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NONLINEAR EVOLUTION AND SATURATION*

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Understanding the high energy behaviour of the hadronic cross sections is a long standing problem. The resummation of leading logarithms in energy [1] leads to a strong power growth of the cross section with center of mass energy $\sqrt{s}$ *i.e.* $\sigma(s) \sim s^{\omega}$ with intercept $\omega = 4\ln 2\frac{N_c}{\pi}\alpha_s$, where α_s is a strong coupling constant and N_c is the number of colours. This is clearly in contradiction with the Froissart bound [2] for the cross section behaviour

$$\sigma(s) \sim \frac{\pi^2}{m_\pi^2}\ln^2 s \, , \tag{1}$$

where m_π^2 is the pion mass. Due to the presence of the hadronic scale m_π it is clear that to satisfy this bound one has to take into account the long range properties of the QCD. In other words the unitarity limit, which manifests itself through the bound (1) has to be satisfied by the *entire* QCD theory, which includes the short and long distance dynamics.

The concept of parton saturation can cure, at least partially, the problem of too strong energy growth of the cross section. Apart from parton splitting one takes into account the parton recombination which results in the nonlinear evolution for the parton densities.

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In this talk we concentrate on the solution to the non-linear Balitsky–Kovchegov (BK) equation [3] which is only one of the infinite set of equations describing the dense partonic system in the high energy limit of QCD. The BK equation has been formulated in the leading $\ln 1/x$ approximation within the dipole picture. Here $x = Q^2/s$ is the Bjorken variable and Q^2 is the scale typical for the process in question (ex. the virtuality of the incoming photon in DIS process). The solution to this equation is the dipole-hadron scattering amplitude $N(\boldsymbol{x},\boldsymbol{y};Y) \equiv N_{\boldsymbol{xy}}$, where $\boldsymbol{x},\boldsymbol{y}$ are the two-dimensional vectors describing the position of the $q\bar{q}$ dipole and $Y \equiv \ln 1/x$ is the rapidity of the process. The BK equation reads

$$\frac{\partial N_{\boldsymbol{xy}}}{\partial Y} = \overline{\alpha}_{\rm s} \int \frac{d^2\boldsymbol{z}}{2\pi} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2} \left\{ N_{\boldsymbol{xz}} + N_{\boldsymbol{yz}} - N_{\boldsymbol{xy}} - N_{\boldsymbol{xz}}\, N_{\boldsymbol{yz}} \right\} . \tag{2}$$

Within the leading $\ln 1/x$ approximation the strong coupling $\overline{\alpha}_{\rm s} = \frac{N_{\rm c}\alpha_{\rm s}}{\pi}$ is fixed. Instead of vectors describing dipole end-points $\boldsymbol{x}$ and $\boldsymbol{y}$ we often use dipole size $\boldsymbol{r} = \boldsymbol{x}-\boldsymbol{y}$ and impact parameter $\boldsymbol{b} = \frac{\boldsymbol{x}+\boldsymbol{y}}{2}$. The kernel in equation (2) possesses the property of conformal symmetry, which means that it is invariant with respect to scale change, translations, rotations and inversions. The general solution depends on two vector variables and the rapidity, and it is very difficult to solve it by means of analytical methods. We have solved this equation numerically [4], starting from the initial conditions which are cylindrically symmetric $N^0(|\boldsymbol{r}|,|\boldsymbol{b}|;Y=0)$ which means that they do not depend on the global position angle of the vector $\boldsymbol{b}$. The form of initial condition has been motivated by the well known Glauber–Mueller formula for the amplitude with multiple interactions

$$N^0(|\boldsymbol{r}|,|\boldsymbol{b}|;Y=0) \;=\; 1 \;-\; {\rm e}^{-10r^2\exp(-b^2/2)}, \qquad |\boldsymbol{r}| \equiv r,\; |\boldsymbol{b}| \equiv b\,, \tag{3}$$

where the second exponential $\exp(-b^2/2)$ is a profile in impact parameter which we assumed to be a Gaussian-like. In Fig. 1 we show the resulting profile in impact parameter for various values of rapidity and a selected small dipole size $r = 0.1$. Variable Θ in Fig. 1 is the angle between the $\boldsymbol{r}$ and $\boldsymbol{b}$ vectors, and $\cos\Theta = 0$ corresponds to the situation when $\boldsymbol{r} \perp \boldsymbol{b}$. First thing to note is the fact that even after very small evolution step, $\Delta Y = 0.1$, the profile in impact parameter changes dramatically for large values of b. The initial strong exponential fall-off is changed into the power-like tail, which behaves initially like $\sim b^{-4}$. This is understandable, because it reflects the general property of the BK equation (2) in which the evolution kernel is also power-like and the integration over $d^2\boldsymbol{z}$ is unlimited. More precisely, power-like tails appear because of the presence of the very large dipoles in the evolution which give rise to a non-factorizable contributions in impact parameter. The tails in b are slightly changed during evolution eventually

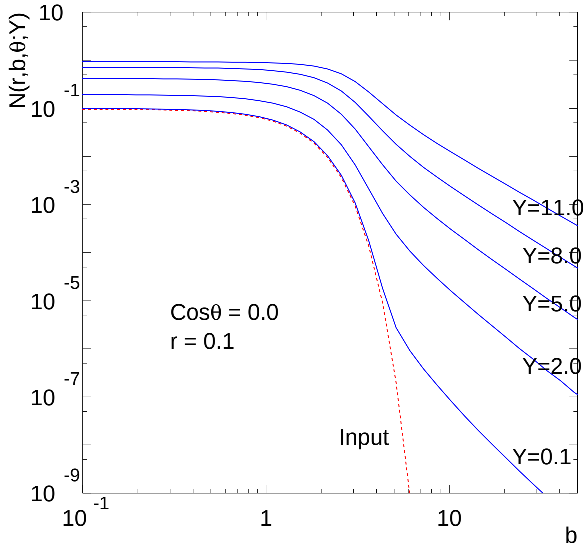

Fig. 1. The amplitude $N(r,b,Y)$ as a function of impact parameter b for different values of rapidity Y. The dipole size and orientation are fixed, $r = 0.1$ and $\cos\theta = 0$. The dashed line is the input distribution (3).

becoming flatter $\sim b^{-2}$ for large values of rapidities ($Y \simeq 11$ in Fig. 1). We note, that for large b where the amplitude N is rather small $N \ll 1$, there is a strong growth in rapidity, which we have estimated to be like $\exp(\omega Y)$ with $\omega \simeq 2.7$ at $b = 10$. This is, of course, well known BFKL rise, which is due to the linear terms in the equation (2) governing the behaviour of the solution in the region where the nonlinear term can be neglected. On the other hand, in the region of small impact parameters, $b < 1$ the growth is highly nonlinear since the value of the amplitude is relatively large $N \sim 0.5$. The nonlinearity plays there crucial role, damping the strong growth of N with rapidity. The resulting solution for amplitude N never exceeds unity. Therefore one can divide the different regions in impact parameter space into the one which is governed by the linear evolution and where the partonic system is relatively dilute

$$N(r,b;Y) \ \ll \ 1\,,$$

and to the one where the amplitude is large and the nonlinearity in Eq. (2) plays a significant role

$$N(r,b;Y) \ \simeq \ 1$$

which we call the black disc region. After integration over the impact parameter b and averaging over the angle Θ one arrives at the dipole–hadron cross section $\sigma(r, Y)$ which however shows an exponential growth with rapidity. It is a direct consequence of the unsuppressed power tails in b. Therefore the solution of the BK equation, although satisfies unitarity condition in the local sense, that means for a given impact parameter, ultimately violates Froissart bound (1) due to the infinite range of interaction.

In a physical situation the profile in the impact parameter would be an exponentially decreasing function with the fall-off governed by the additional mass scale. This scale would break down the conformal symmetry of the equation (2) and cut-off the unphysical power tails in impact parameter. The dipole cross section would then increase at most logarithmically with energy and obey the Froissart bound.

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