JET CALCULUS PROBLEMS OF THE PERTURBATIVE QUANTUM CHROMODYNAMICS*

I.M. Dremin

Lebedev Physical Institute, Moscow, Russia
email: dremin@lpi.ru

(Received November 3, 2003)

The perturbative quantum chromodynamics (pQCD) has been extremely successful in the prediction and description of main properties of quark and gluon jets. There are, however, some problems of the jet calculus with the higher order corrections of the modified perturbative expansion which should be resolved to get more precise statements. Some of them are discussed here.

PACS numbers: 13.65.+i, 13.87.Ce, 13.85.Hd

The numerous achievements of pQCD in the jet calculus are well known and described in the book [1] and many review papers (see, *e.g.*, [2–6]). The leading approximation is perfect and only high order terms need more care. Here I present a critical survey of some problems related to these calculations and rarely discussed. The figures demonstrating the comparison with experiment are omitted to shorten the presentation. They can be found in the above cited review papers.

I mention briefly the following five problems:

1. Different characteristics of jets are differently sensitive to higher order corrections. Therefore, for the comparison with experiment, one should choose those which are not overshadowed by the leading terms of the perturbative expansion and help most efficiently elucidate these corrections.

2. The correction terms are proportional to higher powers of the coupling strength but can get the large numerical coefficients in front of them. Thus, even though in asymptotics this expansion is valid due to the running nature of the coupling strength, at present energies it could fail to provide small corrections. Therefore one should find such characteristics where these coefficients are small enough for corrections to be trusted.

* Presented at the XXXIII International Symposium on Multiparticle Dynamics, Kraków, Poland, September 5–11, 2003.

3. It is very desirable to get the physical interpretation and motivation for the value and nature of the higher order corrections (especially, for cumulant moments).

4. The QCD equations or approximations used in the jet calculus are sometimes not completely precise themselves. Their modifications can be considered or the influence of the omitted terms estimated.

5. Some shortcomings of the analytic approach and numerical solutions are discussed.

I will be mainly concerned with jet characteristics in some sense related to the jet multiplicity distributions that are closer to my personal interests. First, let me remind some simplest definitions [1,2] concerning jet multiplicities in QCD. The generating function G is defined by the formula

$$G(y,u) = \sum_{n=0}^{\infty} P_n(y) u^n \,, \tag{1}$$

where $P_n(y)$ is the multiplicity distribution at the scale $y = \ln(p\Theta/Q_0) = \ln(2Q/Q_0)$, p is the initial momentum, Θ is the angle of the divergence of the jet (jet opening angle), assumed here to be fixed, Q is the jet virtuality, $Q_0 = \mathrm{const.}$, u is an auxiliary variable which is often omitted to shorten notations. The analytic properties of the generating functions in u are of the special interest (see [2,4]) in view of some analogies with the statistical physics, but we will not consider them here.

The moments of the distribution are defined as

$$F_q \;=\; \frac{\sum_n P_n n(n-1)\ldots(n-q+1)}{(\sum_n P_n n)^q} = \frac{1}{\langle n\rangle^q}\,\frac{d^q G(y,u)}{du^q}\bigg|_{u=1}\,, \tag{2}$$

$$K_q \;=\; \frac{1}{\langle n\rangle^q}\,\frac{d^q \ln G(y,u)}{du^q}\bigg|_{u=1}\,. \tag{3}$$

Here, F_q are the factorial moments, and K_q are the cumulant moments, responsible for total and genuine (irreducible to lower ranks) correlations, correspondingly. These moments are not independent. They are connected by definite relations which can easily be derived from moments definitions in terms of the generating function:

$$F_q = \sum_{m=0}^{q-1} C_{q-1}^{m} K_{q-m} F_m \,. \tag{4}$$

The QCD equations for the generating functions are[1]:

$$G'_{\mathrm{G}} = \int_0^1 dx K_{\mathrm{G}}^{\mathrm{G}}(x)\gamma_0^2 \left[G_{\mathrm{G}}(y+\ln x)G_{\mathrm{G}}(y+\ln(1-x)) - G_{\mathrm{G}}(y)\right]$$

$$+n_{\mathrm{f}}\int_0^1 dx K_{\mathrm{G}}^{\mathrm{F}}(x)\gamma_0^2 \left[G_{\mathrm{F}}(y+\ln x)G_{\mathrm{F}}(y+\ln(1-x)) - G_{\mathrm{G}}(y)\right]\,, \quad (5)$$

$$G'_{\mathrm{F}} = \int_0^1 dx K_{\mathrm{F}}^{\mathrm{G}}(x)\gamma_0^2 \left[G_{\mathrm{G}}(y+\ln x)G_{\mathrm{F}}(y+\ln(1-x)) - G_{\mathrm{F}}(y)\right]\,. \quad (6)$$

Here $G'(y) = dG/dy$, n_{f} is the number of active flavors,

$$\gamma_0^2 = \frac{2N_{\mathrm{c}}\alpha_S}{\pi}\,. \quad (7)$$

The running coupling constant in the two-loop approximation is

$$\alpha_S(y) = \frac{2\pi}{\beta_0 y}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\ln 2y}{y}\right) + O(y^{-3})\,, \quad (8)$$

where

$$\beta_0 = \frac{11N_{\mathrm{c}} - 2n_{\mathrm{f}}}{3}\,, \quad \beta_1 = \frac{17N_{\mathrm{c}}^2 - n_{\mathrm{f}}(5N_{\mathrm{c}} + 3C_{\mathrm{F}})}{3}\,. \quad (9)$$

The labels G and F correspond to gluons and quarks, and the kernels of the equations are:

$$K_{\mathrm{G}}^{\mathrm{G}}(x) = \frac{1}{x} - (1-x)[2 - x(1-x)]\,, \quad (10)$$

$$K_{\mathrm{G}}^{\mathrm{F}}(x) = \frac{1}{4N_{\mathrm{c}}}[x^2 + (1-x)^2]\,, \quad (11)$$

$$K_{\mathrm{F}}^{\mathrm{G}}(x) = \frac{C_{\mathrm{F}}}{N_{\mathrm{c}}}\left[\frac{1}{x} - 1 + \frac{x}{2}\right]\,, \quad (12)$$

$N_{\mathrm{c}} = 3$ is the number of colours, and $C_{\mathrm{F}} = (N_{\mathrm{c}}^2 - 1)/2N_{\mathrm{c}} = 4/3$ in QCD.

Hereby, one can get equations for any moment of the multiplicity distribution both for quark and gluon jets. One should just equate the terms

[1] To exclude the nonperturbative region from further consideration, the limits of integration in these equations are often chosen as $\exp(-y)$ and $1 - \exp(-y)$ which tend to 0 and 1 at high energy y.

with the same powers of u in both sides of the equations. In particular, the equations for average multiplicities read

$$\begin{aligned}\langle n_{\mathrm{G}}(y)\rangle' &= \int dx\gamma_0^2[K_{\mathrm{G}}^{\mathrm{G}}(x)(\langle n_{\mathrm{G}}(y+\ln x)\rangle + \langle n_{\mathrm{G}}(y+\ln(1-x)\rangle - \langle n_{\mathrm{G}}(y)\rangle) \\ &\quad + n_{\mathrm{f}}K_{\mathrm{G}}^{\mathrm{F}}(x)(\langle n_{\mathrm{F}}(y+\ln x)\rangle + \langle n_{\mathrm{F}}(y+\ln(1-x)\rangle - \langle n_{\mathrm{G}}(y)\rangle)]\,, \end{aligned} \tag{13}$$

$$\langle n_{\mathrm{F}}(y)\rangle' = \int dx\gamma_0^2 K_{\mathrm{F}}^{\mathrm{G}}(x)\langle n_{\mathrm{G}}(y+\ln x)\rangle + \langle n_{\mathrm{F}}(y+\ln(1-x)\rangle - \langle n_{\mathrm{F}}(y)\rangle)\,. \tag{14}$$

Their solutions can be looked for as

$$\langle n_{\mathrm{G,F}}\rangle \propto \exp\left(\int^{y} \gamma_{\mathrm{G,F}}(y')dy'\right)\,. \tag{15}$$

The lower limit of integration has not been fixed because its variation results in the substitution of a new normalization constant which is not shown in the above relation but is in practice considered as a fitted parameter which depends on the nonperturbative component of the underlying dynamics of a process.

Using the perturbative expansion of the exponent in (15)

$$\gamma_{\mathrm{G}} \equiv \gamma = \gamma_0(1 - a_1\gamma_0 - a_2\gamma_0^2 - a_3\gamma_0^3) + O(\gamma_0^5) \tag{16}$$

one arrives to the so-called modified perturbative expansion of QCD. This means that the perturbative expansion has been used in the exponent of the expression for a physical quantity, *i.e.*, even the first term includes higher power corrections of the ordinary perturbative formulas. Moreover, the expansion parameter is the coupling strength itself and not its squared value α_S as usually happens. The structure of the equations (5), (6) dictates such series. It was first shown in [7] that the systematic expansion can be obtained by considering the Taylor series at low x in Eqs (5), (6). There it was used for higher order calculations in gluodynamics. The ordinary perturbative expansion for mean multiplicity, if boldly attempted, would surely fail because the coupling strength decreases with energy while multiplicities increase mainly due to the enlarged phase space volume. The coefficients a_i are calculable from the Eqs (13), (14).

Let us briefly mention that the equations (5), (6) can be exactly solved [8, 9] for fixed coupling strength, *i.e.*, if γ_0 is set constant. Then the mean multiplicities increase like a power of energy. The comparison with experiment has been done in recent paper [10].

For the running coupling strength the multiplicities increase [1, 11, 12] slower than power-like[2] but stronger than logarithmically, namely

$$\langle n_{\mathrm{G,F}}\rangle = A_{\mathrm{G,F}} y^{-a_1 c^2} \exp\left(2c\sqrt{y} + \delta_{\mathrm{G,F}}(y)\right) , \tag{17}$$

where $c = (4N_{\mathrm{c}}/\beta_0)^{1/2}$,

$$\begin{aligned}\delta_{\mathrm{G}}(y) &= \frac{c}{\sqrt{y}}\left[2a_2c^2 + \frac{\beta_1}{\beta_0^2}(\ln 2y + 2)\right] \\ &\quad + \frac{c^2}{y}\left[a_3c^2 - \frac{a_1\beta_1}{\beta_0^2}(\ln 2y + 1)\right] + O\left(y^{-3/2}\right) .\end{aligned} \tag{18}$$

The corresponding expression for $\delta_{\mathrm{F}}(y)$ can be easily obtained from the formulas for γ_{F}. Usually, in place of γ_{F} the ratio of average multiplicities in gluon and quark jets

$$r = \frac{\langle n_{\mathrm{G}}\rangle}{\langle n_{\mathrm{F}}\rangle} = \frac{A_{\mathrm{G}}}{A_{\mathrm{F}}} \exp(\delta_{\mathrm{G}}(y) - \delta_{\mathrm{F}}(y)) \tag{19}$$

is introduced, and its perturbative expansion

$$r = r_0(1 - r_1\gamma_0 - r_2\gamma_0^2 - r_3\gamma_0^3) + O(\gamma_0^4) \tag{20}$$

is used. The analytic expressions and numerical values of the parameters a_i, r_i for all $i \leq 3$ have been calculated from the perturbative solutions of the above equations. All of them (except $r_0 = N_{\mathrm{c}}/C_{\mathrm{F}} = 9/4$) are at least twice less than 1 (the review is given in [4]).

The relation between the anomalous dimensions of gluon and quark jets is

$$\gamma_{\mathrm{F}} = \gamma - \frac{r'}{r} , \tag{21}$$

where

$$r' \equiv \frac{dr}{dy} = Br_0r_1\gamma_0^3\left[1 + \frac{2r_2}{r_1}\gamma_0 + \left(\frac{3r_3}{r_1} + B_1\right)\gamma_0^2 + O\left(\gamma_0^3\right)\right] \tag{22}$$

with $B = \beta_0/8N_{\mathrm{c}}\,; B_1 = \beta_1/4N_{\mathrm{c}}\beta_0$.

Thus

$$\begin{aligned}\gamma_{\mathrm{F}} &= \gamma_0\left[1 - a_1\gamma_0 - (a_2 + Br_1)\gamma_0^2 - (a_3 + 2Br_2 + Br_1^2)\gamma_0^3\right. \\ &\quad \left. -(a_4 + B(3r_3 + 3r_2r_1 + B_1r_1 + r_1^3))\gamma_0^4\right] .\end{aligned} \tag{23}$$

[2] It is hard to distinguish these dependences at present energies (see [10]) but at 1 TeV their predictions differ by the factor 1.5.

At present, there exist three approaches to treating multiplicities. In analytic solutions of the equations the perturbative approach with approximate energy conservation is used. The numerical solution allows to account accurately for energy conservation. However, the transverse momentum is taken into account only by angular ordering of jets in both approaches. Both energy and momentum are conserved in Monte Carlo QCD models which provide best fits to experimental data at present energies. Their predictions, however, differ at higher energies mainly due to hadronization models used. That is why further studies are needed.

1. Sensitivity to high order terms. The experimental data about the energy dependence of mean multiplicity in e^+e^--annihilation are well described in all approaches. The two leading terms in expressions (17) completely determine it. They are the same for quark and gluon jets. That is why gluodynamics can be used for their estimate as was done in early years. The higher order corrections given by $\delta_{\mathrm{G,F}}$ are almost unnoticeable there. Thus mean multiplicities are not sensitive to these corrections by themselves.

However, if one considers their ratio r, it happens to be really sensitive. This is because in the ratio the two leading terms corresponding to leading order (LO) and next-to-leading order (NLO) cancel since they are the same for both quark and gluon jets. Therefore, only higher order corrections determine the energy behavior of the ratio r. The first term $r_0 = 9/4$ is given by the relative strengths of gluon and quark forces. The next term is proportional to γ_0 and will be called NLO$_r$-correction in distinction to common NLO-terms. Actually, NLO$_r$ corresponds to 2NLO-terms (like a_2) because of the cancellation of the NLO (power-like in y) terms in the ratio of multiplicities (see Eq. (23)). In the same sense the "r_3"-term in r corresponds to 4NLO contribution in γ even though it is proportional to γ_0^3 *etc.* (see [12]). This leads to shift and misuse of the terminology for the anomalous dimensions γ's and for the ratio r.

Thus we have found the characteristic which is more sensitive to higher order corrections than mean multiplicities. The experimental data about the ratio r are described with much lower accuracy about 15–20% in such analytic approach (see [10]). Even though each subsequent perturbative term in r improves the agreement, no precise fit has been achieved yet.

However, one should mention here that the computer solution of the equations [13,14] provides the quantitative fit. This indicates that the higher order uncalculated corrections are still comparatively large for this ratio up to the highest presently available energies. Thus the discrepancy with analytic results is of a purely technical origin.

Another very sensitive characteristic is the behavior of the factorial moments (2) as functions of the size of the phase space bins in which they are measured. Here, one has to deal with a part of the phase space and the above equations are not applicable directly. One has to use the Feynman diagram technique for the treatment of these small bins [15–17]. This complicates the matter. It was impossible to account for high order corrections. Some NLO-terms have only been considered in [16]. From comparison with experiment (see, *e.g.*, [18–20]) it is seen that the qualitative behavior is described but quantitatively the disagreement becomes stronger at smaller bins. This poses the problem of the proper account of higher order corrections. Possible flow of partons from small bins should be considered more precisely. The newly developed technique of the so-called non-global logarithms [21] can be helpful in this respect.

2. High order coefficients. Fortunately, the coefficients a_i and r_i happened to be small enough (see the Table in [4]) so that the subsequent terms in the expansions of γ and r can be trusted even at the rather large values of the expansion parameter $\gamma_0 \approx 0.4$–0.5 at present energies. This is not always the case for some other characteristics. If the high order terms become larger than 1, the expansion can not be trusted. Thus the next problem is to find such characteristics for which it does not happen. Only these features can be reliably compared with experiment.

This criterium becomes crucial, *e.g.*, for the slope r' of the ratio r. The cancellation of two leading terms in the ratio r reveals itself in the proportionality of the scale (energy) derivative r' to γ_0^3. Therefore it can be calculated up to the terms $O(\gamma_0^5)$. The leading term is very small (about 0.02 at the Z^0-resonance). Asymptotically, all corrections vanish. However, at present energies of Z^0, they are so large that calculations become unreliable. The second term in the brackets in (22) is larger than 1 since $2r_2/r_1 \approx 4.9$ and $\gamma \approx 0.45$–0.5. Even the third term is approximately about 0.4. The problem of convergence of the series at Z^0-energies and below becomes crucial.

Therefore, it is desirable to use at present energies such characteristics which are sensitive to these corrections and do not possess large coefficients in front of the expansion parameter. In particular, it has been shown in [12] that these coefficients are smaller in the ratio of derivatives (slopes)

$$r^{(1)} = \frac{\langle n_{\mathrm{G}} \rangle'}{\langle n_{\mathrm{F}} \rangle'}\,. \tag{24}$$

This ratio should be slightly larger than r

$$r^{(1)} \approx r(1 + Br_1\gamma_0^2) \approx r(1 + 0.07\gamma_0^2)\,. \tag{25}$$

The same is true for the ratio of curvatures (or second derivatives)

$$r^{(2)} = \frac{\langle n_{\mathrm{G}} \rangle''}{\langle n_{\mathrm{F}} \rangle''}\,. \tag{26}$$

It is even closer to the asymptotics

$$r^{(2)} \approx r(1 + 2Br_1\gamma_0^2) \approx r(1 + 0.14\gamma_0^2)\,. \tag{27}$$

The QCD predictions for them

$$r < r^{(1)} < r^{(2)} < 2.25 \tag{28}$$

have been confirmed in experiment.

The present experimental accuracy does not allow, unfortunately, to measure these values more accurately. As one sees, in expressions for $r^{(1)}, r^{(2)}$ the coefficients in front of γ_0^2 are slightly decreased compared with r but not in front of γ_0. The last ones cancel in their ratios to r so that the second order terms are left. However, these ratios $r^{(1)}/r$ and $r^{(2)}/r$ have not yet been accurately measured. Further search for such characteristics is needed.

3. Interpretation. Another question I'd like to raise concerns physical interpretation of the high order effects. First of all I mean the oscillations of cumulant moments as functions of their rank in QCD. They have not yet been completely clarified. They were predicted analytically [7] and numerically [22] as the effect of the high order terms of the modified perturbative expansion. Their detailed study was recently performed in [23] by the numerical solution of QCD equations. First experimental confirmation was found in [24, 25].

A peculiar feature of multiplicity distributions has been noticed in [23]. The even order factorial moments F_2, F_4, F_8, F_{16} become equal to 1 at the energy about 20 GeV. This implies that the distribution has a quasi-Poissonian shape. At lower energies it is sub-Poissonian, at higher ones — super-Poissonian. The similar conclusion for gluon jets can be derived from results of [10]. It has been stated that no analytic explanation of it is known. Actually, it is hard to proceed with analytic calculations to high rank moments because the expansion parameter $q\gamma$ becomes large. However, one can answer the question about the energy where F_2 is equal to 1 in NLO-approximation. This moment plays the main role for the distribution since other moments are quite small. The equality $F_2 = 1$ implies $K_2 = 0$, and according to [7] can be written as

$$1 - 4h_1\gamma = 0\,, \tag{29}$$

where $h_1 = 11/24, \gamma$ is the QCD anomalous dimension. The energy E at which this is satisfied is given by

$$\ln\frac{M_Z}{E} = \frac{2\pi}{\beta_0}\left(\frac{1}{\alpha_Z} - \frac{1}{\alpha_E}\right) \tag{30}$$

with $\alpha_E = \pi\gamma^2/6 = \pi/96h_1^2$; $\alpha_Z \approx 0.118$; $\beta_0 = 9$ for $n_{\rm f} = 3$. Hereby one easily estimates

$$E \approx 20 \text{ GeV}. \tag{31}$$

Thus the analytic estimations of the transition region coincide quite well with computer calculations [23] and experiment [10]. Its universality for other collisions would be interesting to check.

Usually exploited phenomenological distributions of the probability theory do not possess any oscillations. *E.g.*, all cumulant moments of the Poisson distribution are identically zero. One interprets this as the absence of genuine correlations irreducible to the lower-rank correlations. For the negative binomial distribution with the parameter k one easily gets

$$H_q = \frac{K_q}{F_q} = kB(q,k) > 0\,. \tag{32}$$

Since F_q are always positive according to their definition, this inequality implies the positive values of K_q.

In the leading order approximation, the gluodynamics equation for the generating function

$$[\ln G(y)]'' = \gamma_0^2(G(y) - 1) \tag{33}$$

transforms in the relation

$$q^2 K_q = F_q \quad \text{or} \quad H_q = \frac{1}{q^2}\,. \tag{34}$$

However, already in the next-to-leading order H_q-moments become negative with a minimum at the rank $q_{\rm min} \approx \frac{24}{11\gamma_0} + 0.5 \approx 5$ [7]. This minimum is rather stable. Nevertheless this is a purely preasymptotic feature. The minimum slowly moves to higher ranks with energy increase and disappears in asymptotics as is required according to the formula (34). At higher orders of the perturbative expansion, the oscillations of higher rank cumulant moments show up [22,23]. They have been confirmed in experiment. Convergence to $1/q^2$-limit with energy increase has been noticed in [23] for low-rank moments. We are interested to get from experiment the data about the energy behavior of the ratios H_q or, better, of the asymptotically normalized ratios $T_q = q^2 H_q$ which should tend to 1 in asymptotics independently of q. It would ask for high precision data at different energies.

The oscillations of cumulants reveal non-trivial collective behavior of particles. The cumulants remind the virial coefficients of statistical physics. The changing character of the genuine correlations implies that repulsion is replaced by attraction (clustering, Van der Waals forces) in particle systems with different number of particles. If the similar behavior of correlators persists at quark level then it reminds, *e.g.*, the theory of superfluidity. Asymptotic disappearance of oscillations would correspond to transition from superfluid to normal component. In superconductivity, it is at the origin of

Cooper pairs. Has it any impact on the hydrodynamical model of multiple production? It would be exciting to find other examples of such a behavior in hadronic systems. From experimental side, it would, perhaps, reveal itself in the irregular behavior of mean multiplicities of subjets.

4. Generalization. Finally, there exists the problem of possible generalization of the equations for the generating functions. As such, the Eqs (5), (6) have only been proved (see, *e.g.*, [1]) up to the NLO-approximation. In principle, their high order treatment is unjustifued. Nevertheless, one can assume that these equations have the status of the kinetic equations of QCD.

From one side, we understand that even if treated as kinetic equations these equations are limited by our ignorance of the four-gluon interaction and non-perturbative effects, by the simplified treatment of conservation laws *etc.* Actually, the energy conservation is accounted by the $\ln x$ and $\ln(1-x)$ terms in the equations. In the perturbative expansion we cut off the Taylor expansions of the generating functions. Thus we approximate the energy conservation. Namely this reveals itself in factorial moments behavior for small bins and in the oscillations of cumulant moments. In the computer solutions [13, 14, 23] the energy (but not $p_{\rm t}$) restrictions are precisely considered and the results show better precision. Thus, probably the inaccuracies of the analytic approach are connected just with the improper treatment of the kinematic boundaries.

The modification of above equations was proposed [26] in the framework of the dipole approach to QCD with more accurate kinematic bounds accounting for the transverse momenta as well. It has been shown that the ratio r can be obtained in good agreement with experimental data. Nevertheless, further study [27] of higher rank moments of the multiplicity distribution predicted by the modified equations has shown their extremely high sensitivity to higher orders of the perturbative expansion. The results become inconclusive.

The more radical phenomenological approaches to generalize these equations were attempted earlier [28–30]. In [28] it was proposed to treat hadronization of partons at the final stage of jet evolution in analogy with the ionization in electromagnetic cascades where it results in their saturation and in the finite length of the shower. This leads to some modified equations if the analogy between ionization losses in QED and confinement in QCD is imposed. Three different stages of the cascade were considered in the modified kinetic equations proposed in [29, 30]. No quantitative results were, however, obtained.

Thus no successful generalization is at work nowadays. Rather, in view of quite satisfactory agreement with experiment, the general theoretical trend has shifted to the direct calculation of non-perturbative effects in some jet characteristics (see, *e.g.*, [31,32]) and to understanding effects described by the non-global logarithms [21].

5. The shortcomings of the analytic and numerical approaches. The success of numerical solutions of QCD equations [8,13,14,23] raises the question if the generalization will give any other noticeable contribution. Our failure to describe more precisely the ratio r in analytic approach could be just due some defects of the purely perturbative expansion at available energies. The high order terms considered above correspond to corrections only due to more accurate treatment of the energy conservation and of the two-loop expression for the coupling strength (the term with β_1 in (8), (18) considered). Also it was claimed recently [33] that the renormalization group improvement of the perturbative results gives rise to good description of experimental data. Even in numerical calculations, it is still impossible to consider in a proper way the transverse momenta. No high order terms have been added to the kernels (10)–(12). The four-gluon vertex has been completely ignored. Also, the non-perturbative effects are disregarded. All these shortcomings provide the problems for further studies in the framework of analytic and numerical approaches as well as for Monte Carlo models.

In conclusion, I would say that the practical accuracy of the pQCD calculations is high enough. This is somewhat surprising in view of the rather large value of the expansion parameter at present energies. They can serve as a good estimate of the background in searches for new physics effects. However, some principal questions concerning the calculation of several properties of quark–gluon jets and the validity of QCD equations for the generating functions at higher orders have not yet been resolved.

This work has been supported in part by the RFBR grants: N 02-02-16779, 03-02-16134, NSH-1936.2003.2.

REFERENCES

[1] Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller, S.I. Troyan, *Basics of Perturbative QCD*, Ed. J. Tran Thanh Van, Editions Frontieres, 1991.

[2] I.M. Dremin, *Phys. Usp.* **37**, 715 (1994).

[3] V.A. Khoze, W. Ochs, *Int. J. Mod. Phys. A* **12**, 2949 (1997).

[4] I.M. Dremin, J.W. Gary, *Phys. Rep.* **349**, 301 (2001).

[5] V.A. Khoze, W. Ochs, J. Wosiek, *At the Frontier of Particle Physics*, Ed. M. Shifman, World Scientific, Singapore, Vol. 2, 2001, p. 1101.

[6] I.M. Dremin, *Usp. Fiz. Nauk* **172**, 551 (2002); *Phys. Usp.* **45** 507 (2002).

[7] I.M. Dremin, *Phys. Lett.* **B313**, 209 (1993).

[8] I.M. Dremin, R.C. Hwa, *Phys. Rev.* **D49**, 5805 (1994).

[9] I.M. Dremin, R.C. Hwa, *Phys. Lett.* **B324**, 477 (1994).

[10] OPAL Collaboration, S. Campana, *Acta Phys. Pol. B* **35**, 407 (2004), J.W. Gary, 365, these Proceedings.

[11] I.M. Dremin, J.W. Gary, *Phys. Lett.* **B459**, 341 (1999).

[12] A. Capella, I.M. Dremin, J.W. Gary, V.A. Nechitailo, J. Tran Thanh Van, *Phys. Rev.* **D61**, 074009 (2000).

[13] S. Lupia, *Phys. Lett.* **B439**, 150 (1998); Proc. of the XXXIII Moriond Conf. *QCD and Strong Interactions*, March 1998, Ed. J. Tran Thanh Van, Editions Frontieres 1998, p. 363.

[14] S. Lupia, W. Ochs, *Phys. Lett.* **B418**, 214 (1998); *Nucl. Phys. (Proc. Suppl.)* **B64**, 74 (1998).

[15] W. Ochs, J. Wosiek, *Phys. Lett.* **B289**, 159 (1992); **304**, 144 (1993).

[16] Yu.L. Dokshitzer, I.M. Dremin, *Nucl. Phys.* **B402**, 139 (1993).

[17] Ph. Brax, J.L. Meunier, R. Peschanski, *Z. Phys.* **C62**, 649 (1994).

[18] E. DeWolf, I.M. Dremin, W. Kittel, *Phys. Rep.* **270**, 1 (1996).

[19] W. Kittel, *Acta Phys. Pol. B* **32**, 3927 (2001).

[20] E. Sarkisyan, *Acta Phys. Pol. B* **33**, 3305 (2002).

[21] M. Dasgupta, G.P. Salam, *Phys. Lett.* **B512**, 323 (2001); *J. High Energy Phys.* **0203**, 017 (2002).

[22] I.M. Dremin, V.A. Nechitailo, *Mod. Phys. Lett.* **A9**, 1471 (1994); *JETP Lett.* **58**, 881 (1993).

[23] M.A. Buican, C. Forster, W. Ochs, *Eur. Phys. J.* **C31**, 57 (2003); W. Ochs,*Acta Phys. Pol. B* **35**, 429 (2004), these Proceedings.

[24] I.M. Dremin, V. Arena, G. Boca *et al.*, *Phys. Lett.* **B336**, 119 (1994).

[25] SLD Collaboration, K. Abe *et al.*, *Phys. Lett.* **B371**, 149 (1996).

[26] P. Eden, Proc. of the XXXVI Moriond Conf. *QCD and Strong Interactions*, March 2001, Ed. J. Tran Thanh Van, Editions Frontieres 2001, p. 129.

[27] I.M. Dremin, P. Eden, Proc. of the 31 Int. Symp. on Multiparticle Dynamics, 1-7 Sept. 2001, Datong, China, Ed. L. Liu, Y. Wu, World Scientific, Singapore 2002, p. 270.

[28] I.M. Dremin, *Pis'ma Zh. Eksp. Teor. Fiz.* **31**, 215 (1980); *JETP Lett.* **31**, 185 (1980).

[29] J. Ellis, K. Geiger, *Phys. Rev.* **D52**, 1500 (1995).

[30] J. Ellis, K. Geiger, *Nucl. Phys.* **A590**, 609c (1995).

[31] Yu.L. Dokshitzer, Talk at the IPPP Workshop on Future Physics at HERA, Durham, England, 6-7 Dec., 2001.

[32] A. Banfi, Yu.L. Dokshitzer, G. Marchesini, G. Zanderighi, *J. High Energy Phys.* **0007**, 002 (2000); *Phys. Lett.* **B508**, 269 (2001); *J. High Energy Phys.* **0103**, 007 (2001).

[33] K. Hamacher, Talk at the IPPP Workshop on Future Physics at HERA, Durham, England, 6-7 Dec., 2001.