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OSCILLATING HADRON AND JET MULTIPLICITY MOMENTS*

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Recently, the moments of multiplicity distributions in e^+e^- annihilation and the ratios H_q (cumulant over factorial moments K_q/F_q) have been determined both for the hadronic final state and for jets at variable resolution. These ratios show an oscillatory behaviour as function of q with strong dependence of the amplitude and length of oscillation on the jet resolution parameter $y_{\rm cut}$. The recent explanation of this phenomenon based on perturbative QCD calculations is discussed.

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1. Introduction

A characteristic feature of high energy collisions is the multiparticle production. It is generally understood as following from a primary hard interaction (here $e^+e^- \to q\bar{q}$) with subsequent evolution of a parton cascade according to the rules of perturbative QCD from the initial scale Q down to a low energy scale Q_0 where colour confinement leads to the formation of hadrons. The parton evolution is dominated by gluon bremsstrahlung with its collinear and soft singularities which causes the jet structure of the partonic final state also characteristic of the final state of hadrons. The transition into hadrons is described by non-perturbative models. In a particularly simple ansatz for hadronization one assumes that observables for the multiparticle final state are proportional to the corresponding quantities for partons, an idea which has been proposed originally for single inclusive energy spectra and is called "Local Parton Hadron Duality" (LPHD). Subsequently, this idea has been applied to a rather wide range of phenomena from inclusive multiparticle correlations to quasi-exclusive processes (for reviews, see [2, 3]). The agreement with data generally increases with the accuracy of the calculations.

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Here we report on a recent application [1] of these duality ideas to the moments of multiplicity distributions which correspond to integrals over multiparticle correlation functions. Experimental results on these quantities have been presented by the L3 collaboration [4, 5].

2. Multiplicity moments

We consider here the unnormalized and normalized factorial moments f_q and F_q

$$f_q = \sum_{n=0}^{\infty} n(n-1)\dots(n-q+1)P_n\,, \quad F_q = \frac{f_q}{N^q}\,, \quad N \equiv f_1 \tag{1}$$

with multiplicity distribution P_n and mean multiplicity N. Furthermore, one introduces the cumulant moments k_q and K_q which are used to measure the genuine correlations without uncorrelated background in a multiparticle sample

$$k_q = f_q - \sum_{i=1}^{q-1} \binom{q-1}{i} k_{q-i} f_i\,, \qquad K_q = \frac{k_q}{N^q}\,, \tag{2}$$

in particular $K_2 = F_2 - 1,\ K_3 = F_3 - 3F_2 + 2$; for a Poisson distribution $K_1 = 1,\ K_q = 0$ for $q > 1$. Of special interest are the ratios

$$H_q = \frac{K_q}{F_q} \tag{3}$$

as there is a prediction for high energies of an oscillatory behaviour [6] with the first minimum near $q \approx 5$ at LEP energies. Such a minimum has been observed indeed in e^+e^- annihilations at SLC [7] and at LEP [4, 5] but the magnitudes of moments are found much smaller than predicted [8].

3. QCD predictions

Predictions in [1] are obtained by solving numerically the evolution equations for the moments which are derived from the evolution equations of the single jet generating function (review [2]) together with the initial condition at threshold at fixed transverse momentum cut-off Q_c. This cut-off characterizes at the same time the hadronization scale $Q_\mathrm{c} = Q_0 \gtrsim \Lambda_\mathrm{QCD}$ in the description of the hadronic final state and also the jet resolution (parameter $y_\mathrm{cut} = (Q_\mathrm{c}/Q)^2,\ Q = 2E$ at jet energy E) for the jet final state obtained with a jet algorithm based on a transverse momentum cut-off ("Durham algorithm"). Different approximations are considered.

In the Double Logarithmic Approximation (DLA) one takes into account the most singular bremsstrahlung singularities. This corresponds to the asymptotic solution and some results can be given in analytic form, for example, $H_q \to 1/q^2$. These asymptotic results are quickly approached above threshold energy $E = Q_0$ where there is only one particle in the jet. Then, $F_q = 0$ for $q > 1$ and K_q oscillates

$$K_q = (-1)^{q-1}(q-1)! \quad \text{for } Y_c = \ln \frac{E}{Q_c} = 0 \tag{4}$$

and consequently also H_q does.

The Modified Logarithmic Approximation (MLLA) takes into account certain non-leading logarithmic terms and gives a more realistic description at finite energies. In Fig. 1 the energy dependence of the parton multiplicity (cut-off Q_0 for hadrons) is seen above the multiplicity for jets at the same ratio E/Q_c because the transverse momentum is in the argument of the coupling constant α_s and $Q_c > Q_0$ (see also [8]). Contrary to the DLA the threshold for higher moments F_q is shifted towards higher energies. Remarkably, there is a Poissonian transition point with $F_q = 1$ for all moments. Below this point the H_q ratios show rapid oscillations as in (4) whereas above there is a transition to oscillations of length increasing with energy.

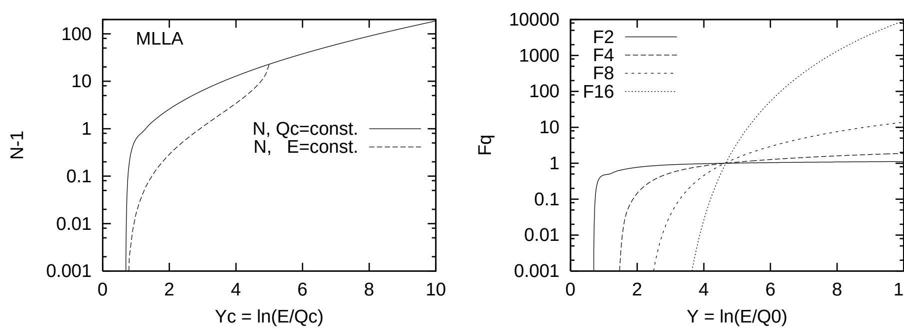

Fig. 1. Multiplicity N in MLLA in single jet vs energy for fixed $Q_c = Q_0$ (representing hadrons) and for fixed energy $Y_0 = \ln(E/Q_0) = 5$ but variable Q_c representing jets at variable resolution Q_c and factorial moments F_q *vs* energy Y.

Finally, a comparison of data is performed with a Monte Carlo calculation of the parton cascade where we use ARIADNE-D as in [9] but with refit parameters Q_0 and Λ_{QCD} to simulate the hadronic final state directly. Here energy momentum conservation is fully taken into account as well as the $O(\alpha_s)$ matrix element for $e^+e^- \to q\bar{q}g$. The results are similar to those from the MLLA evolution equation. In particular, the data on jet and hadron multiplicities with the splitting as in Fig. 1 are rather well described and also the variation of the oscillation pattern of H_q with jet resolution $y_{cut} = (Q_c/Q)^2$, see Fig. 2, in further support of the duality approach.

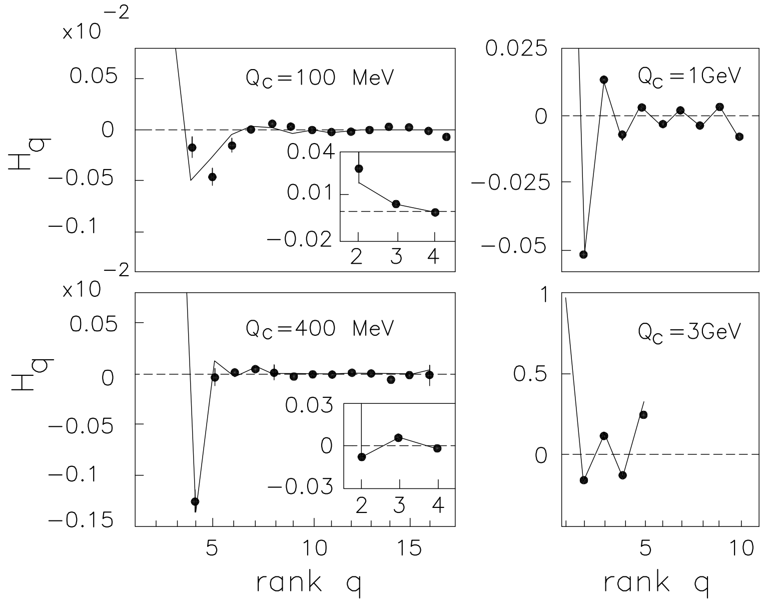

Fig. 2. Ratios H_q of multiplicity moments for jets at different cut-off Q_c as obtained by L3 Collaboration [4] compared with our MC results (ARIADNE-D).

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