

ROTATION CURVES OF SPIRAL GALAXIES: INFLUENCE OF MAGNETIC FIELDS AND ENERGY FLOWS*

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(Received July 21, 2004)

Physical mechanisms that can influence rotation curves of spiral galaxies are discussed. For dark matter studies, possible contributions due to magnetic fields and non-Newtonian gravitational accelerations should be carefully accounted for. We point out that magnetic fields are particularly important in outermost parts of the disk. In the framework of general relativity the physical reason of an enhanced gravity in spiral galaxies depends on the assumed metric. The additional gravity is provided for Schwarzschild metric by nonluminous mass, whereas for Vaidya metric by emission of radiative energy. In the latter case the non-Newtonian acceleration displays $1/r$ behaviour. Also matter flows contribute to non-Newtonian gravity.

PACS numbers: 98.52.Nr, 98.62.Dm

1. Introduction

The best evidence of enhanced gravity in galaxies is provided by flat rotation curves of spiral galaxies which do not decay in a Keplerian way even far from the rotation axis. From simple Newtonian formula for centripetal acceleration

$$\frac{v_{\text{rot}}^2}{r} = \frac{GM(r)}{r^2}, \quad (1.1)$$

one finds that the total mass within radius r grows with r as $M(r) \sim rv_{\text{rot}}^2$. The linear growth of mass is customarily attributed to an invisible component, referred to as dark matter.

* This research is partially supported by the Polish State Committee for Scientific Research (KBN), grants 2P03B 110 24 and PBZ-KBN-054/P03/02

The problem of dark matter has its beginning in the observational determination by Zwicky of dynamical mass of the Coma cluster of galaxies. The gravitational mass inferred by Zwicky from the motion of individual galaxies in the cluster exceeded by a factor of a few hundred the mass obtained by measuring luminosities assuming typical value of mass to light ratio. Later a discrepancy between dynamical and luminous mass has been found in spiral galaxies and galaxy clusters.

Our aim here is to point out that the conclusion as to the existence of dark matter inferred from rotation curves is not inescapable, but based on some un-spelled assumptions. It holds in Newtonian gravity provided any role of magnetic fields is negligible. In the framework of Einstein's gravity for it to hold one *implicite* assumes space-time geometry to be given by the Schwarzschild metric. It is often assumed that galactic gravitational field, as very weak one, is adequately described by Newtonian gravity. We will show, employing Vaidya metric, that the inverse problem, of reconstructing galactic gravity given rotation velocity, has also other solutions. One encounters here ambiguity which can only be resolved by physical input.

Recent observations of dearth of dark matter in elliptical galaxies [2] suggest that there may be more unknowns involved in this problem. One should also consider non-gravitational origin of the above discrepancy, namely due to magnetic fields in galaxies. The role of magnetic fields is likely very important in the outer disk region where the galaxy rotation is detected by tracing hydrogen clouds.

In order to firmly infer the amount of dark matter in spiral galaxies one should subtract contributions to rotation curves generated by other forces (*i.e.* magnetic fields) and processes. It is certain that such contributions exist as in many galaxies rotation curves show wiggly structure, as *e.g.* in our Galaxy. Such a structure cannot be produced by WIMP gravity, as density of WIMPs is a monotonically decreasing function of distance from the center of the distribution. Magnetic influence and energy-flow-generated gravity can easily account for undulations of rotations curves. However, immediately a question arises how much such forces/processes contribute to the bulk of rotation curves. Before gauging this influence the inferred amount of dark matter in a galaxy is subject to substantial uncertainty.

In the next section the problem of flat rotation curves of spiral galaxies is briefly reminded. In Sec. 3 the role of magnetic fields is discussed. In Sec. 4 gravity due to radiation flow is discussed with the use of Vaidya metric [1]. Finally in the last section we summarize important points once again.

2. Enhanced gravity in galaxies

One can formulate the problem of flat rotational curves precisely as follows: there is too much gravity compared to mass we can account for by counting stars and measuring the amount of gas in galaxies. It is a nonrelativistic custom to attribute this enhanced gravity to invisible and (almost) undetectable matter. In a popular Cold Dark Matter model, invisible mass is due to hypothetical Weakly Interacting Massive Particles — WIMPs.

In general relativity, which is supposed to be the theory of gravity, not only mass is capable of generating gravitational field, but also energy or radiation flows induce gravity. Interpreting gravitational acceleration in Newtonian terms, $\mathbf{a} = -GM(r)/r^2\hat{\mathbf{r}}$, for spherical symmetry one *implicite* assumes the Schwarzschild interior metric

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 d\Omega^2, \quad (2.1)$$

with $e^{-\lambda} = e^\nu = 1 - 2M(r)/r$ for matter with negligible pressure, described by a dust equation of state. In the weak field limit, which is appropriate for galactic fields, one obtains then the Newtonian acceleration.

Typical rotational velocities of spiral galaxies, in the flat regime, are of the order of 100 km/s. One can thus infer the mass within radius r to be $M(r) = 2.32 \times 10^9 (v_c/100 \text{ km/s})^2 r/\text{kpc} M_\odot$. For Milky Way galaxy this gives within 30 kpc the mass $M_{\text{MW}} = 3.37 \times 10^{11} M_\odot$ for $v_c = 220 \text{ km/s}$. This high value of mass is thought to show us that the main component of mass in our Galaxy, and in other galaxies, is nonluminous. Astronomers tried hard to detect known nonluminous astrophysical objects that could form an invisible population providing the missing mass. All attempts to account for it by dim stars, dead stars, plasma or other forms of baryon matter have failed. The only viable candidate at the moment is particle dark matter composed of WIMPs (or axions), forming an extended halo around galaxies. The radius of this halo is presently unknown, but some observations suggest that it is of the order of 200 kpc. Also, some cold gas in the form of molecular hydrogen, can contribute to dark matter, as it is very difficult to detect this component.

One can briefly summarize that the dark matter hypothesis is a Newtonian solution of enhanced gravity problem in spiral galaxies with any influence of magnetic field neglected.

3. Magnetic fields and the rotation curves

Magnetic fields are the most common phenomenon in spiral galaxies where we observe fields of regular and chaotic structure. The regular structure has azimuthal and poloidal components. The poloidal component of the magnetic field is produced by the galactic dynamo effect. The azimuthal

component is induced from poloidal component by differential rotation. Regular fields created by such mechanisms may reach intensities of a few to several hundred microgauss.

A question arises [3,4] whether these fields have any influence on the galactic rotation curves. In order to answer this question we must investigate the Navier–Stokes equation with magnetic field,

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \nabla) \vec{v} \right] = -\nabla p - \frac{MG\rho}{r^2} + \eta \Delta \vec{v} + \left(\xi + \frac{\eta}{3} \right) \nabla (\nabla \circ \vec{v}) + \frac{1}{4\pi} \left((\nabla \times \vec{B}) \times \vec{B} \right). \quad (3.1)$$

We assume here the gravitational field of point mass located at the center of galaxy with $M \sim 2 \times 10^{44} \text{g}$, which is a good approximation for our exploratory calculation. In stationary galactic disk we can neglect radial velocities and viscosity and we will compare gravitational and magnetic field forces. Rough estimate shows that for gas clouds of density $\rho \sim 10^{-25} \text{g/cm}^3$ at the radius of a few tens kpc, $r \sim 3 \times 10^{22} \text{cm}$, from the galactic center, and for magnetic fields of a few μG , $B \sim \times 10^{-5} \text{G}$, magnetic forces are comparable with gravitational forces. Gravitational acceleration and acceleration due to magnetic effects are, respectively,

$$\frac{MG}{r^2} \sim 10^{-8} \text{cm s}^{-2}, \quad (3.2)$$

$$\frac{B^2}{\rho r} \sim 3 \times 10^{-8} \text{cm s}^{-2}. \quad (3.3)$$

For magnetic effects to occur the gas must be partially ionized. We know that at least a few percent of the hydrogen in galaxies is ionized. Therefore, we may expect, that the magnetic fields' influence on rotational curves is not negligible.

The above order-of-magnitude estimate shows that the magnetic influence is particularly important in the outermost regions of the galactic disk, where the density of hydrogen is the lowest. From Eq. (3.3) we find that when density decreases by a factor of 100, magnetic fields on the scale of $1\mu\text{G}$ can overwhelm gravity! Let us remind that most of the dark matter contribution to galaxy rotation comes from the outskirts. Any unaccounted for magnetic field contribution can completely corrupt dark mass measurement. One should also keep in mind that magnetic fields of μG order are expected to be ubiquitous in the intergalactic space in clusters of galaxies.

Taking only azimuthal component of magnetic field into account and assuming that it depends only on its radial coordinate we can get a simple analytical form of this component which will flatten the rotation curve:

$$\frac{(v_\phi)^2}{r} = \frac{MG}{r^2} + \frac{1}{4\pi\rho r} B_\phi \frac{\partial}{\partial r} (r B_\phi), \quad (3.4)$$

where $v_\phi = v_c = \text{const}$ is the rotational velocity of a galaxy. The solution of this equation reads:

$$B_\phi = +/ - \frac{2\sqrt{2\rho\pi}\sqrt{0.5(v_\phi)^2 r^2 - MGr - C}}{r}, \quad (3.5)$$

where C is the integration constant.

The real magnetic fields in spiral galaxies have all the components, poloidal and azimuthal, nonvanishing. Therefore, the radial part of Navier–Stokes equation has the following form:

$$\frac{(v_\phi)^2}{r} = \frac{MG}{r^2} - \frac{1}{4\pi\rho} \left(\left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} \right) B_z - B_\phi \left(\frac{1}{r} \frac{\partial}{\partial r} (r B_\phi) - \frac{1}{r} \frac{\partial B_r}{\partial \phi} \right) \right). \quad (3.6)$$

To assess a possible influence of such magnetic fields on rotation curves one should take into account the whole 3D structure of magnetic field.

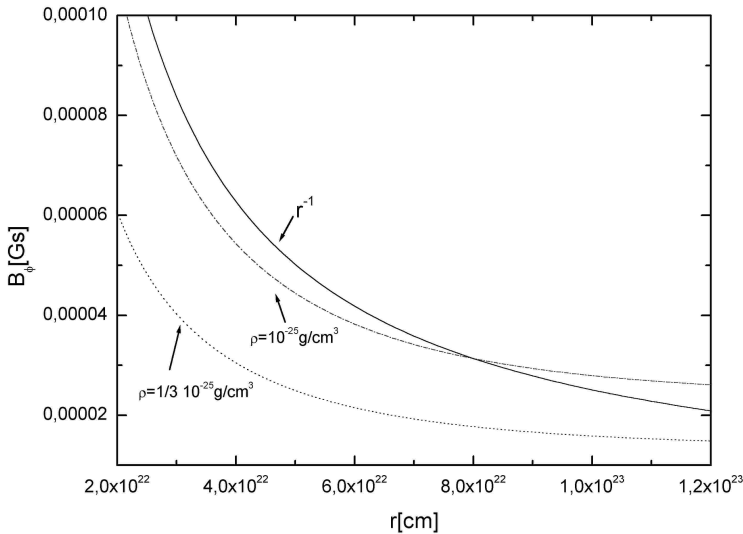


Fig. 1. The dashed–dotted line is a solution of Eq. (3.8) with $M = 2 \times 10^{44} \text{g}$, $v_\phi = 220 \text{ km/s}$, $\rho = 10^{-25} \text{ g/cm}^3$, dotted line is a solution corresponding to density $\rho = 0.333 \times 10^{-25} \text{ g/cm}^3$. Solid curve is a function $B_\phi \sim 1/r$.

4. Gravity generated by radiation flow: the Vaidya metric

The Schwarzschild metric describes strictly speaking gravity of cold spherically symmetric astrophysical body, such as a planet, dead star or a black hole. Gravity of radiating objects such as normal stars is only approximately

described by the Schwarzschild metric. There exists, however, exact solution of Einstein's field equations corresponding essentially to a real star which emits radial flux of radiation, found by Vaidya [1]. In its original form, the Vaidya metric is

$$ds^2 = \left(\frac{\dot{m}}{m'}\right)^2 \left(1 - \frac{2m}{r}\right)^{-1} dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\Omega^2. \quad (4.1)$$

It corresponds to the space-time region outside the star, $r > r_0$, where r_0 is the stellar radius, and $m \equiv m(r, t)$. This metric can be cast in a very elegant form employing the retarded time variable $u = t - r$,

$$ds^2 = -\left(1 - \frac{2m}{r}\right) du^2 - 2du dr + r^2 d\Omega^2, \quad (4.2)$$

as shown by Vaidya in Ref. [1].

The energy tensor corresponding to the Vaidya metric has non-zero T_0^1 component which describes the energy outflow carried away by massless fields. Let us consider the energy tensor for directed flow of radiation [5] in the form

$$T_\mu^\nu = \rho v_\mu v^\nu, \quad (4.3)$$

where ρ is the energy density of radiation, and the 4-vector v^μ is null, $v^\mu v_\mu = 0$. For the radial outflow, $v^2 = v^3 = 0$, and $T_2^2 = T_3^3 = 0$. The metric (4.1) is a particular example of a general non-static spherically symmetric metric [5],

$$ds^2 = e^{\nu(r,t)} dt^2 - e^{\lambda(r,t)} dr^2 - r^2 d\Omega^2. \quad (4.4)$$

The Einstein field equations are [5]:

$$-8\pi T_0^0 = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right), \quad (4.5)$$

$$-8\pi T_1^1 = -\frac{1}{r^2} + e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right), \quad (4.6)$$

$$\begin{aligned} -8\pi T_2^2 &= -\frac{1}{4} e^{-\nu} (2\ddot{\lambda} + \dot{\lambda}(\dot{\lambda} - \dot{\nu})) + \frac{1}{4} e^{-\lambda} \left(2\nu'' + \nu'^2 - \lambda'\nu' + 2\frac{\nu' - \lambda'}{r} \right) \\ &= -8\pi T_3^3, \end{aligned} \quad (4.7)$$

$$-8\pi T_{01} = -\frac{1}{r} \dot{\lambda}. \quad (4.8)$$

Let us introduce the mass function $m(r, t)$ through $e^{-\lambda(r,t)} = 1 - 2m(r, t)/r$. From the null condition, $v_\mu v^\mu = 0 = -e^\lambda (v^1)^2 + e^\nu (v^0)^2$ we find $e^{(\nu-\lambda)/2} T_1^0 + T_0^0 = 0$ which gives

$$e^{-\lambda'/2} m' + e^{-\nu/2} \dot{m} = 0. \quad (4.9)$$

This allows us to express the function e^ν through $m(r, t)$ and its derivatives, and to write the metric (4.1) in the form (4.2) given by Vaidya.

The physical interpretation of Vaidya's metric is straightforward. In the weak field limit we find $\dot{m} + m' = 0$ and the energy flux flowing out of a sphere of radius r is

$$T^{01} = -\frac{m'^2}{4\pi r^2 \dot{m}} = -\frac{\dot{m}}{4\pi r^2}. \quad (4.10)$$

Hence $m(r, t)$ is the radiation energy inside this sphere

$$m(r, t) = \int 4\pi r^2 T_0^0 dr. \quad (4.11)$$

The function \dot{m} is the rate of energy emission, or total luminosity, and $m' = 4\pi r^2 T_0^0$. We should also include the radiation source, located at the origin, which loses energy at a rate $\dot{M}(0, t - r) = \dot{m}(r, t)$.

The most important result is the gravitational acceleration in the weak field limit, $e^\nu \approx 1 + \nu$. From general expression we find for the metric (4.4) the Einstein's formula

$$\ddot{r} = -\frac{1}{2}\nu'. \quad (4.12)$$

When applied to the Schwarzschild metric in the weak field limit, with $\nu = -2M/r$, it gives the Newtonian acceleration.

For the Vaidya metric from Eq. (4.6) we have

$$\nu' = \frac{2}{r} \left[(e^\lambda - 1) + e^\lambda \left(m' - \frac{m}{r} \right) \right]. \quad (4.13)$$

In the weak field limit we find

$$\ddot{r} = -\frac{m'}{r} - \frac{m}{r^2}. \quad (4.14)$$

This expression shows that there appears a non-Newtonian acceleration

$$a_L = -\frac{m'}{r} = G \frac{\dot{m}}{rc} \quad (4.15)$$

which is inversely proportional to the distance. Far from the center, a_L becomes dominating. For radiating body, with energy flowing out of the central mass, $\dot{m} < 0$ and the acceleration (4.14) produces an attractive force which becomes stronger than the usual Newtonian gravitation.

The additional gravitational attraction due to radiation emission implied by the Vaidya metric was first discussed by Lindquist *et al.* [6]. It gives an explicit example of non-Newtonian gravitational force resulting from Einstein's gravity theory for a realistic metric in the weak field limit. Thus the

notion that Newtonian acceleration is the only weak-field limit of general relativity is inaccurate.

The formula (4.14) can be generalized to the case of galactic wind which is a radial matter outflow,

$$a_{\text{wind}} = G \frac{\dot{m} v_{\text{wind}}}{r c^2}. \quad (4.16)$$

Here v_{wind} is the radial velocity of the wind and $\dot{m} < 0$ is the mass loss rate due to wind. Please note that the above formula (4.15) is also valid for radial accretion, with radial infall velocity $v_r < 0$. Since for accretion the mass increases, $\dot{m} > 0$, the induced acceleration is also directed inward, as for the wind. One can thus conclude that radially oscillating shell of gas would always produce gravitational attraction, both in expansion and contraction phase.

5. Discussion

Astronomers tend to consider the Newtonian solution of the enhanced gravity problem in spiral galaxies to be the only one compatible with general relativity. One can encounter statements that any non-Newtonian gravitational acceleration in galaxies, as *e.g.* employed by Milgrom in his model of galaxy gravity [7], would necessarily require modifications of Einstein's gravity theory. We have given here an example that the statement that non-Newtonian gravitational acceleration, $a \sim 1/r$, is incompatible with general relativity, is not true. The acceleration (4.14) can be shown to produce flat rotational curves of spiral galaxies. The centripetal acceleration when the non-Newtonian acceleration dominates, is

$$\frac{v_{\text{rot}}^2}{r} = \frac{G\dot{m}}{cr} \quad (5.1)$$

which allows us to calculate the source luminosity $L = -\dot{m}$. For $v_{\text{rot}} = 100 \text{ km/s}$ $L \sim 10^{52} \text{ erg/s}$. Hence the problem of enhanced gravity in spiral galaxies with the Vaidya metric changes to the problem of the energy source and the physical nature of its emission. Physically, it is very different from the Newtonian solution, which is the nonluminous matter.

Presently it is a standard assumption that galactic dynamics is governed by dark matter. To prove the dark matter hypothesis a number of experiments start to search for neutralino, the best supersymmetry candidate for WIMP. Also, astrophysical observations of dark matter in elliptical galaxies have been attempted by P.N.S. collaboration [2] with planetary nebulae as a tracer of gravity. Surprisingly, gravity of those galaxies is adequately described by luminous matter only, a result described as a "missing missing

mass" problem [2]. If the results obtained by P.N.S collaboration are correct than dark matter in elliptical galaxies is at least differently distributed than in spiral galaxies, with only trace amount inside inner 5–6 effective radii.

A radical proposal is the Milgrom's Modified Newtonian Dynamics (MOND) hypothesis, which postulates new gravitation law for very weak accelerations. This proposal, employed as a phenomenological model, is quite successful in explaining spiral galaxies dynamics. MOND also explains the dearth of dark matter in elliptical galaxies [8] observed by Romanowsky *et al.* [2]. The radial dependence of the MOND acceleration is the same as in non-Newtonian acceleration a_L (4.14) for the Vaidya metric.

One can notice that the Vaidya metric is an example of metric considered recently by Lake [9] that can give flat rotation curves of spiral galaxies.

We have shown here that magnetic fields in spiral galaxies can play a crucial role in determining the rotation curves. The solution (3.6) shows that magnetic field which fully accounts for a flat rotation curve has toroidal component compatible with observed magnetic fields in spiral galaxies. It may not be a good approximation to completely suppress magnetic field influence when studying the physical origin of flat rotation curves. In realistic description observed magnetic field influence should be subtracted before fitting gravitational potential generated by assumed dark matter halo. The importance of magnetic field contribution to flat rotational curves of spiral galaxies has been recently discussed in Ref. [10].

It is also worth to notice that pure magnetic mechanism of flat rotation curves in spiral galaxies could explain simultaneously why these curves in elliptical galaxies are Keplerian. It is because there are no regular magnetic fields in elliptical galaxies.

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