Vol. 35 (2004) ACTA PHYSICA POLONICA B No 1

EMISSION ANGLE DEPENDENCE OF HBT RADII: THEORETICAL BACKGROUND AND INTERPRETATION* **

Ulrich Heinz

The Ohio State University, Physics Department
Columbus, OH 43210, USA

(Received November 14, 2003)

The Wigner function formalism which relates source size parameters to experimental "HBT radii" extracted from two-particle Bose–Einstein correlations is generalized to azimuthally deformed and longitudinally tilted sources. It is explained how this can be used to complement anisotropic flow measurements with relevant space-time information on the source.

PACS numbers: 25.75.–q, 25.75.Gz, 25.75.Ld, 24.10.Nz

1. Source size parameters and HBT radii

Two-particle interferometry, which exploits the Bose–Einstein symmetrization effects on the production cross section for pairs of identical bosons, has become a powerful tool to extract detailed space-time information about the freeze-out configuration of the hot and dense fireballs formed in relativistic heavy-ion collisions [1]. Recent exciting data on transverse flow anisotropies in non-central heavy-ion collisions and their interpretation as evidence for early and efficient thermalization at RHIC energies [2] have generated new interest in a better understanding of the space-time structure of the deformed sources created in these collisions. This requires the generalization of the HBT interferometry tool from azimuthally symmetric sources (reviewed in [1]) to deformed situations. Applications of this new formalism to recent RHIC data are reported in the following article by M. Lisa.

For chaotic sources (*i.e.* independent particle emission), the two-particle Bose–Einstein correlation function $C(\boldsymbol{p}_1, \boldsymbol{p}_2) = C(\boldsymbol{q}, \boldsymbol{K})$ can be expressed

* Presented at the XXXIII International Symposium on Multiparticle Dynamics, Kraków, Poland, September 5–11, 2003.

** This work was supported by the U.S. Department of Energy under contract DE-FG02-01ER41190.

through a Fourier transform of the *emission function* $S(x,\boldsymbol{K})$ which describes the single-particle phase-space distribution at freeze-out [1]. $\boldsymbol{K}$ is the average momentum of the pair while $\boldsymbol{q}$ is the relative momentum between the two particles. The Fourier transform in $\boldsymbol{q}$ is restricted by the *mass-shell constraint* $q^0 = E_1{-}E_2 = \boldsymbol{\beta}\cdot\boldsymbol{q}$ (where $\boldsymbol{\beta} = \boldsymbol{K}/K^0$ is the pair velocity) and therefore not fully invertible. The extraction of the emission function $S(x,\boldsymbol{K})$ from the measured correlation function $C(\boldsymbol{K},\boldsymbol{q})$ thus requires additional theoretical input [1].

If the source is dominated by a single length scale (its "size"), the emission function can, for every momentum $\boldsymbol{K}$, be approximated by a Gaussian in space-time whose width parameters form a symmetric tensor, the *spatial correlation tensor* $S_{\mu\nu}(\boldsymbol{K}) = [\langle x_\mu x_\nu\rangle - \langle x_\mu\rangle\langle x_\nu\rangle](\boldsymbol{K}) \equiv \langle\tilde{x}_\mu\tilde{x}_\nu\rangle$. The correlation function is then also a Gaussian in the relative momentum $\boldsymbol{q}$, $C(\boldsymbol{q},\boldsymbol{K}) = 1 + \exp[-\sum_{i,j=o,s,l} q_i q_j R^2_{ij}(\boldsymbol{K})]$, where l denotes the beam direction, o the transverse emission direction $\boldsymbol{K}_\perp$ of the pair, and s the third Cartesian direction perpendicular to l and o. The transverse emission direction $\boldsymbol{K}$ is characterized by an azimuthal angle Φ with respect to the reaction plane formed by the beam axis and the impact parameter $\boldsymbol{b}$. In collisions between spherical nuclei the emission function is reflection symmetric with respect to the reaction plane. This and other, more specific symmetries of the emission function are most easily expressed in a reaction-plane-fixed coordinate system where z points along the beam direction, x along the impact parameter, and y perpendicular to the reaction plane. On the other hand, the correlation radii $R^2_{ij}(\boldsymbol{K})$ are more easily interpreted in the (o,s,l) system because the q_s-dependence, being transverse to the pair velocity $\boldsymbol{\beta}$, is not affected by the mixing of space and time induced by the mass-shell constraint into the Fourier transform. The (o,s,l) and (x,y,z) systems are rotated with respect to each other by the azimuthal emission angle Φ.

The Fourier transform between the emission and correlation functions the leads to the following relations between the "HBT radii" $R^2_{ij}(\boldsymbol{K})$ and the components of the spatial correlation tensor $S_{\mu\nu}(\boldsymbol{K})$ [3]:

$$
\begin{aligned}
R_s^2 &= \tfrac{1}{2}(S_{xx}{+}S_{yy}) - \tfrac{1}{2}(S_{xx}{-}S_{yy})\cos(2\Phi) - S_{xy}\sin(2\Phi)\,,\\
R_o^2 &= \tfrac{1}{2}(S_{xx}{+}S_{yy}) + \tfrac{1}{2}(S_{xx}{-}S_{yy})\cos(2\Phi) + S_{xy}\sin(2\Phi)\\
&\quad -2\beta_\perp(S_{tx}\cos\Phi{+}S_{ty}\sin\Phi) + \beta_\perp^2 S_{tt}\,,\\
R_{os}^2 &= S_{xy}\cos(2\Phi) - \tfrac{1}{2}(S_{xx}{-}S_{yy})\sin(2\Phi) + \beta_\perp(S_{tx}\sin\Phi{-}S_{ty}\cos\Phi)\,,\\
R_l^2 &= S_{zz} - 2\beta_l S_{tz} + \beta_l^2 S_{tt}\,,\\
R_{ol}^2 &= (S_{xz}{-}\beta_l S_{tx})\cos\Phi + (S_{yz}{-}\beta_l S_{ty})\sin\Phi - \beta_\perp S_{tz} + \beta_l\beta_\perp S_{tt}\,,\\
R_{sl}^2 &= (S_{yz}{-}\beta_l S_{ty})\cos\Phi - (S_{xz}{-}\beta_l S_{tx})\sin\Phi\,.
\end{aligned}
\tag{1}
$$

These relations display the *explicit* Φ-dependence arising from the mentioned rotation between the (x, y, z) and (o, s, l) systems, but hide the *implicit* Φ-dependence of the spatial correlation tensor components $S_{\mu\nu}(\boldsymbol{K}) = S_{\mu\nu}(Y, K_\perp, \Phi)$. The total emission angle dependence of the HBT radii results from a combination of both explicit and implicit Φ-dependences [3, 4].

The implicit Φ-dependence of the spatial correlation tensor is restricted by symmetries of the source [4]. It is a relativistic effect associated with an azimuthal spatial source deformation superimposed by strong transverse collective flow [3, 5] which vanishes with the 4^{th} power of the transverse flow velocity $v_{\rm T}/c$ for weak or no collective expansion [5, 6].

2. Azimuthal oscillations and harmonic analysis

A full analysis of the symmetry constraints on $S_{\mu\nu}(Y, K_\perp, \Phi)$ for symmetric collisions between spherical nuclei and for pairs detected in a symmetric rapidity window around $Y = 0$ can be found in Ref. [4]. One finds the following *most general* form for the azimuthal oscillations of the HBT radii:

$$
\begin{aligned}
R_s^2 &= R_{s,0}^2 + 2 \sum_{n=2,4,6,\dots} R_{s,n}^2 \cos(n\Phi)\,, \\
R_{os}^2 &= \qquad\; 2 \sum_{n=2,4,6,\dots} R_{os,n}^2 \sin(n\Phi)\,, \\
R_o^2 &= R_{o,0}^2 + 2 \sum_{n=2,4,6,\dots} R_{o,n}^2 \cos(n\Phi)\,, \\
R_{ol}^2 &= \qquad\; 2 \sum_{n=1,3,5,\dots} R_{ol,n}^2 \cos(n\Phi)\,, \\
R_l^2 &= R_{l,0}^2 + 2 \sum_{n=2,4,6,\dots} R_{l,n}^2 \cos(n\Phi)\,, \\
R_{sl}^2 &= \qquad\; 2 \sum_{n=1,3,5,\dots} R_{sl,n}^2 \sin(n\Phi)\,.
\end{aligned}
\qquad (2)
$$

We see that only even *or* odd sine *or* cosine terms occur, but no mixtures of such terms. Statistical errors in the resolution of the reaction plane angle as well as finite angular bin sizes in Φ tend to reduce the actually measured oscillation amplitudes; fortunately, these dilution effects can be fully corrected by a model-independent correction algorithm [4]. A Gaussian fit to the thus corrected correlation function, binned in Y, $K_\perp$ and emission angle Φ, then yields the "true" HBT radius parameters $R_{ij}^2(Y, K_\perp, \Phi)$ from which the n^{th} order azimuthal oscillation amplitudes are extracted via $R_{ij,n}^2(Y, K_\perp) = \frac{1}{n_{\rm bin}} \sum_{j=1}^{n_{\rm bin}} R_{ij}^2(Y, K_\perp, \Phi_j) \mathrm{osc}(n\Phi_j)$. Here $n_{\rm bin}$ indicates the number of (equally spaced) Φ bins in the data and $\mathrm{osc}(n\Phi_j)$ stands for $\sin(n\Phi_j)$ or $\cos(n\Phi_j)$ as appropriate, see Eqs. (2). (Note that Nyquist's theorem limits the number of harmonics that can be extracted to $n \leq n_{\mathrm{bin}}$.)

3. HBT oscillation amplitudes and source shape

We would like to relate the azimuthal oscillation amplitudes of the 6 HBT radius parameters to the geometric and dynamical anisotropies of the source, as reflected in the azimuthal oscillations of the 10 independent components of the spatial correlation tensor. Their allowed oscillation patterns at midrapidity $Y = 0$ are given by [4]

$$
\begin{aligned}
A(\Phi) \equiv \frac{1}{2}\langle \tilde{x}^2 + \tilde{y}^2 \rangle &= A_0 + 2 \sum_{n\geq 2,\mathrm{even}} A_n \cos(n\Phi)\,, \\
B(\Phi) \equiv \frac{1}{2}\langle \tilde{x}^2 - \tilde{y}^2 \rangle &= B_0 + 2 \sum_{n\geq 2,\mathrm{even}} B_n \cos(n\Phi)\,, \\
C(\Phi) \equiv \langle \tilde{x}\tilde{y} \rangle &= 2 \sum_{n\geq 2,\mathrm{even}} C_n \sin(n\Phi)\,, \\
D(\Phi) \equiv \langle \tilde{t}^2 \rangle &= D_0 + 2 \sum_{n\geq 2,\mathrm{even}} D_n \cos(n\Phi)\,, \\
E(\Phi) \equiv \langle \tilde{t}\tilde{x} \rangle &= 2 \sum_{n\geq 1,\mathrm{odd}} E_n \cos(n\Phi)\,, \\
F(\Phi) \equiv \langle \tilde{t}\tilde{y} \rangle &= 2 \sum_{n\geq 1,\mathrm{odd}} F_n \sin(n\Phi)\,, \\
G(\Phi) \equiv \langle \tilde{t}\tilde{z} \rangle &= 2 \sum_{n\geq 1,\mathrm{odd}} G_n \cos(n\Phi)\,, \\
H(\Phi) \equiv \langle \tilde{x}\tilde{z} \rangle &= H_0 + 2 \sum_{n\geq 2,\mathrm{even}} H_n \cos(n\Phi)\,, \\
I(\Phi) \equiv \langle \tilde{y}\tilde{z} \rangle &= 2 \sum_{n\geq 2,\mathrm{even}} I_n \cos(n\Phi)\,, \\
J(\Phi) \equiv \langle \tilde{z}^2 \rangle &= J_0 + 2 \sum_{n\geq 2,\mathrm{even}} J_n \cos(n\Phi)\,.
\end{aligned} \tag{3}
$$

The missing terms in the sums over n have amplitudes which are odd functions of Y and vanish at midrapidity. They do, however, contribute to the HBT radii if the data are averaged over a finite, symmetric rapidity window around $Y = 0$ [4]. Their contributions can be eliminated by varying the width ΔY of this rapidity window and extrapolating quadratically to

$\Delta Y \to 0$. Note that $C_0 = E_0 = F_0 = G_0 = I_0 = 0$ by symmetry, *i.e.* the corresponding components of $S_{\mu\nu}$ oscillate around zero.

The oscillation amplitudes of the HBT radii relate to the oscillation amplitudes of the source parameters as follows: For the odd harmonics $n = 1, 3, 5, \ldots$ we have

$$\begin{aligned}
R^2_{ol,n} &= \tfrac{1}{2}\langle H_{n-1}+H_{n+1}-I_{n-1}+I_{n+1} - \beta_l(E_{n-1}+E_{n+1}-F_{n-1}+F_{n+1})\rangle \\
&\quad -\langle \beta_\perp G_n - \beta_l D_n\rangle\,, \\
R^2_{sl,n} &= \tfrac{1}{2}\langle -H_{n-1}+H_{n+1}+I_{n-1}+I_{n+1} - \beta_l(-E_{n-1}+E_{n+1}+F_{n-1}+F_{n+1})\rangle\,,
\end{aligned} \tag{4}$$

whereas the even harmonics $n = 0, 2, 4, \ldots$ satisfy

$$\begin{aligned}
R^2_{s,n} &= \langle A_n\rangle + \tfrac{1}{2}\langle -B_{n-2}-B_{n+2}+C_{n-2}-C_{n+2}\rangle, \\
R^2_{o,n} &= \langle A_n\rangle + \tfrac{1}{2}\langle B_{n-2}+B_{n+2}-C_{n-2}+C_{n+2}\rangle \\
&\quad -\beta_\perp\langle E_{n-1}+E_{n+1}-F_{n-1}+F_{n+1}\rangle + \beta_\perp^2\langle D_n\rangle, \\
R^2_{os,n} &= \tfrac{1}{2}\langle -B_{n-2}+B_{n+2}+C_{n-2}+C_{n+2}+\beta_\perp(E_{n-1}-E_{n+1}-F_{n-1}-F_{n+1})\rangle, \\
R^2_{l,n} &= \langle J_n\rangle - 2\langle \beta_l G_n\rangle + \langle \beta_l^2 D_n\rangle.
\end{aligned} \tag{5}$$

In these relations it is understood that all negative harmonic coefficients $n < 0$ as well as C_0, E_0, F_0, G_0 and I_0 are zero. The angular brackets $\langle\ldots\rangle$ indicate an average over a finite, symmetric rapidity window around $Y = 0$. The terms involving the longitudinal pair velocity β_l vanish quadratically as the width ΔY of that window shrinks to zero.

Even after extrapolating to $Y = 0$ in this way, we have still many more source parameters than measurable HBT amplitudes. One counts easily that up to $n = 2$ there are 9 measurable Fourier coefficients which (at $Y = 0$) depend on 19 source amplitudes. From there on, increasing n by 2 yields 6 additional measured amplitudes which depend on 10 additional source amplitudes. This lack of analysis power is an intrinsic weakness of the HBT microscope and due to the fundamental restrictions arising from the mass-shell constraint $q^0 = \boldsymbol{\beta}\cdot\boldsymbol{q}$. The reconstruction of the source thus must necessarily rely on additional assumptions.

One such assumption which may not be too unreasonable is that the emission duration $D = \langle\tilde{t}^2\rangle$ is approximately independent of emission angle and that the source is sufficiently smooth that higher order harmonics $n \geq 3$ of $S_{\mu\nu}$ can be neglected. Such source properties would result in the "Wiedemann sum rule" [3]

$$R^2_{o,2} - R^2_{s,2} + 2R^2_{os,2} = 0 \tag{6}$$

which can be experimentally tested. If verified for all $K_\perp$ it would provide strong support for the underlying assumptions on the source. In this case we can measure 3 azimuthally averaged HBT radii and 5 independent oscillation amplitudes with $n \leq 2$, depending on 14 source parameters of which 5 can be eliminated by going to $K_\perp = \beta_\perp = 0$ (see [4] for explicit expressions). This makes the geometry of the effective source for particles with $K_\perp = 0$ "almost solvable" [7], as confirmed by hydrodynamical calculations [9] which show that at $K_\perp = 0$ the effective emission region closely tracks the overall geometry of the source even if it is strongly and anisotropically expanding.

The source geometry can be completely reconstructed from HBT data if transverse flow is so weak that all implicit Φ-dependence (*i.e.* all higher harmonics $n \geq 1$) of $S_{\mu\nu}$ can be neglected. In this case one obtains at $Y = 0$ the "geometric relations" [6]

$$
\begin{aligned}
&R^2_{s,0} = A_0 = \tfrac{1}{2}\langle \tilde{x}^2 + \tilde{y}^2\rangle_0\,,\\
&R^2_{o,0} - R^2_{s,0} = \beta^2_\perp D_0 = \beta^2_\perp \langle \tilde{t}^2\rangle_0\,,\\
&R^2_{l,0} = J_0 = \langle \tilde{z}^2\rangle_0\,,\\
&R^2_{ol,1} = -R^2_{sl,1} = \tfrac{1}{2} H_0 = \tfrac{1}{2}\langle \tilde{x}\tilde{z}\rangle_0\,,\\
&R^2_{o,2} = -R^2_{s,2} = -R^2_{os,2} = \tfrac{1}{2} B_0 = \tfrac{1}{4}\langle \tilde{x}^2 - \tilde{y}^2\rangle_0\,.
\end{aligned}
\tag{7}
$$

A_0 describes the average transverse size and B_0 (which generates a second-order harmic in the transverse HBT radii) the transverse deformation of the source. H_0 generates a first-order harmonic in the *ol* and *sl* cross terms and describes a longitudinal tilt of the source away from the beam direction [6]. Such a tilt was found in Au + Au collisions at the AGS [10]. Its sign yielded important information on the kinetic pion production mechanism [6, 10].

4. Conclusions

Azimuthally sensitive HBT interfrometry is a powerful tool for analyzing the dynamic origin and space-time manifestations of the strong anisotropic collective flow seen in single-particle spectra at RHIC. Symmetries strongly constrain the azimuthal Fourier series of the emission angle dependent HBT radii. This is helpful but not sufficient for a fully model independent reconstruction of the source deformations from such data. To separate temporal from geometric contributions to the HBT radii, the source must satisfy special properties. One can test for them experimentally, albeit not in a fully model-independent way. Hydrodynamic simulations [9] (which were reported at the meeting but are not included here) show that at RHIC and LHC energies emission from the source at non-zero transverse momentum is very strongly surface dominated ("source opacity"). To obtain an unambiguous estimate of the transverse deformation of the source at freeze-out one should study the Φ-oscillations of the HBT radii at very small $K_\perp$. A possible longitudinal tilt of the source away from the beam direction manifests itself through first-order harmonics in R^2_{ol} and R^2_{sl}. The $K_\perp$-dependence of the oscillation patterns of the transverse HBT radius parameters can be analyzed to obtain evidence for or against a faster expansion of the source in-plane than out-of-plane due to anisotropic collective flow [9].

REFERENCES

[1] U.A. Wiedemann, U. Heinz, *Phys. Rep.* **319**, 145 (1999); U. Heinz, B.V. Jacak, *Ann. Rev. Nucl. Part. Sci.* **49**, 529 (1999); B. Tomášik, U.A. Wiedemann, `hep-ph/0210250`.

[2] P.F. Kolb, U. Heinz, `nucl-th/0305084`.

[3] U.A. Wiedemann, *Phys. Rev.* **C57**, 266 (1998).

[4] U. Heinz, A. Hummel, M.A. Lisa, U.A. Wiedemann, *Phys. Rev.* **C66**, 044903 (2002).

[5] A. Hummel, M.Sc. Thesis, Ohio State University, 2002, unpublished.

[6] M.A. Lisa, U. Heinz, U.A. Wiedemann, *Phys. Lett.* **B489**, 287 (2000).

[7] Note that the limit $K_\perp \to 0$ eliminates all influence from the temporal structure of the source; the emission duration and correlations between position and time at freeze-out must be extracted from correlation data at non-zero $K_\perp$ where, however, the explicit $\beta_\perp$-dependence associated with factors t in the variances $\langle \tilde{x}_\mu \tilde{x}_\nu \rangle$ interferes with the implicit $K_\perp$-dependence in the spatial variances which result from collective expansion flow [1]. Models for disentangling these different contributions to the $K_\perp$-dependence have been extensively studied for azimuthally symmetric sources [1], but must be generalized for azimuthally deformed sources [8].

[8] F. Retiére, M.A. Lisa, `nucl-th/0312024`.

[9] U. Heinz, P.F. Kolb, *Phys. Lett.* **B542**, 216 (2002).

[10] M.A. Lisa *et al.*, [E895 Collaboration], *Phys. Lett.* **B496**, 1 (2000).