# ON THE INVARIANCE OF SCALED FACTORIAL MOMENTS WHEN ORIGINAL DISTRIBUTION IS FOLDED WITH THE BINOMIAL

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It is shown, that the Scaled Factorial Moments of any rank do not change if the original distribution is folded with a binomial one.

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Scaled Factorial Moments (SFMs) have been used for some time in the analysis of event-by-event correlations and fluctuations, as they allow for a "direct" access to the "dynamical" fluctuations of the multiplicity [1]. They were used (among others) for the intermittency analysis [1], and recently they were also suggested as a tool for testing the assumption of chemical equilibration in the nuclear collision [2]. For a selected class of particles produced with low multiplicity — ones that carry produced and conserved in the reaction charge-like quantity — the SFM would be close to $1/2$ if chemical equilibrium is reached.

One example of particles, that are good candidates for such an analysis, are kaons when observed at SIS energies (beam kinetic energy up to $2A$GeV). They carry positive strangeness, and as the beam energies are close to the production threshold, their multiplicities are small.

In this case the kaons are produced either as $K^+$ or $K^0$. However only one (usually charged) type is registered in the detector with reasonable (but still smaller then one) probability. This led to the question — how much the SFMs of the measured distributions differ from the "original" SFMs?

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Factorial Moment of rank $j$ is defined as follows:

$$F_j = \langle I(I-1)\ldots(I-j+1)\rangle = \sum_I^{\infty} I(I-1)\ldots(I-j+1)P_I\,, \qquad (1)$$

where $I$ is a number of particles, and $P_I$ is the probability of producing $I$ particles in an event. $P_I$ has a normalized distribution.

Scaled Factorial Moment of rank $j$ is then defined as

$$SF_j = \frac{F_j}{\langle I\rangle^j}\,. \qquad (2)$$

Capital symbols, like $I$ and $P_I$ denote the "original" distribution of produced kaons. Now let us assume that for each produced kaon the probability of registering it is equal to $q$. The resulting, "measured" distribution will then be a fold of the "original" distribution with the binomial. Denoting with small symbols $i$ the number of registered kaons per event and $p_i$ probability of an event with $i$ registered kaons one obtains:

$$p_i = \sum_{I=i}^{\infty} \binom{I}{i} q^i (1-q)^{I-i} P_I\,. \qquad (3)$$

The distribution of $p_i$ is normalized. The following proof illustrates the technique of reordering sums, which is used in the later part of the paper. One needs to note, that for any individual case $i \le I$, so each $I$ contributes only to terms with $i \le I$.

$$\begin{aligned} \sum_i^{\infty} p_i &= \sum_i^{\infty}\sum_{I=i}^{\infty} \binom{I}{i} q^i (1-q)^{I-i} P_I = \sum_{I=0}^{\infty}\sum_{i=0}^{I} \binom{I}{i} q^i (1-q)^{I-i} P_I \\ &= \sum_{I=0}^{\infty} P_I \sum_{i=0}^{I} \binom{I}{i} q^i (1-q)^{I-i} = \sum_{I=0}^{\infty} P_I = 1\,. \end{aligned} \qquad (4)$$

The normalization of $P_I$ and normalization of the binomial distribution were used in Eq. (4).

Factorial Moment of the measured distribution ($f_j$) is equal to:

$$\begin{aligned} f_j &= \sum_i^{\infty} i(i-1)\ldots(i-j+1)p_i \\ &= \sum_i^{\infty} i(i-1)\ldots(i-j+1) \sum_{I=i}^{\infty} \binom{I}{i} q^i (1-q)^{I-i} P_I\,. \end{aligned} \qquad (5)$$

To calculate $f_j$ explicitly, the same reordering of the summation as in Eq. (4) is used. One notes, that terms with $i < j$ are equal to zero and do not contribute to the sum, so in reality the summation starts not with $i = 0$, but with $i = j$. At a point $k$ is substituted for $i - j$ and $K$ for $I - j$. Note, that $(I - i) = (K - k)$.

$$\begin{aligned} f_j &= \sum_{i=0}^{\infty} i(i-1)\dots(i-j+1) \sum_{I=i}^{\infty} \binom{I}{i} q^i (1-q)^{I-i} P_I \\ &= \sum_{I=0}^{\infty} P_I \sum_{i=j}^{I} i(i-1)\dots(i-j+1) \\ &\quad \times \frac{I(I-1)\dots(I-j+1)(I-j)!}{i(i-1)\dots(i-j+1)(i-j)!(I-i)!} q^j q^{i-j} (1-q)^{I-i} \\ &= \sum_{I=0}^{\infty} P_I I(I-1)\dots(I-j+1) q^j \sum_{k=0}^{K} \frac{K!}{k!(K-k)!} q^k (1-q)^{K-k} \\ &= q^j \sum_{I=0}^{\infty} P_I I(I-1)\dots(I-j+1) = q^j F_j \,. \end{aligned} \tag{6}$$

In order to calculate the scaling denominator one needs to calculate $\langle i \rangle$. As $\langle i \rangle$ is equal to $f_1$ (and $\langle I \rangle = F_1$), one can use formula (6)

$$\langle i \rangle = f_1 = q^1 F_1 = q \langle I \rangle \,. \tag{7}$$

Finally the measured Scaled Factorial Moment, $sf_j$ appears equal to the original one.

$$sf_j = \frac{f_j}{\langle i \rangle^j} = \frac{q^j F_j}{(q\langle I \rangle)^j} = \frac{q^j F_j}{q^j \langle I \rangle^j} = \frac{F_j}{\langle I \rangle^j} = SF_j \,. \tag{8}$$

## REFERENCES

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