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ANOMALOUS PHENOMENA IN TERMS OF THE QUANTUM STATISTICS*

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We argue that data coming from the Japanese–American cooperative emulsion experiment (JACEE) on the Si + AgBr event at 4 TeV/nucleon reveal a remarkable regularity (without adjusting any free parameter) which is in accord with the quantum statistical expectation.

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1. Normal phenomena

Through many years the dynamical as well as the statistical properties of the multiparticle production phenomena observed in the field of the high energy physics were studied and described quite often in terms of the Pascal or negative binomial distribution, $P^{(\mathrm{NBD})}(n)$,

$$P^{(\mathrm{NBD})}(n) = \begin{pmatrix} n+M-1 \\ n \end{pmatrix} \left(1+\frac{\langle n\rangle}{M}\right)^{-M} \left(1+\frac{M}{\langle n\rangle}\right)^{-n}, \qquad (1)$$

$\langle n\rangle$ being the average multiplicity and M the number of modes, cells or, in our case, of bins. For instance, the charge ratio of cosmic ray muons [1] as well as distributions and correlations in multiparticle production processes [2] were successfully described essentially in terms of (1) already many years ago (some details can be found *e.g.* in [3]). Let us mention that the NBD (1) is related to the case when n bosons are produced stochastically (thermally) from the vacuum, without any coherence.

In the present contribution we consider as *normal* those phenomena whose analysis can start quite naturally by applying a reasonable number

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of (superimposed, convoluted or in any way combined) continuous functions (say, with smooth derivatives) which might involve acceptably many free parameters. However, in general there should be achieved some consensus about the answer to the question: to what extent such a procedure still might be considered as a sufficiently reliable one? (Also during the Tihany symposium (in 2000) it was concluded that the NBD describes reasonably well all measured multiplicity distributions.)

Moreover, it was remarked [4] that *the multiplicity distributions convolute as the bin size is extended. And the fact that the measured multiplicity distributions are excellently fit by NBD with well known properties under convolution enables this observation to be made by inspection. This result explains and demystifies intermittency. Intermittency is nothing more than statistical independence of the multiplicity in rapidity bins of size about 0,1 (in central O+Cu collisions)!* (For this type of collisions very good fits are seen in Fig. 1 of Ref. [4].)

2. Inappropriateness of the negative binomial distribution

Factorial moments of the NBD (1) aquire the form

$$F^{(\mathrm{NBD})}(q,M) = \frac{(M+q-1)!}{M^q\ (M-1)!}\,. \tag{2}$$

Search for fractality (or intermittency) means, according to Bialas and Peschanski, [5], that we are looking for the exponentiation indices, a_q, in the relation

$$F^{(\dots)}(q,M) \sim M^{a_q^{(\dots)}}\,. \tag{3}$$

In the present contribution the indices $a_q^{(\dots)}$ in relation (3) are called "the scaling indices" **only** in the case when they are independent of the number of bins, M.

In the case when we take into account the NBD, relation (3) gives,

$$a_q \equiv a_q^{(\mathrm{NBD})} \sim \frac{\ln F^{(\mathrm{NBD})}(q,M)}{\ln M} \tag{4}$$

and the plot $a_q^{(\mathrm{NBD})}$ *versus* $\ln M$ is seen in Fig. 1.

It is clear that the indices $a_q^{(\mathrm{NBD})}$ depend on the number of bins. This observation allows to conclude that the quantities $a_q^{(\mathrm{NBD})}$ should not be called "the scaling indices". And the factorial moments $F^{(\mathrm{NBD})}(q,M)$ are not appropriate for description of the case where the realistic intermittency phenomena appear.

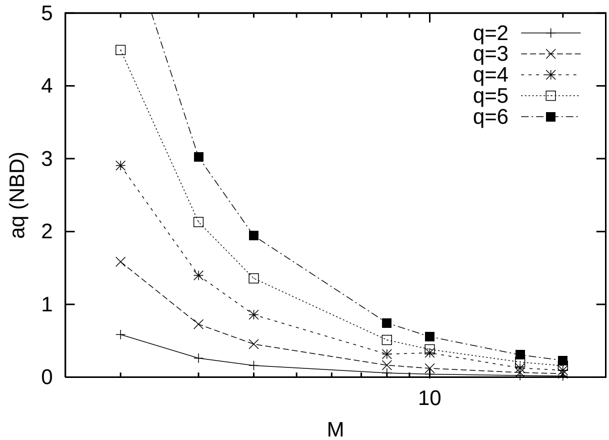

Fig. 1. Since the curves on this figure depend essentially on the number of bins, M, application of the factorial moments $F^{(\mathrm{NBD})}(q, M)$ is inappropriate for investigating the presence of intermittency.

3. Anomalous phenomena

As to the physically acceptable description of the JACEE phenomenon (published in Fig. 1 of Ref. [6]) no distribution was successful, so far (in general, it is pretended that there is "nothing" interesting in the JACEE data). Even more, there is not known a reliable answer to the question: when should we conclude that there is a reasonable agreement between the experimental data (as in [6]) and a theoretical model? In such a case let us speak about the *anomalous* phenomena.

To overcome the difficulty just appeared let us start by the experimental evidence, essentially by the number of secondaries, n_m observed in every bin $(m = 1, 2, \ldots, M)$.

We construct the factorial moments according to their very definition[1]

$$F^{(\ldots)}(q, M) = \left[\sum_{m=1}^{M} n_m\right]^{-q} \left[\sum_{m=1}^{M} n_m(n_m - 1)\ldots(n_m - q + 1)\right] . \quad (5)$$

Inclusion of the JACEE data into the r.h.s. of relation (5) leads to the result that the left hand side of the relation

$$a_q^{(\mathrm{JAC})} \sim \frac{\ln F^{(\mathrm{JAC})}(q, M)}{\ln M} \quad (6)$$

[1] This procedure can be directly generalized to the case when there are available data from arbitrary number of events and several kinds of quantities (like pseudorapidity, azimuthal angle and transverse momentum, compare [7]).

(JAC means shortened JACEE) is to a good degree of accuracy, independent of the number of bins, admitting, say, about some few percent errors in location of the points entering Fig. 2.

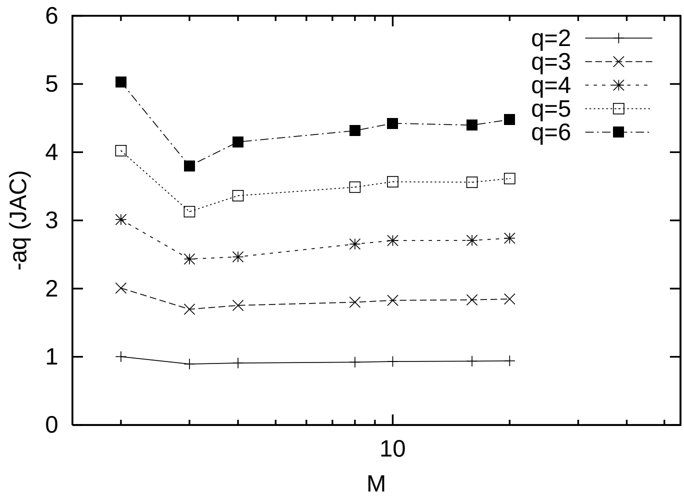

Fig. 2. The dependence of the JACEE scaling indices on $\ln M$; their independence of the number of bins, M, is crucial (the figure is drawn for $M = 2, 3, 4, 8, 10, 16$ and 20).

The quantities $a_q^{(\mathrm{JAC})}$ are really scaling indices. Let us note that relation (6) can be expressed in an alternative way, namely,

$$\ln F^{(\mathrm{JAC})}(q, M) \sim a_q^{(\mathrm{JAC})} \ln M\,, \tag{7}$$

with the number of bins, M being independent of the rank q of the factorial moment involved.

4. Generalized distribution

To find a way out of the difficulties let us generalize the approach allowing to derive the NBD in the field of the quantum statistics. Concretely, let us assume that in every bin $m(= 1, 2, \ldots, M)$ there are produced $(n_T)_m$ particles stochastically **and** $(n_C\kappa^2)_m$ particles coherently. Moreover, let us introduce also M bins where $(n_C)_m - n_C\kappa^2)_m \equiv B_m$ particles are produced only coherently. In this case the quantum statistics gives the factorial moments which are somehow cumbersome, [8], however, they can be presented also in the following form, [9],

$$F^{(\ldots)}(q, M) = X^q\,, \tag{8}$$

where

$$X^q \equiv \left[\sum_{m=1}^{M} B_m + \sum_{m=1}^{M} (n_T)_m L_{(\omega_m)}\left(-\frac{(n_C)_m \kappa_m^2}{(n_T)_m}\right)\right]^q \left[\frac{1}{\sum_{m'=1}^{M}(n_T+n_C)_{m'}}\right]^q \tag{9}$$

and L are the Laguerre' polynomials (with vanishing whole coherency the NBD is restored). As it is seen, relation (8) can be expressed in the form,

$$\ln F^{(\dots)}(q,M) \sim q \ln X \tag{10}$$

with the expression X being independent of the rank q.

Now, comparing relation (7) with relation (10) we observe that in the JACEE case the validity of the following relation might be qualitatively expected,

$$a_q^{(\mathrm{JAC})} \sim q\,. \tag{11}$$

As it is seen, the data (in Fig. 3) approve the linear dependence between the scaling indices $a_q^{(\mathrm{JAC})}$ and the rank q of the factorial moments.

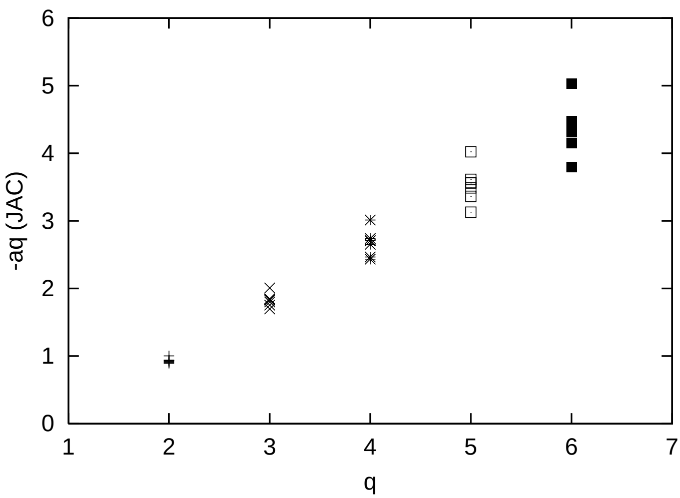

Fig. 3. Linear dependence of the JACEE scaling indices on the rank q of the factorial moments involved is in accord with the quantum statistical expectation. (The points are slightly scattered with different values of the number of bins, however, the linear dependence is not violated.)

5. Intermittency of the JACEE data

In conclusion, it is seen that some experimental results (like *e.g.* those coming from JACEE) cannot be understood in frame of the NBD. To describe at least a portion of the data correctly, also the coherent production of the secondaries is to be introduced. **If** the case when the genuine scaling indices do not vanish, is interpreted in terms of the intermittency, **then** the JACEE data represent a clear evidence of its presence also at very high energies.

It is nice to formulate those conclusions at the twentieth anniversary since publication of the JACEE phenomenon and to show that probably there is "something" encoded in it.

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