; other approaches in the semiclassical framework have also been essayed [32]. With some ansatz for f , a simple formula is derived [33], $\frac{dN}{dy} \propto s^{\lambda/2} N_{\text{part}} e^{-\lambda|y|} \ln[Q_s^2/\Lambda_{\text{QCD}}^2]$, $Q_s^2 \sim A^{1/3}$, $\lambda \sim 0.3$ (GBW model). This formula successfully reproduces multiplicities, their evolution with centrality and pseudorapidity distributions at RHIC. See that the deviation from the scaling with the number of participants N_{part} is due just to the log (coming from a factor $1/\alpha_s(Q_s^2)$), at variance with what is found in other approaches [34]. So the question arises whether RHIC data can be explained by initial state effects⁵ [36].

⁵ Elliptic flow in saturation models has also been analyzed [8, 35].

4.2. Transverse momentum distributions

The transverse momentum distribution of partons and particles is expected to be suppressed by the presence of a medium, the so-called jet quenching. This is usually quantified through the ratios

$$R_{pA} = \frac{\frac{dN_{pA}}{dy d^2p_T d^2b}}{N_{\text{coll}} \frac{dN_{pp}}{dy d^2p_T d^2b}}, \quad R_{AA} = \frac{\frac{dN_{AA}}{dy d^2p_T d^2b}}{N_{\text{coll}} \frac{dN_{pp}}{dy d^2p_T d^2b}}. \quad (5)$$

Normalized in this way, this ratio goes to 1 at large p_T according to the usual QCD factorization expectation, with N_{coll} the number of binary nucleon-nucleon collisions. Indeed, such suppression has been observed in AuAu collisions at RHIC [36]. Contrary to the jet quenching interpretation, it has been argued in [37] that such suppression can be explained by initial state, saturation effects, so it should also be present in dAu collisions. These collisions have been studied at RHIC and an enhancement has been found [38], the so-called Cronin effect measured long ago. This leads to the conclusion that the depletion in AuAu is due to final state effects.

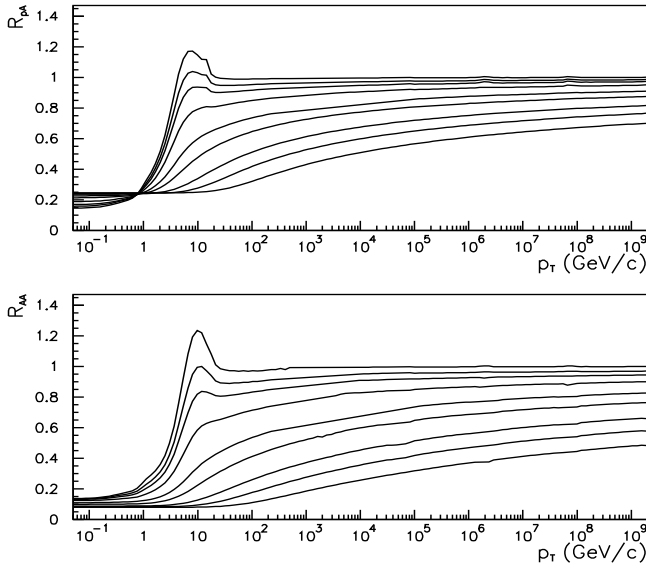


Fig. 3. Ratios (5) in pA (upper plot) and AA (lower plot). In each plot, lines from top to bottom correspond to rapidities $y = \bar{\alpha}_s Y = 0, 0.05, 0.1, 0.2, 0.4, 0.6, 1.0, 1.4, 2.0$. See [30] for further explanations.

Let us examine in more detail the result in [37]. It has been shown [39] that the behavior of f above Q_s is crucial to get either suppression or enhancement in the ratios (5). The naive expectation is that saturation effects are important only for $k < Q_s$. Still, the behavior of f in the region $k > Q_s$ is driven by evolution [22, 23, 30]. Non-evolved forms lead generally to enhancement [39–41]. On the contrary, in [37] a form which contains evolution features [22] has been used. After numerical studies [30], it has become clear [30, 39–41] that quantum evolution does not generate enhancement but very efficiently erases any that may be present in the initial conditions (see Fig. 3). Then a prediction is that the Cronin effect will disappear at higher energies (LHC) or for forward rapidities in pA , corresponding to smaller x in the nucleus. Preliminary BRAHMS data [42] in dAu collisions at $\eta \sim 3$ suggest such effect. Nevertheless, other effects like running coupling [28, 43] may be important for a quantitative comparison.

Summarizing, the concept of saturation in small x physics has been introduced. Some features of the solutions of the non-linear BK equation which arises in this context, have been analyzed. The relevance of saturation for the initial stage of a nuclear collision has been discussed. Assuming that x at RHIC is small enough to apply saturation ideas, the importance of non-linear small x evolution for the interpretation of enhancement or suppression of the p_T distributions measured there has been shown.

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