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TWO-MODE SQUEEZED AND ENTANGLED GLUON STATES*

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We study non-perturbative evolution of the gluon states during small time. Fluctuations of the gluons are less than those for coherent states that is indication of the gluon squeezed states. We show that the two-mode squeezed and entangled states of the gluon fields can appear as a result of the non-perturbative gluon selfinteraction.

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Analogies between multiple hadron in HEP and photon production in quantum optics (QO) discussed long ago [1–4], in particular, the general distribution that characterizes $e^+e^-, p\bar{p}$ is a k-mode squeezed state distribution [5, 6], squeezed states (SS) can have both sub-Poissonian (for coincide phases) and super-Poissonian (for antiphases) statistics corresponding to antibunching and bunching of photons [7, 8], characteristic behaviour of the factorial and cumulant moments [9]. Studying non-perturbative evolution of gluon states prepared by perturbative cascade stage in jets [10] we have proved within quantum chromodynamics (QCD) that this stage of jet evolution can be a source of single-mode gluon SS by analogy with nonlinear devices in QO for photon [11–13].

At the same time two-mode photon SS [7] defined as

$$|\mathrm{f}\rangle = S(r)|0\rangle_1|0\rangle_2 = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n\,|n\rangle_1|n\rangle_2 \qquad (1)$$

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are simultaneously the entangled states [14]. Here $S(r) = \exp\{r(a_1^+ a_2^+ - a_1 a_2)\}$ is operator of two-mode squeezing. In fact, rewriting the expression (1) at small squeezing parameter r

$$|\mathrm{f}\rangle = |0\rangle_1 |0\rangle_2 + r\,|1\rangle_1 |1\rangle_2, \tag{2}$$

it is easy to show that the state vector $|\mathrm{f}\rangle$ describes the entangled state. The entanglement condition of the photon state can be verified by investigation of the conditional probability $P\,(Y_j/X_i)\;(i,j=1,2)$

$$P\,(Y_j/X_i) = \frac{\|\langle Y_j|\,\langle X_i\,|\mathrm{f}\rangle\|^2}{\langle \mathrm{f}|\,X_i\rangle\,\langle X_i\,|\mathrm{f}\rangle} = \begin{cases} \delta_{ij} \\ \text{or} \\ 1-\delta_{ij}, \end{cases} \tag{3}$$

where $|X_1\rangle = |0\rangle_1, |X_2\rangle = |1\rangle_1$, $|Y_1\rangle = |0\rangle_2, |Y_2\rangle = |1\rangle_2$.

In order to verify whether the gluon state vector describes the two-mode squeezed states on colours h and g, it is necessary to introduce the phase-sensitive Hermitian operators $(X_\lambda^{h,g})_1 = [b_\lambda^h + b_\lambda^g + b_\lambda^{h+} + b_\lambda^{g+}]/(2\sqrt{2})$ and $(X_\lambda^{h,g})_2 = [b_\lambda^h + b_\lambda^g - b_\lambda^{h+} - b_\lambda^{g+}]/(2i\sqrt{2})$ by analogy with quantum optics [7] and to establish conditions under which the variance of one of them can be less than the variance of a coherent state. Here $b_{(\lambda)}^h, b_\lambda^g (b_\lambda^{h+}, b_\lambda^{g+})$ are the operators annihilating (creating) a gluons of colours $h,g = \overline{1,8}$ and polarization index $\lambda = \overline{1,3}$.

The condition of two-mode squeezing for colours h,g is

$$\left\langle N\left(\Delta (X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \left\langle \left(\Delta (X_\lambda^{h,g})_{\frac{1}{2}}\right)^2\right\rangle - \frac{1}{4} <\, 0. \tag{4}$$

Averaging of operators $\left(\Delta (X_\lambda^{h,g})_{\frac{1}{2}}\right)^2$ for gluons with specified colours and polarization index are taken over the vector $|\mathrm{f}\rangle$ which describes the evolution of the virtual gluon field during small time interval Δt within interaction representation

$$|\mathrm{f}\rangle \simeq |\mathrm{in}\rangle -\, i\,\Delta t\, H_{\mathrm{I}}(t_0)\,|\mathrm{in}\rangle, \tag{5}$$

where $H_{\mathrm{I}}(t_0) = H_{\mathrm{I}}^{(3)}(t_0) + H_{\mathrm{I}}^{(4)}(t_0)$ is the Hamiltonian three-gluon $(H_{\mathrm{I}}^{(3)})$ and four-gluon $(H_{\mathrm{I}}^{(4)})$ self-interactions in momentum representation, $|\mathrm{in}\rangle$ is an initial state vector of the virtual gluon field, $\Delta t = t - t_0$ (below we will assume $t_0 = 0$ and consequently $\Delta t = t$).

We choose product of the coherent states of the gluons with different colour and polarization indexes as initial state vector that is $|\mathrm{in}\rangle \equiv |\,\alpha\rangle = \prod\limits_{\lambda=1}^{3}\prod\limits_{b=1}^{8} |\alpha_\lambda^b\rangle$ because any vector may be decomposed on these basis vectors and coherent states are widely used in QO [7,8]. Gluon coherent state vector $|\alpha^b_\lambda\rangle$ is the eigenvector of the corresponding annihilation operator b^b_λ with eigenvalue $\alpha^b_{\lambda_1} = |\alpha^b_{\lambda_1}|\mathrm{e}^{i\,\gamma^b_{\lambda_1}}$, where $|\alpha^b_{\lambda_1}|$ is the gluon coherent field amplitude and γ^b_λ is the phase of the given gluon field.

It is easy to show that the three-gluon self-interaction (as in single-mode case [13]) does not lead to squeezing effect and only the four-gluon self-interaction can yield to the two-mode squeezing effect. Indeed, the two-mode squeezing condition can be written in explicit form as

$$\begin{aligned}
&\left\langle N\left(\Delta(X^{h,g}_\lambda)_{\frac{1}{2}}\right)^2\right\rangle = \pm\frac{i\,t}{8}g^2(2\pi)^3\int d\tilde{k}_1 d\tilde{k}_2\sum_{\lambda_1,\lambda_2}\Bigg\{\langle\alpha|b^{b+}_{\lambda_1}(k_1)b^{c+}_{\lambda_2}(k_2)\\
&-b^b_{\lambda_1}(k_1)b^c_{\lambda_2}(k_2)|\alpha\rangle\left[\delta(2\vec{k}-\vec{k_1}-\vec{k_2})-\delta(2\vec{k}+\vec{k_1}+\vec{k_2})\right]\Big[(f_{ahb}f_{ahc}+f_{agb}f_{agc}\\
&+2f_{ahb}f_{agc})\Big(\varepsilon^{\lambda_1}_\mu(k_1)\,\varepsilon^\mu_{\lambda_2}(k_2)\varepsilon^\lambda_\nu(k)\,\varepsilon^\nu_\lambda(k)-\varepsilon^{\lambda_1}_\mu(k_1)\varepsilon^\mu_\lambda(k)\varepsilon^{\lambda_2}_\nu(k_2)\,\varepsilon^\nu_\lambda(k)\Big)\\
&-2f_{ahg}f_{abc}\,\varepsilon^{\lambda_1}_\mu(k_1)\varepsilon^\mu_\lambda(k)\varepsilon^{\lambda_2}_\nu(k_2)\varepsilon^\nu_\lambda(k)\Big]+2\langle\alpha|b^{b+}_{\lambda_1}(k_1)b^c_{\lambda_2}(k_2)|\alpha\rangle\\
&\times\left[\delta(2\vec{k}-\vec{k_1}+\vec{k_2})-\delta(2\vec{k}+\vec{k_1}-\vec{k_2})\right](f_{ahb}f_{ahc}+f_{agb}f_{agc}+f_{ahc}f_{agb}\\
&+f_{ahb}f_{agc})\Big(\varepsilon^{\lambda_1}_\mu(k_1)\,\varepsilon^\mu_{\lambda_2}(k_2)\varepsilon^\lambda_\nu(k)\,\varepsilon^\nu_\lambda(k)-\varepsilon^{\lambda_1}_\mu(k_1)\,\varepsilon^\mu_\lambda(k)\varepsilon^{\lambda_2}_\nu(k_2)\,\varepsilon^\nu_\lambda(k)\Big)\Bigg\}<0.
\end{aligned}\tag{6}$$

Here g is a self-interaction constant, $d\tilde{k} = \dfrac{d^3k}{(2\pi)^3 2k_0}$, k_0 is a gluon energy, ε^μ_λ is a polarization vector, f_{ahb} are a structure constants of $\mathrm{SU_c}(3)$ group.

For simplicity let us investigate the obtained two-mode squeezing condition (4) for collinear gluon

$$\begin{aligned}
\left\langle N\left(\Delta(X^{h,g}_\lambda)_{\frac{1}{2}}\right)^2\right\rangle &= \pm\,t\,\frac{\alpha_s\pi}{4k_0}(f_{ahb}f_{ahc}+f_{agb}f_{agc}+f_{ahb}f_{agc}+f_{agb}f_{ahc})\\
&\times\sum_{\lambda_1\neq\lambda}|\alpha^b_{\lambda_1}||\alpha^c_{\lambda_1}|\sin(\gamma^b_{\lambda_1}+\gamma^c_{\lambda_1})<0.
\end{aligned}\tag{7}$$

The two-mode squeezing condition (7) is fulfilled in any cases apart from $\gamma^b_{\lambda_1}+\gamma^c_{\lambda_1}=0,\pi$. In particular, if all initial gluon coherent fields are real or imaginary then the two-mode squeezing condition is not fulfilled as in the single-mode case. Obviously, the larger are both amplitudes of the initial gluon coherent fields with different colour and polarization indexes and coupling constant, the larger is two-mode squeezing effect. Thus non-perturbative gluon evolution is very significant under investigation of the squeezing effect.

We regard that two-mode gluon SS with fixed colours h,g are also connected with corresponding entangled states of the gluons. By analogy with

QO as initial states we take vector including the states $|\,0^h_\lambda\rangle\;|\,0^g_\lambda\rangle$ evolution of which leads to the next final vector

$$|\mathrm{f}\rangle=|0^h_\lambda\rangle|0^g_\lambda\rangle+r\,|1^h_\lambda\rangle|1^g_\lambda\rangle,\qquad r=2\left|\left\langle N\left(\Delta(X^{h,g}_\lambda)_{\frac{1}{2}}\right)^2\right\rangle\right|. \tag{8}$$

The entanglement condition of the gluon states with colours h,g and polarization λ can be verified by calculation of the conditional probability $P\,(Y_j/X_i)\,(i,j=1,2)$ by analogy with condition for photons (3) assuming that $|X_1\rangle = |0^h_\lambda\rangle$, $|X_2\rangle = |1^h_\lambda\rangle$, $|Y_1\rangle = |0^g_\lambda\rangle$, $|Y_2\rangle = |1^g_\lambda\rangle$. It can be shown that the condition (3) is fulfilled for the final gluon state vector $|\mathrm{f}\rangle$ (8).

Thus we show that two-mode gluon squeezed and entangled states appear as a result of four-gluon self-interaction. Two-mode gluon entangled states with two different colours can lead to $q\bar{q}$-entangled states. Interaction (quantum measurement) of the quark entangled states with stochastic vacuum (reservoir in QO) has a remarkable property, namely, as soon as some measurement projects one quark onto a state with definite colour, the other quark also immediately obtains opposite colour that can lead to coupling of quark-antiquark pair, string tension between q and $\bar{q}$ (confinement [15]) and free propagation of colourless hadrons.

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