

ANALYSIS OF TRANSVERSE MOMENTUM
DISTRIBUTIONS OBSERVED AT RHIC
BY A STOCHASTIC MODEL
IN HYPERBOLIC SPACE*

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To describe large momentum distributions of charged hadrons observed at RHIC, a diffusion equation in the three dimensional hyperbolic space is introduced.

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1. Introduction

We are much interested in events with high multiplicity at RHIC, where thousands of particles are produced per event. To describe such many particle system, a sort of collective approach will be useful. One-particle rapidity or pseudo-rapidity distributions observed at RHIC are well described by the Ornstein–Uhlenbeck process with two or three sources [1]. In this paper, we would extend our approach to include an expansion for transverse direction. We can derive the same formula with that proposed by Voloshin [2] from the three-dimensional Ornstein–Uhlenbeck process using the variables, longitudinal rapidity and two transverse momenta. The fundamental solution is Gaussian in these variables. Even if the formula is integrated over the azimuthal angle in the transverse momentum plane, the distribution is only slightly modified from the Gaussian.

However, observed p_T distributions at RHIC have long tails compared with an exponential distribution, and cannot be described by the formula.

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Therefore, a relativistic approach, in other words, a stochastic model in the three dimensional hyperbolic space is adopted for the production processes, and p_T distributions observed at RHIC are analysed by the model.

2. Diffusion equation in the three dimensional hyperbolic space

For simplicity, we consider the diffusion equation in the geodesic polar coordinate system. In this system, 4 energy-momentum (E, p_L, \mathbf{p}_T) of particle with mass m is specified by radial rapidity ρ , polar angle θ and azimuthal angle ϕ . Then energy and absolute value of momentum are expressed respectively as

$$E = m \cosh \rho, \quad p = \sqrt{p_L^2 + \mathbf{p}_T^2} = m \sinh \rho. \quad (1)$$

The metric in the geodesic polar coordinate system is written as

$$ds^2 = d\rho^2 + \sinh^2 \rho (d\theta^2 + \sin^2 \theta d\phi^2).$$

The diffusion equation,

$$\frac{\partial f}{\partial t} = \frac{D}{\sinh^2 \rho} \left[\frac{\partial}{\partial \rho} \left(\sinh^2 \rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2} \right], \quad (2)$$

with initial condition

$$f(\rho, \theta, \phi, t = 0) = \frac{\delta(\rho)}{4\pi \sinh^2 \rho \sin \theta} \quad (3)$$

is taken as a model of particle production processes: After a collision of nuclei, particles are produced at the origin of rapidity space expressed by Eq. (3). Then those particles propagate according to the diffusion equation (2). In the course of the space-time development, energy is supplied from the leading particle system (composed of collided nuclei) to the produced particle system. The number density of particles becomes lower and at some (critical) density particles become free.

An approximate solution of Eq. (2) for $Dt \ll 1$ is given as [3]

$$f(\rho, t) = C \frac{\rho}{\sinh \rho} \exp \left[-\frac{\rho^2}{2\sigma(t)^2} \right], \quad \sigma(t)^2 = 2Dt. \quad (4)$$

Equation (4) is the solution of the radial symmetric diffusion equation [4],

$$\frac{\partial f}{\partial t} = \frac{D}{\sinh^2 \rho} \frac{\partial}{\partial \rho} \left(\sinh^2 \rho \frac{\partial f}{\partial \rho} \right) \quad (5)$$

if $C = (4\pi Dt)^{-3/2}e^{-Dt}$. In the geodesic cylindrical coordinate system, 4 energy-momentum is expressed by longitudinal rapidity y , transverse rapidity ξ and azimuthal angle ϕ . Energy is given as

$$E = m \cosh y \cosh \xi = m_T \cosh y, \quad (6)$$

where $m_T = \sqrt{p_T^2 + m^2}$. From Eqs. (1) and (6), we have an identity,

$$\cosh \rho = \cosh y \cosh \xi. \quad (7)$$

Radial rapidity coincides with transverse rapidity, $\rho = \xi$, when $y = 0$.

Therefore, we can analyse transverse momentum (rapidity) distributions at fixed longitudinal rapidity ($y = 0, 2.2, \dots$), using Eqs. (4) and (7) with parameters, C and $\sigma(t)^2$.

3. Analysis of p_T distributions observed at RHIC

Transverse momentum distributions of charged hadrons, h^+ and h^- , observed by the STAR [5] and BRAHMS [6] collaborations are analysed. Data are taken at fixed pseudo-rapidity η . Particles are not identified in these experiments. Therefore, we use mass m as a parameter in addition to C and $\sigma^2(t)$. To estimate the values of parameters, only the statistical error is used.

The results at $\eta = 0$ by the STAR collaboration are shown in Fig. 1 and Table I. Estimated value of m decreases as the centrality cut increases, where as variance $\sigma^2(t)$ decreases.

TABLE I

Estimated parameters of p_T distributions of $\frac{h^+ + h^-}{2}$ at $\eta = 0$ in Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV [5].

centrality	m	C	$\sigma(t)^2$	$\chi^2_{\min}/\text{n.d.f}$
00–05%	0.57	316.7 ± 5.2	0.332 ± 0.001	244.9/32
05–10%	0.56	266.5 ± 4.7	0.334 ± 0.001	129.6/32
10–20%	0.53	202.8 ± 3.1	0.354 ± 0.001	144.9/32
20–30%	0.52	143.5 ± 2.5	0.378 ± 0.001	97.94/32
30–40%	0.50	101.2 ± 2.5	0.360 ± 0.001	98.3/32
40–60%	0.44	62.7 ± 1.1	0.402 ± 0.001	52.1/32
60–80%	0.37	25.6 ± 0.5	0.444 ± 0.001	26.6/32
min-bias	0.32	3.03 ± 0.08	0.457 ± 0.004	16.1/29

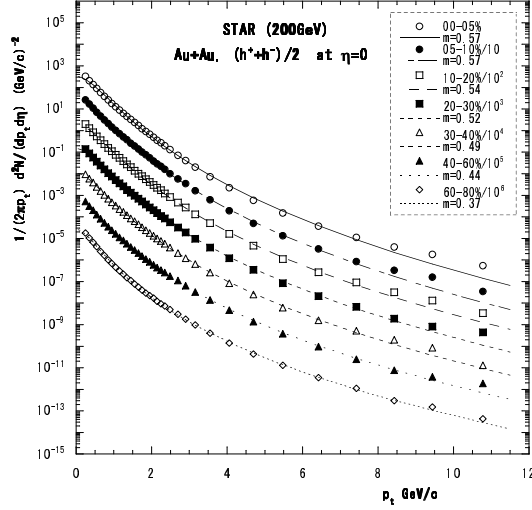


Fig. 1. p_T distributions of $\frac{h^+ + h^-}{2}$ at $\eta = 0$ in Au+Au collisions [5].

The results at $\eta = 0$ by the BRAHMS collaboration are shown in Fig. 2 and Table II. Estimated value of m increases as centrality cut increases. However, the value is somewhat higher than that by the STAR collaboration at the same centrality cut. Variance $\sigma^2(t)$ decreases as the centrality cut increases. In this case the value is smaller than that by the STAR collaboration.

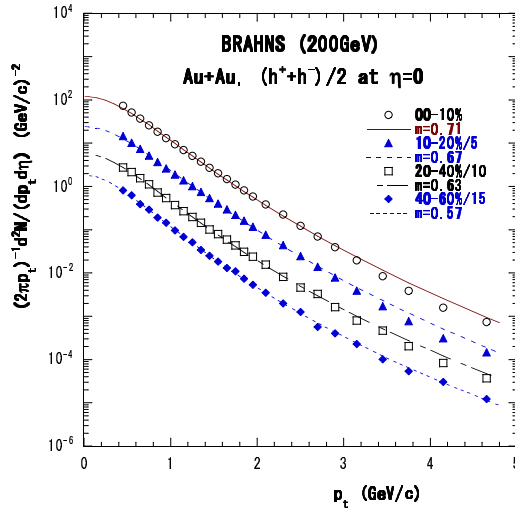


Fig. 2. p_T distributions of $\frac{h^+ + h^-}{2}$ at $\eta = 0$ in Au+Au collisions [6].

TABLE II

Estimated parameters of p_T distributions of $\frac{h^+ + h^-}{2}$ at $\eta = 0$ in Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV by the BRAHMS collaboration [6].

centrality	m	C	$\sigma(t)^2$	$\chi^2_{\min}/\text{n.d.f}$
00–10%	0.71	121.0 ± 0.7	0.307 ± 0.001	351.2/23
10–20%	0.67	88.7 ± 0.6	0.325 ± 0.001	233.5/23
20–40%	0.63	56.9 ± 0.7	0.340 ± 0.001	97.8/23
40–60%	0.57	27.5 ± 0.5	0.358 ± 0.002	28.7/23

The results at $\eta = 2.2$ by the BRAHMS collaboration are shown in Fig. 3 and Table III. Data are well described by Eq. (4). In this analysis, we assume $y = 2.2$, and use Eq. (7).

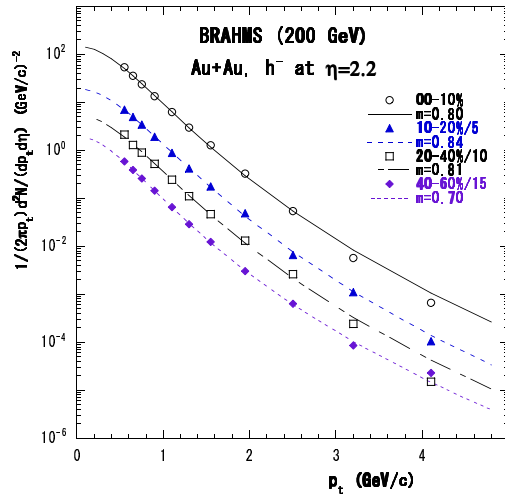


Fig. 3. p_T distributions of h^- at $\eta = 2.2$ in Au+Au collisions [6].

TABLE III

Estimated parameters of p_T distributions of h^- at $\eta = 2.2$ in Au+Au collisions at 200 GeV by the BRAHMS collaboration [6].

centrality	m	C	$\sigma(t)^2$	$\chi^2_{\min}/\text{n.d.f}$
00–10%	0.80	52286 ± 1575	0.470 ± 0.002	93.7/8
10–20%	0.84	41669 ± 1561	0.452 ± 0.002	30.5/8
20–40%	0.81	19364 ± 862	0.469 ± 0.003	68.9/8
40–60%	0.70	6070 ± 418	0.524 ± 0.005	3.1/8

4. Summary and discussions

In order to analyse large p_T distributions of charged hadrons observed at RHIC, a stochastic process in the three dimensional hyperbolic space is taken as a model of particle production process. The solution is Gaussian-like in radial rapidity.

Transverse momentum distributions at $\eta = 0$ observed by the STAR collaboration and those at $\eta = 0$ and 2.2 by the BRAHMS collaboration are well described by Eq. (4), if the value of mass contained in rapidity variable is used as a parameter.

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