# OPEN QUESTIONS RELATED TO BOSE–EINSTEIN CORRELATIONS IN $e^+e^- \to$ HADRONS*

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Questions concerning the Bose–Einstein (BEC) and Fermi–Dirac (FDC) correlations of hadrons produced in $e^+e^-$ collisions are discussed. Among them the emitter dimension $r$ as a function of $\sqrt{s_{ee}}$ and the hadron mass, the extension of the BEC by including isospin invariance and the proposed relation between $r$ and the inter-atomic separation in Bose condensates.

## 1. Introduction

The present report covers a few of the open questions which are related to the Bose–Einstein and Fermi–Dirac Correlations (BEC and FDC) of hadrons in $e^+e^-$ annihilations. The topics which are dealt here include the dependence of the hadron emitter dimension on the $\sqrt{s_{ee}}$ and on the mass of the hadron; the interpretation of the 2-dimensional BEC analysis results obtained in annihilations as compared to those found in heavy ion collisions and finally the status of the so called Generalised BEC. The issue whether one may relate some features of the BEC at high energies with those present in Bose condensates is also briefly discussed.

## 2. The dependence of the dimension $r$ on the $\sqrt{s_{ee}}$

As is well known the Bose–Einstein interferometry method rests on the fact that the density of identical boson pairs is enhanced when they are emitted near in momentum and phase space. Assuming the emitter to be a sphere of a Gaussian distribution, the correlation of pairs of identical bosons can be parametrised by $C_2(Q) = 1 + \lambda_2 e^{-r^2 Q^2}$ where $Q^2 = -(p_1 - p_2)^2$ and $p_1$ and $p_2$ are the four momenta of the two bosons. An early compilation of

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the $r_{\pi\pi}$ values measured in heavy nuclei $(AA)$ reactions is seen in Fig. 1 to be rather well described by $r = 1.2\,A^{1/3}$ fm, the known nucleus radius dependence on the atomic number $A$. This behaviour is taken to be the result of the increase of pion sources as $A$ gets larger. As for the $r_{\pi\pi}$ values obtained from the $e^+e^-$ annihilation analyses, they are seen in Fig. 1 to be essentially independent of $\sqrt{s_{ee}}$ as it is generally accepted that the small deviations from a constant value are due to the non-uniformity of the chosen experimental procedures. Since $e.g.$ the average hadron multiplicity increases with energy this flat $r$ distribution poses a theoretical challenge. In an effort to
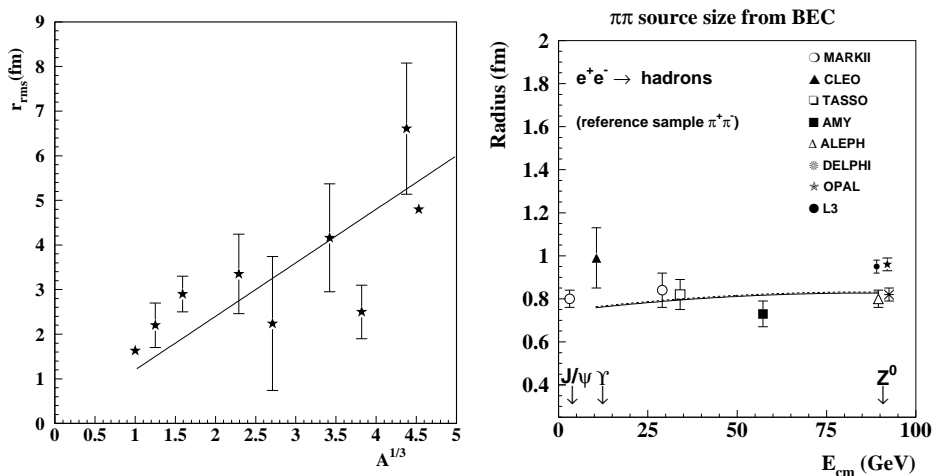


Fig. 1. Left: $r_{\pi\pi}$ *versus* $A^{1/3}$ found in heavy ion collisions. The line represents $r = 1.2\,A^{1/3}$ fm. Right: $r_{\pi\pi}$ *versus* $\sqrt{s_{ee}}$ found in $e^+e^-$ annihilations. The line is the expectation of a hadron jet source model (see text).

understand the origin of this flat $r$ distribution we adopt here, for the $e^+e^-$ annihilations, the hadron jets as the pion sources and further assume that only pion pairs emerging from the same jet may be correlated. Now the effect of multi-hadron sources on particles' correlation strength has been studied by several authors [1] in the frame work of the factorial cumulants moments $K_q$ where $q = 2$ in the case of pion–pair correlations. Specifically, it has been shown that the cumulant is diluted by a factor $D_q$, so that $K_q^D = D_q \cdot K_q^{D=1}$ where $D_q$ is defined as $D_q = N_q^{\text{same}}/(N_q^{\text{same}} + N_q^{\text{different}})$. Here $N_q^{\text{same}}$ and $N_q^{\text{different}}$ denote respectively the number of $q$ pions combinations coming from the same and from different hadron jets. Now for $q = 2$ there exists the relation $C_2(Q) = 1 + \lambda_2 e^{-Q^2 r^2} = 1 + K_2$ which can further be extended to include the dilution factor $D(= D_2)$ so that

$$C_2^D(Q) = 1 + K_2^D = 1 + DK_2^{D=1} = 1 + \lambda_D D e^{-Q^2 r_{D=1}^2} = 1 + \lambda_D e^{-Q^2 r_D^2}\,.$$

The dependence of $Q$ in this expression is then removed by integrating the quantity $C_2(Q) - 1$ over $Q$ from zero to infinity to finally obtain

$$r_D = \frac{1}{D} \frac{\lambda_D}{\lambda_{D=1}} r_{D=1} \,.$$

To apply this formula to the $e^+e^- \rightarrow$ hadrons reaction there is the need to evaluate the dilution factor $D$ as a function of $\sqrt{s_{ee}}$. As is well known the determination of the number of hadron jets in a particle reaction does depend on the chosen jet identification method [2]. Here for simplicity we will assume that the $e^+e^-$ annihilation proceeds via three hadron jets, two coming from the quark and anti-quark pair and the third from a radiated gluon[1]. The total $\sqrt{s_{ee}}$ is then divided in the following way. The first quark jet gets on the average the energy $\sqrt{s_{ee}}/2$ and thus the second quark jet has the energy of $\sqrt{s_{ee}}/2 - E_{\text{gluon}}$. The final number of outgoing charged pions assigned to each of the three jets is determined from the known total average charged hadron multiplicity dependence on $\sqrt{s_{ee}}$ and the known average charged multiplicity of quarks and gluon jets parametrised as a function of their jet energy [2]. From this pion division the dilution factor $D_2$ is estimated and the resulting $r$ dependence on $\sqrt{s_{ee}}$ is shown in Fig. 1 by the continuous line which is normalised at 40 GeV while setting $\lambda_D = \lambda_{D=1}$. As seen, the $r$ behaviour extracted from this simplified model, which rises by only $\sim 3.5\%$ from 10 to 90 GeV CM energy, already describes nicely experimental situation.

## 3. The $r$ dependence on the hadron mass

The very high statistics of the $Z^0$ hadronic decay events accumulated by the LEP experiments and the extension of the BEC analysis to baryon pairs via the FDC method opened the way to study the emitter dimension $r_{\pi\pi}$ as a function of the hadron mass, from pions to $\Lambda$ baryons. The results of these analyses are shown in Fig. 2 together with two recent, L3 and OPAL, measured $r_{\pi^0\pi^0}$ values which are seen to be inconsistent and therefore are presently disregarded. The emitter dimension $r(m)$ is seen to decrease with the hadron mass, from $\sim 0.75$ fm at $m_\pi$ to $\sim 0.15$ fm at $m_\Lambda$. It has further been shown [4] that this behaviour can be derived from the Heisenberg uncertainty relations which yield

$$r(m) = c\sqrt{\Delta t \, \hbar}/\sqrt{m} \,. \tag{1}$$

Choosing $\Delta t = 10^{-24}$ second, as a representative time scale of the strong interaction sector, $r(m)$ is shown in Fig. 2 by the continuous line. An almost

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[1] Even at the $Z^0$ the fraction of events with more than three hadron jets is very small if one uses *e.g.* $y_{\text{cut}} > 0.01$ in the Durham jet identification scheme [3].

identical expectation for $r(m)$ is also forthcoming from the Local Parton Hadron Duality and a general QCD potential [4]. On the other hand, the decrease of $r$ with the hadron mass poses a challenge to a large variety of hadronisation models, including the Lund one, which expect $r$ to increase with the hadron mass. In particular at present, there is no satisfactory explanation for the very small $r_\Lambda$ value.
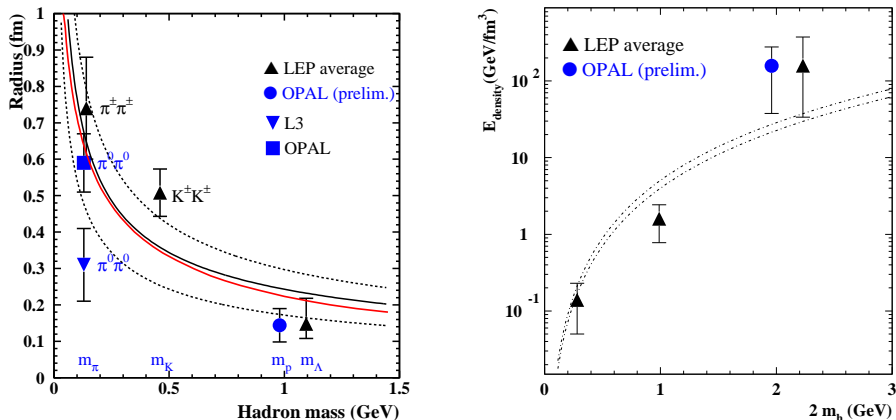


Fig. 2. Left: The $r$ dependence on the hadron mass in hadronic $Z^0$ decays. Right: The minimum energy density of the hadron emitter [1] *versus* the hadron mass.

In heavy ion collisions part of the emitter survives after the emission of the identical bosons used in the BEC analysis. This is not the case in $e^+e^-$ annihilation where the emitter collapses with the emission of its hadrons used for the BEC analysis. Consequently one may try and estimate an approximate minimum energy density of the emitter of the two hadrons by the relation $E_{\text{density}} = 2m_h/V = 6m_h/(4\pi r^3)$ which is shown in Fig. 2. Whereas the energy density for the pion and kaon emitters still lies within a reasonable range, the energy densities of the protons and $\Lambda$'s emitters are lying in the vicinity of 100 GeV/fm$^3$, way above the values anticipated from the current models for hadron production.

## 4. The 2-dimensional BEC analyses

In recent years the correlations of pions emerging from heavy ion collisions and from $e^+e^-$ annihilations have been analysed in terms of the 2-dimensional BEC analysis. This analysis is carried out in the Longitudinal Centre of Mass System, the description of which can be found *e.g.* in Ref. [1]. In this system the direction of the event thrust is referred to as the longitudinal direction and the two other directions are referred to as the outgoing and side axes. To these directions one associates three four momentum

differences $Q_z$, $Q_{out}$ and $Q_{side}$ which in a 2-dimensional analysis are reduced to $Q_z$ and the transverse momentum difference $Q_T = \sqrt{Q_{out}^2 + Q_{side}^2}$. In this method the emitter is not anymore restricted to a spherical configuration but can also assume the shape of an ellipsoid. Furthermore, one has an additional variable, the so called transverse mass $m_T$, defined as $m_T = 0.5 \times \left( \sqrt{m^2 + p_{T1}^2} + \sqrt{m^2 + p_{T2}^2} \right)$ where the $p_{Ti}$ are the transverse momenta of the two hadrons. From a fit to the data one is able to determine the longitudinal $r_L$ and the transverse $r_T$ dimensions. The dependence of $r_L$ on $m_T$ are shown in Fig. 3 for $e^+e^-$ annihilations, as measured by DELPHI at the $Z^0$ mass, and for S+Pb collisions at 200 GeV/$A$ as measured by the NA44 Collaboration. First to note is that $r_L(m_T)$ coming from $e^+e^-$ annihilations is very similar to $r(m)$ shown in Fig. 2 which was obtained from the 1-dimensional BEC analyses of the $Z^0$ hadronic decay. Moreover it has indeed been shown [5] that, as in the case of $r(m)$, also $r_L(m_T)$ can be deduce from the Heisenberg uncertainty relations to yield

$$r_L(m_T) \approx c\sqrt{\Delta t\ \hbar}/\sqrt{m_T} \qquad (2)$$

which is the Eq. (1) expression where $m$ and $r(m)$ are substituted respectively by $m_T$ and $r_L(m_T)$. The dependence of $r_L$ on $1/\sqrt{m_T}$ is repeated also in the heavy ion collisions with the difference that the proportionality factor is equal to 2.0 as compared to 0.354 found for $e^+e^-$ annihilations. To note is that the ratio 2.0/0.354 is not far from the ratio of the average radius of the S and Pb nuclei to the radius of $\sim$ 0.85 fm obtained from the 1-dimensional BEC of the hadronic $Z^0$ decays. At this point the obvious question arises as to what extend one is permitted to extract from the 2-dimensional BEC analyses information on the underlying dynamics of heavy ion collisions when $r_L(m_T)$ behaves essentially the same as the in $e^+e^-$ annihilations which are void of nuclear matter.

## 5. Generalised Bose–Einstein Correlations (GBEC)

In analogue to the generalised Pauli principle, which extends that principle from two identical nucleons to the proton–neutron system, one may consider an inclusions of the isospin invariance to extend the BEC to a generalised BEC (GBEC). Such an extension has been proposed by several authors [1] with the aim to establish BEC relations between *e.g.* the $\pi^\pm\pi^\pm$ and $\pi^\pm\pi^0$ systems. One relation of this kind, which was proposed by [6], states that

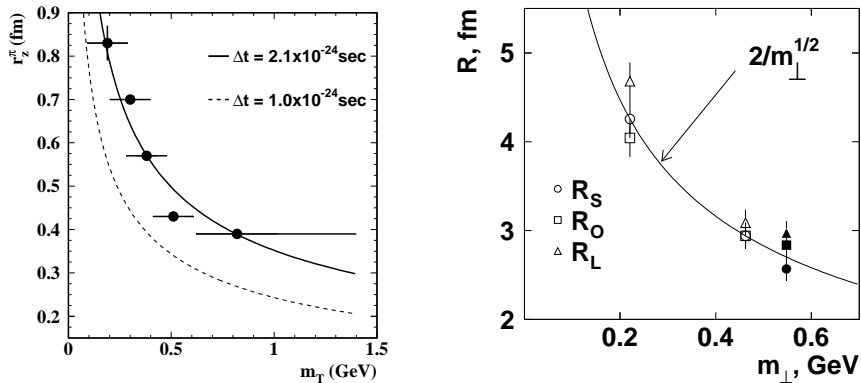$$\sum_X \sqrt{P\left[i_0 \rightarrow (\pi^\pm\pi^\pm)X\right]} = \sum_X \sqrt{P\left[i_0 \rightarrow (\pi^\pm\pi^0)_{\text{even } \ell}\ X\right]}.$$

Fig. 3. The $r$ as a function of $m_T$. Left: DELPHI preliminary results from $e^+e^- \to Z^0 \to$ hadrons. The line represents $r = 0.354/\sqrt{m_T}$. Right: NA44 results from S+Pb collisions. The line represents $r = 2/\sqrt{m_T}$.

Here $P$ stands for probability, $i_0$ stands for an $I = 0$ initial state, like $\Upsilon \to b\bar{b} \to$ hadrons, and $X$ stands for the rest of the final state associated with the specified two pion system. The GBEC validity at high energy physics is still an open question due to the missing of an experimental verification.

## 6. Conclusions

The $r(m)$ behaviour should be investigated also in other than $e^+e^-$ particles reactions and hopefully the GBEC applicability to hadron production will soon be tested experimentally. In trying to gain an insight into the $r(m)$ behaviour and the energy density of the hadron emitter it may be instructive to turn to other phenomena related to the Bose–Einstein statistics. One of them are the Bose condensates where it was shown [5] that at equal very low temperature their inter-atomic separations are proportional to $1/\sqrt{m_{\text{atom}}}$ and not to their dimension. Moreover, in as much that the Heisenberg relations are applicable to condensates, as most of their atoms are in the energy ground state, the inter-atomic separation formula can be transformed to Eq. (1) so that a comparison with the BEC dimension is not unreasonable. This comparison coupled to the behaviour of $r(m)$ and the high emitter energy densities may point to the need to re-examine the interpretation given to the BEC derived dimension $r$.

# REFERENCES

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