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EVENT-BY-EVENT FLUCTUATIONS*

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An overview of the physics of event by event fluctuations in heavy ion collisions is provided. Several observables are discussed.

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1. Introduction

The study and analysis of fluctuations are an essential method to characterize a physical system. In general, one can distinguish between several classes of fluctuations. On the most fundamental level there are quantum fluctuations, which arise if the specific observable does not commute with the Hamiltonian of the system under consideration. These fluctuations probably play less a role for the physics of heavy ion collisions. Second, there are "dynamical" fluctuations reflecting the dynamics and responses of the system. They help to characterize the properties of the bulk (semi-classical) description of the system. Examples are density fluctuations, which are controlled by the compressibility of the system. Finally, there are "trivial" fluctuations induced by the measurement process itself, such as finite number statistics *etc.* These need to be understood, controlled and subtracted in order to access the dynamical fluctuations which tell as about the properties of the system.

Fluctuations are also closely related to phase transitions. The well known phenomenon of critical opalescence is a result of fluctuations at all length scales due to a second order phase transition. First order transitions, on the other hand, give rise to bubble formation, *i.e.* density fluctuations at the extreme.

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The most efficient way to address fluctuations of a system created in a heavy ion collision is via the study of event-by-event (E-by-E) fluctuations, where a given observable is measured on an event-by-event basis and the fluctuations are studied over the ensemble of the events. In most cases (namely when the fluctuations are Gaussian) this analysis is equivalent to the measurement of two particle correlations over the same region of acceptance [1]. Consequently, fluctuations tell us about the 2-point functions of the system, which in turn determine the response of the system to external perturbations.

In the framework of statistical physics, which appears to describe the bulk properties of heavy ion collisions up to RHIC energies, fluctuations measure the susceptibilities of the system. These susceptibilities also determine the response of the system to external forces. For example, by measuring fluctuations of the net electric charge in a given rapidity interval, one obtains information on how this (sub)system would respond to applying an external (static) electric field. In other words, by measuring fluctuations one gains access to the same fundamental properties of the system as "table top" experiments dealing with macroscopic probes. In the latter case, of course, fluctuation measurements would be impossible.

2. Transverse momentum and charge fluctuations

The field of event-by-event fluctuations is relatively new to heavy ion physics and ideas and approaches are just being developed. So far, most of the analysis has concentrated on transverse momentum and the net charge fluctuations.

The pioneering event-by-event studies have been carried out by the NA49 collaboration. They have analyzed the fluctuations of the mean transverse momentum [2] and the kaon to pion ratio [3]. Both measurements have been carried out at the CERN SPS at slightly forward rapidities. In Fig. 1 the resulting distributions are shown together with that from mixed events (histograms). In both cases the mixed event can essentially account for the observed signal, leaving little room for genuine dynamical fluctuations.

Transverse momentum fluctuations should be sensitive to temperature/energy fluctuations [4, 5]. These in turn provide a measure of the heat capacity of the system [6] since

$$\left\langle \delta E^2 \right\rangle = \frac{\partial^2}{\partial \beta^2} \log Z = -T^3 \frac{\partial^2}{\partial T^2} F = T^2 C_V \,. \tag{1}$$

As the QCD phase transition is associated with a maximum of the specific heat, the temperature fluctuations should exhibit a minimum in the excitation function. It has also been argued [7, 8] that these fluctuations may

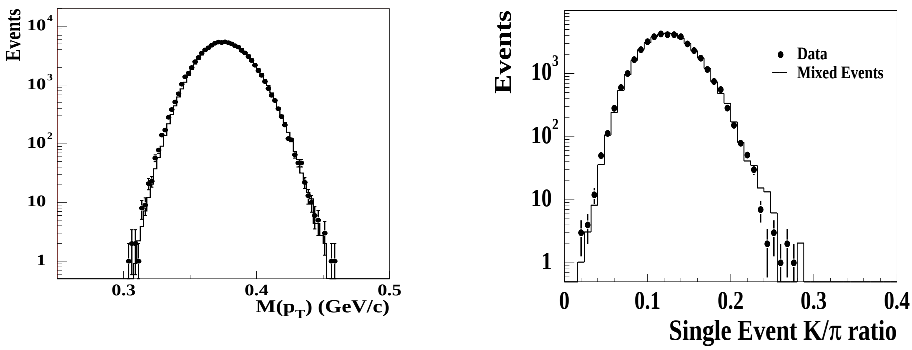

Fig. 1. Results for the fluctuations of the mean transverse momentum (left) and kaon to pion ratio (right). Both results are from the NA49 collaboration.

provide a signal for the long range fluctuations associated with the tri-critical point of the QCD phase diagram. In the vicinity of the critical point the transverse momentum fluctuations should increase, leading to a maximum of the fluctuations in the excitation function.

Transverse momentum fluctuations have been analyzed by several experiments at different bombarding energies. At SPS energies the NA49 collaboration measured transverse momentum fluctuations in the forward rapidity region and found no significant deviation from pure statistics [2] (see Fig. 1). Similarly, at RHIC energies, the PHENIX collaboration also reports no significant non-statistical transverse momentum fluctuations [9]. In contrast to that the CERES collaboration finds fluctuations larger than those from mixed events [10] at SPS energies and at RHIC the STAR collaboration reports significant deviations from mixed events [11]. To which extent this can be attributed to the different acceptance regions covered by these experiments remains to be investigated.

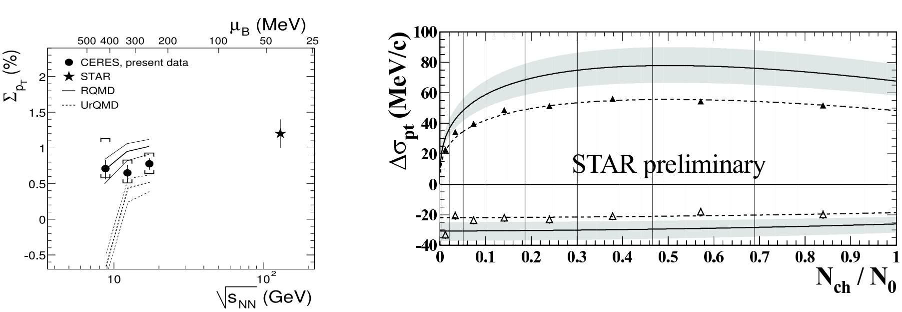

Fig. 2. Left: Excitation function for transverse momentum fluctuations from [10]. Right: Centrality dependence of p_t fluctuations (from [11]).

Another observable of interest are so-called charge fluctuations, since they provide a signature for the existence of a de-confined Quark Gluon Plasma phase [12,13]. The essential idea is that in a QGP the charge carriers are the quarks, which posses fractional charge. Since charge fluctuations are proportional to the square of the charge

$$\left\langle \delta Q^2 \right\rangle \;=\; q^2 \left\langle (\delta N)^2 \right\rangle \,, \tag{2}$$

the ratio of charge fluctuation over entropy

$$\frac{\left\langle \delta Q^2 \right\rangle}{S} \sim \frac{\left\langle \delta Q^2 \right\rangle}{\langle N_{\rm charge} \rangle} \tag{3}$$

is sensitive to the fractional charges in a QGP. In Ref. [13] the observable

$$D \equiv 4 \frac{\left\langle \delta Q^2 \right\rangle}{\langle N_{\rm charge} \rangle} \tag{4}$$

has been proposed and it has been shown that $D = 4$ for an uncorrelated pion gas, $D \simeq 3$ for a resonance gas [14] and $D \simeq 1$–1.5 for a Quark Gluon Plasma, respectively. Since the electric charge is conserved globally, and thus does not fluctuate, experimental measurements need to be corrected for charge conservation effects. These become significant once a sizeable fraction of the final state particles are taken into account. Several prescriptions for these corrections have been proposed [15, 16] which all agree in the limit of small acceptance. A detailed discussion can be found in [17].

In the mean time charge fluctuation have been analyzed by several experiments. PHENIX [18] at RHIC which measures with a small rapidity acceptance, finds charge fluctuations consistent with a resonance gas, if extrapolated to larger acceptance. STAR, which has a large acceptance also finds charge fluctuations consistent with a resonance gas [19]. CERES [20] and NA49 [21], which both measure at SPS energies, report preliminary results on charge fluctuations, which are consistent with a pure pion gas. However, at the SPS the overall rapidity distribution is rather narrow, so that the correlation effect of the resonance gets lost when correcting for charge conservation [22]. But certainly, none of the measurements is even close to the prediction for the QGP.

These findings have prompted ideas, that possibly a constituent quark plasma, without gluons, has been produced [23]. However, the measurement of additional observables would be needed in order to distinguish this from a hadronic gas.

But maybe the present range of Δy is so small, that the charge fluctuations have time to assume the value of the resonance gas. As shown in [13] and [24], the larger the rapidity interval considered, the longer the relaxation time for the charge fluctuations. Thus, maybe even larger acceptance is needed to recover the QGP value. This is also suggested by a model calculation using several event generators. As shown in Fig. 3, the results for the parton cascade arrive at the predicted value for the QGP only for $\Delta y \geq 3$. None of the present experiments has such a coverage yet and thus a detailed analysis of D as a function of Δy is needed, before any firm conclusions can be drawn.

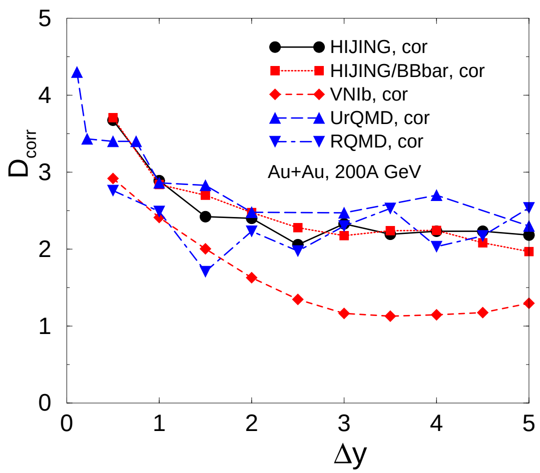

Fig. 3. Charge fluctuation from several event generators [25].

Another way to access the non-trivial correlations of the system is the so called balance function [26,27]. The balance function for charged particle for instance is defined as

$$B(\eta|\Delta\eta) = \frac{1}{2}\left[\frac{\langle N_{+-}(\eta|\Delta\eta)\rangle}{\langle N_{-}(\Delta\eta)\rangle} + \frac{\langle N_{-+}(\eta|\Delta\eta)\rangle}{\langle N_{+}(\Delta\eta)\rangle} - \frac{\langle N_{++}(\eta|\Delta\eta)\rangle}{\langle N_{+}(\Delta\eta)\rangle} - \frac{\langle N_{--}(\eta|\Delta\eta)\rangle}{\langle N_{-}(\Delta\eta)\rangle}\right], \tag{5}$$

where $N_{+-}(\eta|\Delta\eta)$ is the number of unlike-sign pairs which are η apart from each other within the rapidity window $\Delta\eta$. It essentially measures the average distance in rapidity over which a given charge is neutralized (balanced). It is related to the above charge fluctuations in that the latter can be expressed as an integral over the charge balance function [27]. The Balance function measurement at $\sqrt{s} = 130$ GeV has been reported by the STAR collaboration [28]. Going from peripheral to central collisions, the width of balance function steadily decreases. The trend is what one would expect if more of the system is filled with a QGP as the collision becomes more central. However, since the reduction is only about 20 % going from most peripheral to most central, it is not yet clear whether this signals the presence of a QGP, constituent quark clusters [29] or more mundane effect such as the strong flow. For instance in [30] the measured balance functions, along with particle ratios and spectra, could be explained in an expanding hadron gas model.

As detailed in [17] all these event-by-event fluctuation observables can be derived from underlying basic correlator

$$\Delta_{\alpha,\beta}(p,q) = \langle n_\alpha(p) n_\beta(q) \rangle \tag{6}$$

which gives the correlation between particles with quantum numbers α and β and momenta p and q respectively. The difference between momentum fluctuation charge fluctuations *etc.* is then simply the choice of α and β and the weighting functions this correlator is folded with. Also, in order to remove effects from finite number statistics, so called dynamical fluctuations are extracted by either subtracting [31] or dividing [8] the result obtained with an uncorrelated basic "correlator",

$$\Delta^0_{\alpha,\beta}(p,q) = \delta_{\alpha,\beta}\delta_{p,q}\langle n_\alpha(p)\rangle \,. \tag{7}$$

Recently is has been pointed out [32, 33] that this can also be achieved by generalized factorial moments.

3. Equilibrium

Another important question, which might be addressed by the study of fluctuations is equilibration. While measured particle abundances are well described by a hadron gas in chemical equilibrium [34] this is also the case for collisions of elementary particle such as proton-proton or e^+–e^-. Thus simple phase-space dominance *a la* Fermi [35] needs to be ruled out [36]. In other words, how can we distinguish between a superposition of essentially independent nucleon–nucleon collisions as depicted in Fig. 4(a) and a system which equilibrated over the entire volume (Fig. 4(b))? In the absence of any correlations, the partition functions factorizes and thus the two systems are undistinguishable.

At low energies ($\sim 1\,A$GeV) strangeness conservation introduces such correlations and leads to unique predictions for the second factorial moment of the kaon abundance [37]. At higher energies, however, explicit strangeness conservation becomes less relevant and has only a small effect on the single particle yield. Only if, for some reason, the domain over which strangeness (or any other conserved quantum number) is conserved is so small that it

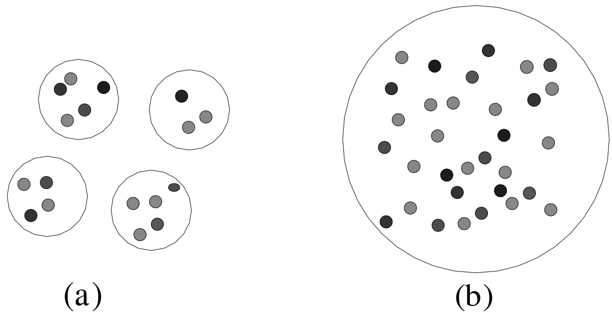

Fig. 4. Individual nucleon–nucleon collisions (a) and "true" matter generated in a nucleus–nucleus collision (b).

contains of the order of one conserved quantum, conservation laws still affect particle abundances. In case of strangeness this has been demonstrated in [38]. However, once the strangeness correlation volume, *i.e.* the volume over which strangeness is conserved is larger than twenty times that of a nucleon, the particle abundances are simply a superposition of the sub-domains and sensitivity to the size of the correlation volume is lost. In order to establish strangeness correlation volumina comparable with the system size, many particle correlations need to be measured. As demonstrated in [39] a definitive measurement of equilibration at RHIC energies would require the measurement of five Omega-baryon coincidences. To which extend this is feasible remains to be seen. Two particle correlations, which are often discussed as a possible means to establish the degree of equilibrium, are dominated by Poisson statistics and thus are misleading.

4. Conclusions

In this contribution we have discussed the physics of fluctuations in the context of heavy ion collisions. As this is a developing field, this should be considered as a snapshot of our present understanding. We have argued that fluctuations are indeed a new tool to investigate the properties of the matter created in these collisions. As an example we have shown how charge fluctuations can be utilized to detect the presence of a Quark Gluon Plasma. The measurement of momentum fluctuations, on the other hand should give us an idea about the heat capacity of the system. Furthermore, if the system is created close to a second order phase transition point, the associated long range fluctuations should be observable in event-by-event observables. Also the question of equilibration can be addressed via fluctuations. At RHIC energies, however, this requires rather difficult measurements of many (>5) particle correlations.

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