

## SATURATION AND DIFFRACTIVE DIS\*

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We review QCD based descriptions of diffractive deep inelastic scattering emphasizing the role of models with parton saturation. These models provide natural explanation of such experimentally observed facts as the constant ratio of  $\sigma^{\text{diff}}/\sigma^{\text{tot}}$  as a function of the Bjorken variable  $x$  and Regge factorization of diffractive parton distributions.

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**1. Introduction**

Around 10% of deep inelastic scattering (DIS) events observed at HERA at small value of the Bjorken variable  $x$  are diffractive events [1, 2], when the incoming proton stays intact losing only a small fraction  $x_P$  of its initial momentum. A large rapidity gap is formed between the scattered proton (or its low mass excitation) and the diffractive system. The ratio of diffractive to total DIS cross sections is to a good approximation constant as a function of  $Q^2$ . Thus in a first approximation, DIS diffraction is a leading twist effect with logarithmic scaling violation. Moreover, the same ratio as a function of  $x$  (or energy) is also constant. Theoretical models of diffraction should explain these facts.

**2. Diffractive parton distributions**

In addition to the Bjorken variable  $x = Q^2/(Q^2 + W^2)$ , there are two dimensionless variables used in the description of DIS diffraction

$$x_P = \frac{Q^2 + M^2}{Q^2 + W^2}, \quad \beta = \frac{x}{x_P} = \frac{Q^2}{Q^2 + M^2}, \quad (1)$$

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where  $M^2$  is invariant mass squared of the diffractive system and  $W^2$  is the center-of-mass energy squared of the  $\gamma^*p$  system, see Fig. 1. In analogy to the inclusive DIS, the diffractive structure functions are defined:  $F_{2,L}^D(x, x_P, Q^2, t)$ , where  $t = (p - p')^2$  is the four momentum squared transferred from the proton into the diffractive system.

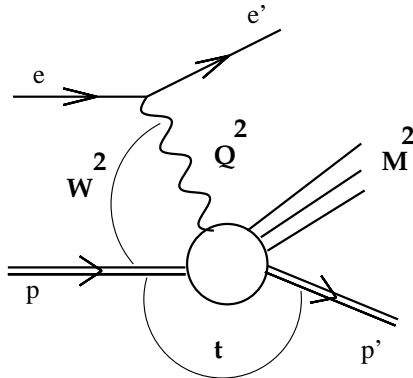


Fig. 1. Kinematic invariants in DIS diffraction.

The leading twist description of diffractive DIS is realized using the diffractive parton distributions (DPD)  $q^D$  in terms of which

$$F_2^D = \sum_{i=1}^{N_f} e_i^2 \beta \{ q_i^D(x_P, t; \beta, Q^2) + \bar{q}_i^D(x_P, t; \beta, Q^2) \} , \quad (2)$$

where  $i$  enumerates quark flavours. Eq. (2) is an example of the collinear factorization formula proven for DIS diffraction in [3]. In the infinite momentum frame, the DPD have an interpretation of conditional probabilities to find a parton in the proton with the momentum fraction  $x = \beta x_P$  under the condition that the incoming proton stays intact and loses the fraction  $x_P$  of its momentum. The collinear factorization fails in hadron–hadron hard diffractive scattering due to initial state soft interactions [4, 5]. Thus, unlike inclusive scattering, the diffractive parton distributions are no universal quantities. They can be used, however, for different diffractive processes in lepton–nucleon scattering, *e.g.* for diffractive dijet production.

The collinear factorization theorem of [3] allows to use the Altarelli–Parisi (DGLAP) evolution equations to find the  $Q^2$  dependence of DPD, provided the initial conditions for evolution are known. They are found from fits to diffractive DIS data in full analogy to the determination of inclusive parton distributions [1, 2]. In the evolution equations only  $(\beta, Q^2)$  are relevant variables while  $(x_P, t)$  play the role of external parameters. Thus a modelling of the latter dependence for the DPD is necessary. This

is done using physical ideas about the nature of interactions leading to DIS diffraction.

Traditionally, diffraction is related to the exchange of a pomeron: a vacuum quantum number exchange, described by the linear Regge trajectory  $\alpha_P(t) = \alpha_P(0) + \alpha' t$  with  $\alpha_P(0) \geq 1$ , which dominates at high energy ( $s \rightarrow \infty$ ,  $t = \text{const.}$ ). This is the basis of the Ingelman–Schlein (IS) [6] model in which the pomeron is exchanged between the proton and the diffractive system. In this case  $F_2^D$  factorizes into a pomeron flux  $f(x_P, t)$  and pomeron parton distributions  $q_P(\beta, Q^2)$  obeying the DGLAP equations

$$F_2^D = f(x_P, t) \sum_{i=1}^{N_f} e_i^2 \beta \{2 q_P(\beta, Q^2)\}, \quad (3)$$

where  $q_P = \bar{q}_P$  reflects the vacuum nature of the pomeron. In this model  $\beta$  is a fraction of the pomeron momentum carried by a quark. The QCD analysis of the early HERA data using the IS model was done in [7] with the pomeron flux  $f(x_P, t) \sim x_P^{1-2\alpha_P(t)}$ , where the parameters of the Regge trajectory and initial parton distributions were determined from analyses of soft hadronic reactions, *e.g.* the soft pomeron value,  $\alpha_P(0) = 1.1$ , was used. More recent analyses of inclusive DIS diffraction [1, 2, 8] assume that the DPD exhibit the same factorization as in the IS model, called *Regge factorization*,

$$q^D(x_P, t; \beta, Q^2) = f(x_P, t) \tilde{q}(\beta, Q^2), \quad (4)$$

and parameters in (4) (including  $\alpha_P(0)$ ) are determined from fits to the diffractive DIS data using the DGLAP evolution equations.

In all cases a good description of data is found. However, the basic experimental facts: the constant ratio  $\sigma^{\text{diff}}/\sigma^{\text{tot}}$  as a function of energy and Regge factorization, are described but not understood.

### 3. Dipole models and saturation

In these models, see [5, 9, 10], the diffractive final state is built starting from a  $q\bar{q}$  pair in the color singlet state, and subsequently higher Fock components ( $q\bar{q}g$  being the first one) are added (Fig. 2). The colorless interaction of such a diffractive state with the proton is also modelled. This could be two gluons in the color singlet state (which leads to no energy dependence) or more complicated gluon exchanges, *e.g.* the BFKL ladder with much stronger than the soft pomeron energy dependence. In the simplest case of the  $q\bar{q}$  system, the interaction is encoded in a dipole cross section  $\hat{\sigma}(x, r)$ . The diffractive  $\gamma^* p \rightarrow q\bar{q}p'$  cross section is given in this case by

$$\frac{d\sigma^{\text{diff}}}{dt} \Big|_{t=0} = \frac{1}{16\pi} \int d^2r dz |\Psi^\gamma(r, z, Q^2)|^2 \hat{\sigma}^2(x, r), \quad (5)$$

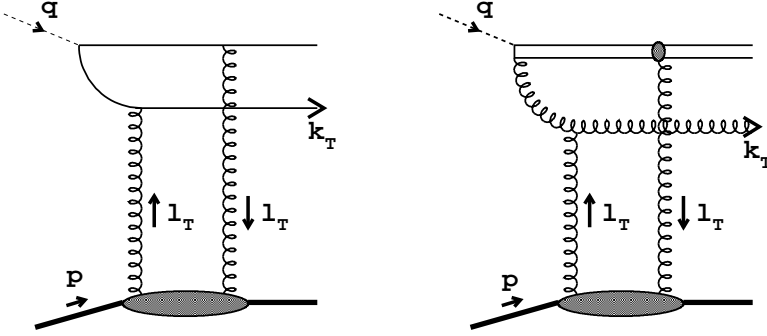


Fig. 2. The  $q\bar{q}$  and  $q\bar{q}g$  components of the diffractive system.

where  $r$  is the transverse separation of the  $q\bar{q}$  pair (dipole),  $z$  is the photon longitudinal momentum fraction carried by a quark and  $\Psi^\gamma$  is the light-cone wave function of the virtual photon.

In [11] the following form of the dipole cross section was proposed

$$\hat{\sigma}(x, r) = \sigma_0 \{1 - \exp(-r^2 Q_s^2(x))\}, \quad (6)$$

where the *saturation scale*  $Q_s^2(x) \sim x^{-\lambda}$  with  $\lambda \approx 0.3$ . Three parameters of (7) were found from a fit to DIS data on  $F_2$  since the same dipole cross section is involved in the description of  $\sigma^{\text{tot}} \sim F_2/Q^2$ . Formula (7) captures essential features of parton saturation, see [12] and references therein. In particular, it is important that  $\hat{\sigma} \approx \sigma_0$  for  $r \gg 1/Q_s(x)$ , and that the boundary of this region  $1/Q_s(x) \rightarrow 0$  for decreasing  $x$ . Thus with increasing energy, the dipole cross section saturates (proton is black) for smaller dipole sizes. In the dual momentum space, the dipole cross section corresponds to the number of gluons per unit of rapidity and transverse momentum. With the form (6), the number of gluons with transverse momenta  $k_T \gg Q_s(x)$  is proportional to  $1/k_T^2$  and gluons are dilute, while for small momenta,  $k_T \leq Q_s(x)$ , the number of gluons is tamed by their fusion in a dense system [13]. In such a case, in the dipole space,  $\hat{\sigma} \approx \sigma_0$ . With decreasing  $x$ , this effect occurs for transverse momenta  $k_T \gg \Lambda_{\text{QCD}}$ . This is the region where nonlinear QCD evolution equations appear [14].

The DIS diffraction is an ideal process to study parton saturation since it is especially sensitive to the large dipole contribution,  $r > 1/Q_s(x)$ . Unlike inclusive DIS, the region below is suppressed by an additional power of  $1/Q^2$ . Moreover, saturation leads in a natural way to the constant ratio [15]

$$\frac{\sigma^{\text{diff}}}{\sigma^{\text{tot}}} \sim \frac{1}{\ln(Q^2/Q_s^2(x))}. \quad (7)$$

A good description of diffractive DIS was obtained in this approach without additionally fitted parameters [15]. For other parameterization of  $\hat{\sigma}$  which describes diffractive data but does not use the saturation form, see [16].

The description which is based on the high energy formula (5) contains all powers of  $1/Q^2$  (twists). Extracting the leading twist contribution from both  $q\bar{q}$  and  $q\bar{q}g$  components, the quark and gluon DPD can directly be computed in the saturation model [17]. An exciting aspect of this calculation is the Regge factorization of the DPD,

$$x_P q^D(x_P, \beta) = Q_s^2(x_P) \bar{q}(\beta) \sim x_P^{-0.3}, \quad (8)$$

due to the form (6) in which  $r$  and  $x$  (or  $x_P$ ) are combined into one dimensionless variable  $rQ_s(x)$ . This also leads to the geometric scaling for inclusive DIS [18]. The dependence:  $F_2^D \sim x_P^{1-2\alpha_P}$  with  $\alpha_P \approx 1.15$ , resulting from (8), is in remarkable agreement with the data [1, 2]. Thus the Regge type behaviour and the dependence on energy of the diffractive DIS data are naturally explained.

#### 4. Diffractive vector meson production

Diffractive vector meson production gives an access to more detailed structure of the  $q\bar{q}$  dipole interaction with the proton. Namely, the dipole–proton scattering amplitude  $N(r, b, x)$  can be studied, for which

$$\hat{\sigma}(r, x) = 2 \int d^2b N(r, b, x), \quad (9)$$

where  $b$  is the impact parameter of the dipole, see Fig. 3. Through the  $t$ -dependence (at small  $t$ ) of the vector meson production cross section, the impact parameter dependence of this amplitude can be analyzed since

$$\frac{d\sigma^{VM}}{dt} = \frac{1}{16\pi} \left| \Psi^V \otimes \int d^2b e^{ib \cdot \Delta} N(r, b, x) \otimes \Psi^\gamma \right|^2, \quad (10)$$

where  $\Delta$  is a two-dimensional vector of transverse momentum transferred into a vector meson:  $t = -\Delta^2$ . Formula (10) reflects the three step factorization, shown in Fig. 3, and involves a nonperturbative vector meson wave function  $\Psi^V$ , which needs to be modelled. The first studies of the diffractive  $J/\psi$  production in the presented approach has already been performed [19].

The amplitude  $N$  can also be obtained from the QCD nonlinear evolution equation of Balitsky and Kovchegov [20], resulting from Color Glass Condensate, an effective theory of dense gluon systems with saturation [12]. This is an exciting program to confront the theoretical studies of saturation using the BK equation with the phenomenological analysis of data.

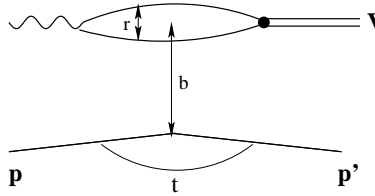


Fig. 3. Diffractive vector meson production.

I dedicate this presentation to the memory of Professor Jan Kwieciński, my teacher and master.

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